



# The influence of surface roughness on postcritical flow over circular cylinders revisited

Anil Pasam<sup>1,2,3,†</sup>, Daniel Tudball Smith<sup>1,2,3</sup>, John D. Holmes<sup>4</sup>,  
David Burton<sup>1,2,3</sup> and Mark C. Thompson<sup>1,2,3</sup>

<sup>1</sup>Monash Wind Tunnel Research Platform (MWTRP), Department of Mechanical and Aerospace Engineering, Monash University, Clayton, VIC 3800, Australia

<sup>2</sup>Fluids Laboratory for Aeronautical and Industrial Engineering (FLAIR), Department of Mechanical and Aerospace Engineering, Monash University, Clayton, VIC 3800, Australia

<sup>3</sup>Department of Mechanical and Aerospace Engineering, Monash University, Clayton, VIC 3800, Australia

<sup>4</sup>JDH Consulting, Mentone, VIC 3194, Australia

(Received 2 June 2023; revised 1 September 2023; accepted 3 October 2023)

This work investigates the effect of surface roughness on cylinder flows in the postcritical regime and reexamines whether the roughness Reynolds number ( $Re_{k_s}$ ) primarily governs the aerodynamic behaviour. It has been motivated by limitations of many previous investigations, containing occasionally contradictory findings. In particular, many past studies were conducted with relatively high blockage ratios and low cylinder aspect ratios. Both of these factors appear to have non-negligible effects on flow behaviour, and particularly fluctuating quantities such as the standard deviation of the lift coefficient. This study employs a 5 % blockage ratio and a span-to-diameter ratio of 10. Cylinders of different relative surface roughness ratios ( $k_s/D$ ), ranging from  $1.1 \times 10^{-3}$  to  $3 \times 10^{-3}$ , were investigated at Reynolds numbers up to  $6.8 \times 10^5$  and  $Re_{k_s}$  up to 2200. It is found that the base pressure coefficient, drag coefficient, Strouhal number, spanwise correlation length of lift and the standard deviation of the lift coefficient are well described by  $Re_{k_s}$  in postcritical flows. However, roughness does have an effect on the minimum surface pressure coefficient (near separation) that does not collapse with  $Re_{k_s}$ . The universal Strouhal number proposed by Bearman (*Annu. Rev. Fluid Mech.*, vol. 16, 1984, pp. 195–222) appears to be nearly constant over the range of  $Re_{k_s}$  studied, spanning the subcritical through postcritical regimes. Frequencies in the separating shear layers are found to be an order of magnitude lower than the power law predictions for separating shear layers of smooth cylinders.

**Key words:** vortex shedding, wakes

† Email address for correspondence: [anil.pasam@monash.edu](mailto:anil.pasam@monash.edu)

## 1. Introduction

For more than a century, the flow over circular cylinders has garnered the attention of fluid dynamics researchers. This is due to a combination of their geometric simplicity and axisymmetry, and their relevance to real-world applications. A host of studies on smooth circular cylinders has helped advance our understanding of how the aerodynamic forces and wake change as the Reynolds number is varied over a wide range. However, engineering structures seldom have perfectly smooth surfaces due to factors such as imperfections in surface finish, paint and corrosion. At Reynolds numbers of  $O(10^5)$  and above, a range of considerable relevance to many applications, the surface roughness has a strong influence on flow separation and the wake and, in turn, the lift and drag. This motivates further study of the flow over cylinders focusing on the effects of surface roughness, also noting that this Reynolds number range has been difficult to explore in practice with standard wind-tunnel-based experiments. Therefore, a reliable, consistent and comprehensive data set addressing these areas is necessary to unravel how the flow behaviour and underlying flow physics vary over this crucial range.

Broadly, the flow over circular cylinders can be classified into subcritical, critical, supercritical and postcritical regimes (Roshko 1961; Achenbach 1971; Güven, Farrell & Patel 1980). These regimes are represented schematically in figure 1. They reflect key changes in the location of turbulent transition in the separating shear layer or boundary layer as the Reynolds number ( $Re = U_\infty D/\nu$ , with  $U_\infty$  the free-stream velocity,  $D$  the cylinder diameter and  $\nu$  the kinematic viscosity) is increased. The regimes can also be identified by the variation of the drag coefficient,  $C_D$ . In subcritical flow ( $10^4 \lesssim Re \lesssim 10^5$  for a smooth cylinder), the transition to turbulence occurs in the shear layers in the near wake of the cylinder and  $C_D$  remains fairly constant with increasing Reynolds number. As the Reynolds number is increased further ( $Re \sim O(3\text{--}4 \times 10^5)$ ), this transition moves upstream to occur in the separating shear layers very close to the separation point. When transition occurs in this key region, it allows the separated shear layers to reattach to the surface and separate at a later position further towards the rear of the cylinder. This separation zone near reattachment, marked by laminar separation, transition to turbulent flow and reattachment, is known as the *laminar separation bubble* (LSB).

The formation of the LSB delays the eventual final separation of the boundary layer allowing the pressure to recover and, hence, results in a significant drop in  $C_D$  from that in the subcritical state. It is in this range that the minimum  $C_D$  with respect to Reynolds number is observed. Further increase in Reynolds number ( $Re \gtrsim 4 \times 10^5$  for a smooth cylinder) causes upstream movement of the transition in the boundary layer that results in intermittent and asymmetric formation of the LSB followed by no LSB and, thus, leads to increasing  $C_D$  in the supercritical regime. In the postcritical regime ( $Re \gtrsim 6 \times 10^6$  for a smooth cylinder), the transition to turbulence in the boundary layer occurs sufficiently upstream on the cylinder surface that  $C_D$  becomes nearly insensitive to Reynolds number. However, the higher momentum in the turbulent boundary layer enables it to remain attached for longer than the laminar counterpart, resulting in a lower  $C_D$  than for the subcritical case. The Reynolds numbers at which these transitions occur are sensitive to the turbulence intensity of the incoming flow and the surface roughness.

In addition, an increase in cylinder surface roughness causes (i) a decrease in the critical Reynolds number, i.e. the Reynolds number of minimum  $C_D$ , (ii) an increase in minimum  $C_D$  and (iii) an increase in  $C_D$  in the postcritical regime (see Fage & Warsap 1929; Achenbach 1971). These changes are illustrated in figure 1.

## Influence of surface roughness on postcritical flow

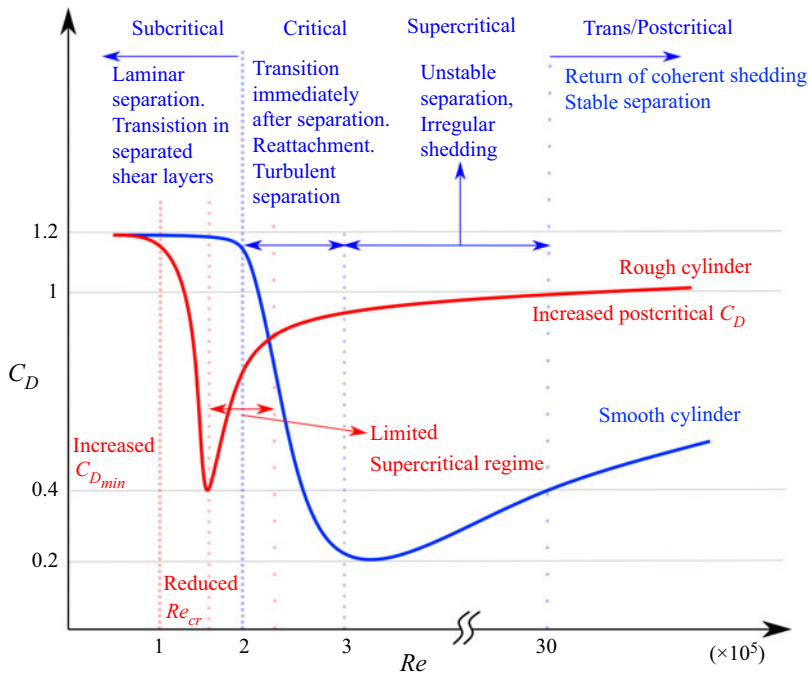


Figure 1. Schematic illustrating the four distinct high-Reynolds-number flow regimes over a smooth circular cylinder and the changes induced by increasing surface roughness. (Plot not to scale. Values are estimated from Schewe (1983) and the current investigation.)

Owing to the difficulty of reaching Reynolds numbers corresponding to large engineered cylindrical structures subject to atmospheric winds (i.e. postcritical Reynolds numbers  $\sim 10^7$ ) in wind tunnels, Szechenyi (1975) proposed the use of surface roughness to simulate postcritical flows since increasing surface roughness decreases the critical Reynolds number. Quantitatively, this is based on a curve collapse achieved when the mean  $C_D$  and the fluctuating lift coefficient,  $\sigma_{C_L}$ , of cylinders of different surface roughness are plotted against a Reynolds number based on the degree of roughness ( $Re_k = \rho U_\infty k / \mu$  where  $k$  denotes roughness height); see figures 3 and 4 from Szechenyi (1975). This Reynolds number is referred to in the current work as the *roughness Reynolds number*.

Supporting this collapse, Batham (1973) observed that mean pressure distributions on a rough cylinder at postcritical Reynolds numbers are similar to those of a smooth cylinder in turbulent incident streams at  $Re \sim 10^7$ . Following this, Güven *et al.* (1980) attempted to develop a theoretical framework to explain the effect of roughness on the boundary-layer development and separation points. Although they found an asymptotic postcritical  $C_D$  at large Reynolds numbers and roughness, they opined that the flow phenomena cannot sufficiently be determined through a single parameter,  $Re_{k_s}$ , and that a (joint) description of Reynolds number and surface roughness is necessary. This conclusion was based on the large scatter in the collapse of the  $C_D$  and  $\sigma_{C_L}$  curves when results from different works (Achenbach 1971; Szechenyi 1975; Güven *et al.* 1980) were taken into account. Nakamura & Tomonari (1982) then posited that appropriately placed roughness strips are more effective than distributed roughness for simulating postcritical flows since the postcritical  $C_D$  of a cylinder with roughness strips is closer to that of a smooth cylinder. In contrast to the  $Re(k/D)$  dependence proposed in Szechenyi (1975), a different similarity

Experiment	Data	Aspect ratio	Blockage	Relative roughness $k/D \times 10^3$
Achenbach (1971)	P, $\tau$	3.3	16.67 %	1.1, 4.5, 9 <sup>+</sup>
Batham (1973)	P, C	6.6	5 %	2.2
Szechenyi (1975)	P	4–9.3	7–23 %	0.15–2
Güven <i>et al.</i> (1980)	P, B	3.08	17.8 %	2.5–6.2 <sup>+</sup>
Buresti (1981)	P, C	6.6–23	3–10 %	0.91–12.35
Nakamura & Tomonari (1982)	P	3.3	15.5 %	0.9–10
Ribeiro (1991)	P, C	6	12 %	1.8
Chakroun, Rahman & Quadri (1997)	P	5.98	16.6 %	1.1–2.3
Zan & Matsuda (2002)	F	10	2.5 %*	0.1
Eaddy (2019)	P, C	5.7, 9	12 %, 8 %	0.74, 2.48, 4.22
van Hinsberg (2015)	P, F	10	10 %	1.2 <sup>+</sup>
Current	P, C, W	9.8	5 %	1.1–3 <sup>+</sup>

Table 1. A non-exhaustive list of previous investigations concerning rough cylinders and the measurement techniques used. P,  $C_D$ ,  $C_L$  obtained from integration of pressure distribution; F,  $C_D$ ,  $C_L$  obtained from total force measurements;  $\tau$ , skin-friction measurements; B, boundary-layer measurements; W, wake measurements; C, spanwise correlations. \*Porous walls and a suction plenum were also used to reduce blockage effects. <sup>+</sup>Values given are the equivalent sand grain roughness measures (Nikurdase 1933).

parameter,  $Re(k/D)^{0.6}$ , was found to result in a better collapse of the base pressure and pressure recovery obtained from cylinders of different roughness.

More recently, Eaddy (2019) measured the fluctuating lift coefficient and axial correlation length of lift for two different roughnesses and concluded that Roughness Reynolds number falls short in collapsing the fluctuating properties. While a comparison of results from different works (given in Güven *et al.* 1980) does raise concerns about the validity of a collapse based on roughness Reynolds number, the considerable differences in blockage and aspect ratios amongst these studies weaken the significance of such a conclusion. The sensitivity of the flow to geometrical features of the set-up has already been discussed in Güven *et al.* (1980) and Nakamura & Tomonari (1982), and is particularly evident in the difference in the fluctuating lift and spanwise correlation obtained from cylinders of two different aspect ratios in Eaddy (2019) (see figure 3b). These differences in the set-ups of previous studies are highlighted in table 1.

Also evident from table 1 is the high wind-tunnel blockage ratio that has been a feature of the majority of previous works. Results were corrected for blockage effects through a technique originally proposed by Allen & Vincenti (1944), and later substantiated for cylinder flows by Farrell *et al.* (1977). However, support for the assumption that blockage effects can be fully corrected comes from the collapse of only the mean drag coefficient and to a lesser extent, the surface pressure post correction. The applicability of this correction to the more sensitive fluctuating parameters, e.g. fluctuating lift, is less certain. The majority of the previous studies did not measure the fluctuating lift, and the few that did (Szechenyi 1975; Eaddy 2019) report scatter owing to differences in geometry of the set-up. Moreover, the conclusions of Eaddy (2019) and Szechenyi (1975) are inconsistent.

Fox & West (1990), with experiments on smooth cylinders at a subcritical Reynolds number, concluded that a minimum aspect ratio of 7 is required to isolate the flow over the mid-span of the cylinder from the influence of end effects. Although their work does not involve rough cylinders at postcritical Reynolds numbers, it underlines the need for

using high aspect-ratio cylinders with low wind-tunnel blockage. While more recent works (e.g. Zan & Matsuda 2002; van Hinsberg 2015) incorporate such a set-up, these were limited to only a single surface roughness. Thus, despite a long history, there is still a need for a more comprehensive and consistent data regarding flow over cylinders of different roughnesses at postcritical Reynolds numbers. Not only to ascertain the validity of simulating postcritical flows with larger roughness, but also to explain the influence of surface roughness on flow parameters in general.

Moreover, measurements of pressures at a single cross section and the consequent derived coefficients of pressure drag and lift for the cylinder are incomplete without information about the extent of the spanwise correlation of the flow. As mentioned previously, in the subcritical and the postcritical regimes, (spanwise) organised vortex shedding is observed in cylinder wakes. Through the critical and supercritical regimes, however, this vortex shedding is less organised because of the variation of separation angle across the span caused by the high sensitivity of the LSB (Bearman 1984). This results in lower spanwise coherence through the critical regime. One measure of the spanwise correlation is the axial correlation length of the lift coefficient,  $\Lambda$  given by

$$\Lambda = \int_0^\infty R(s) ds, \quad (1.1)$$

where  $R(s)$  is the correlation coefficient between the coefficients of lift measured at spanwise locations,  $z = 0$  and  $z = s$  (Norberg 2003). For smooth cylinders, the axial correlation length varies from  $4\text{--}5D$  in the subcritical regime to  $\sim 0.5\text{--}1.5D$  in the critical and supercritical regimes (Duarte Ribeiro 1992). Axial correlation lengths in the postcritical regime for smooth cylinders are unknown due to the difficulty in reaching high Reynolds numbers. In addition, estimates of  $\Lambda$  for rough cylinders are only available for a few Reynolds numbers and roughnesses. Of studies reporting on this, Ribeiro (1991) determined the correlation length for cylinder wrapped with a sandpaper roughness of  $k/D = 1.8 \times 10^{-3}$  at a Reynolds number of  $4 \times 10^5$  as  $\sim 3.9D$ , and Batham (1973) determined it to be  $\sim 3.2D$  for a relative roughness of  $k/D = 2.2 \times 10^{-3}$  at a Reynolds number of  $2.8 \times 10^5$ . Eaddy (2019) measured the axial correlation length in postcritical flows for two different roughnesses at two aspect ratios. It is of note that the higher aspect ratio (span to diameter,  $AR = 9$ ) set-up led to  $\Lambda$  between  $2D$  and  $3D$ , while an aspect ratio of 5.7, which is closer to that of Batham (1973) ( $AR = 6.6$ ) and Ribeiro (1991) ( $AR = 6$ ) was found to have a larger correlation length ( $\Lambda \sim 6D - 8D$ ). On the other hand, Buresti (1981) with hot-wire measurements in the wake, estimated the correlation length to be  $\sim 4D$  in postcritical flows for rough cylinders. Information about the variation, if any, of this correlation length with roughness is limited but is required to infer the net fluctuating lift force across some length of the cylinder.

Shear-layer frequency content,  $f_{KH}$ , corresponding to Kelvin–Helmholtz waves for a smooth cylinder in the subcritical regime is known to follow a power-law dependence based on Reynolds number, and the theoretical framework behind such a dependency was given in Prasad & Williamson (1997) (also see Thompson & Hourigan 2005). As Reynolds number increases, the energy in the shear layer becomes more distributed and only a broadband signature is found (Lehmkuhl *et al.* 2014). The existence of such frequencies at postcritical Reynolds numbers and/or in the wake of rough cylinders has not been established, with table 1 highlighting the scarcity of wake and shear-layer measurements for rough cylinder flows.

Through the current study, we aim to provide a more comprehensive and consistent data set of lift and drag coefficients, surface pressure distributions, and spanwise correlation

lengths, providing the variation of these properties with roughness and Reynolds number in postcritical flows. This data tests the validity of whether flow parameters collapse based on roughness Reynolds number alone. With the help of various measurement techniques, we aim to explore similarities and differences between the wakes from cylinders of different surface roughnesses at the same postcritical Reynolds number, and also at the same Roughness Reynolds number. This exploration augments our understanding of the influence of surface roughness on the flow over and forces on circular cylinders, enabling the prediction of aerodynamic loads at Reynolds numbers often relevant to large engineered structures. In addition, information about wake width and wake frequencies helps in understanding and predicting the aerodynamics of multiple cylindrical structures in close proximity.

For the remainder of this paper, further discussion is split into three sections. Section 2 gives the geometric details of the set-up and describes the data acquisition and postprocessing. Results are discussed in §§ 3, and 4 summarises the key conclusions from the current work.

## 2. Methodology

### 2.1. Experimental set-up

Experiments were conducted in a closed circuit wind tunnel with a 2000 mm tall and 4000 mm wide rectangular cross section. The turbulence intensity of the incoming flow is 1.35 % at the centre of the cylinder, which is positioned 4600 mm downstream from the start of the working section. At  $35 \text{ m s}^{-1}$  (corresponding to a Reynolds number of  $\sim 4.5 \times 10^5$  based on cylinder diameter), displacement thickness and momentum thickness of the vertical boundary layers are  $\sim 12 \text{ mm}$  and  $\sim 10 \text{ mm}$ , respectively, near both the floor and the roof of the test section at the point where cylinder is installed. Outside the boundary layers, variation in the local mean velocity is within 0.5 % of the global mean measured along the axis of the cylinder.

The diameter of the cylinder,  $D$ , is 204 mm including the thickness of the sandpaper backing cloth and the average roughness height. It spans the height of the tunnel (2000 mm), thereby resulting in an aspect ratio of  $\sim 9.8$  and blockage ratio of  $\sim 5 \%$ . It consists of two separate spanwise sections to facilitate the installation of pressure taps. The lower section is made of steel to lend overall strength, while the section with pressure taps is made of aluminium. To increase the uniformity in diameter, the steel section is formed by turning a cylinder of larger diameter while machined bulkheads are inserted inside the aluminium section at regular intervals. The resulting variation in the diameter of the cylinder over the entire span is  $< 1 \%$ . These two sections are joined from inside in order to reduce the degree of non-uniformity on the outer surface. The seam due to the joint between these two sections is  $\sim 0.5 \text{ mm}$  and  $1.4D$  away from the nearest measurement plane and, hence, is expected to have negligible influence on the measurements. The cylinder is also fixed at the roof of the tunnel to further reduce the vibrations and the influence of any gap flow.

The desired uniform surface roughness is achieved by wrapping sandpaper around the bare cylinder. The spanwise seam from joining the two ends of the sandpaper is at  $180^\circ$  (i.e. in the wake region). The types of sandpaper used, and the corresponding average roughness heights  $k$  are given in table 2.

Equivalent sand grain roughness,  $k_s$ , is the uniform sand grain height that generates a similar frictional velocity deficit in a fully rough flow determined from Nikurdase (1933). Achenbach & Heinecke (1981) conducted experiments to determine values of  $k_s$  of

## *Influence of surface roughness on postcritical flow*

---

Sandpaper grade	P40	P60	P80	P100
Average grain size, $k$ ( $\mu\text{m}$ )	412	262	196	157
Relative roughness, $k/D \times 10^3$	2.01	1.28	0.96	0.76
Equivalent sand grain roughness, $k_s/D \times 10^3$	3	1.9	1.4	1.1

---

Table 2. Different grades of sandpaper used and the corresponding roughness measures.

$4.5 \times 10^{-3}$  and  $1.1 \times 10^{-3}$  for 40 grit and 120 grit sandpapers thereby yielding  $k_s/k$  of 1.57 and 1.43, respectively. A similar average conversion factor of  $k_s/k \sim 1.5$  for sandpaper grits was found in Speidel (1954). In the absence of experiments to determine the exact value of  $k_s$  for the sandpapers used in the current work, the conversion factor  $k_s/k = 1.5$  is used for all of the grits examined to ascertain the equivalent sand grain roughness.

Four cross sections along the span are chosen for pressure measurements, located at heights of  $0D$ ,  $-1D$ ,  $1D$  and  $2D$ , where  $0D$  is for the spanwise mid-plane of the cylinder and the axis is positive towards the roof. Each of these cross sections consists of 30 uniformly distributed pressure measurement locations that were installed after wrapping the sandpaper onto the surface. These locations contain a 20-mm-long hypodermic tube of 2 mm outer diameter and 1.5 mm inner diameter, which is attached from inside of the cylinder and positioned carefully to end at the backing cloth of the sandpaper so not to protrude above the trough of the roughness elements. Flexible PVC tubes of length 2200 mm were then used to connect these hypodermic tubes to the pressure measurement system. Cross-sectional measurement planes, the joint between the sections, and the pressure ports are illustrated in [figure 2](#).

### *2.2. Data acquisition and postprocessing*

Pressure at the above-mentioned locations was sampled at 2000 Hz for at least 120 s using a TFI synchronous Differential Pressure Measurement System. An amplitude and phase correction was applied to the instantaneous pressure data to account for the length and diameter of pressure tubing. The recorded pressure data was transformed into the frequency domain and a transfer function was applied before transforming the data back into the time domain. This transfer function was computed based on the length and the diameter of the pressure tube following the procedure given in Bergh & Tijdeman (1965) and is reported in [figure 18](#) in [Appendix A](#). The amplitude response of the theoretical transfer function falls below 0.4 at  $\sim 500$  Hz and below 0.25 at  $f \sim 630$  Hz. The pressure signals were cut off at  $\sim 630$  Hz and spectra from these signals reported in this investigation were only plotted until 250 Hz. The test duration amounts to more than 600 shedding cycles at the lowest Reynolds number presented here and more than 2000 shedding cycles at postcritical Reynolds numbers, which form the crux of the discussion. Sectional mean and fluctuating pressures presented here were obtained from tests of longer duration ( $\sim 5000$  shedding cycles) and these tests were also used to establish the convergence of  $C_D$  and  $\sigma_{C_L}$  for the shorter tests.

Flow parameters at each Reynolds number were measured at least three times and the data reported includes 90 % confidence intervals based on  $t$ -distribution statistics. The

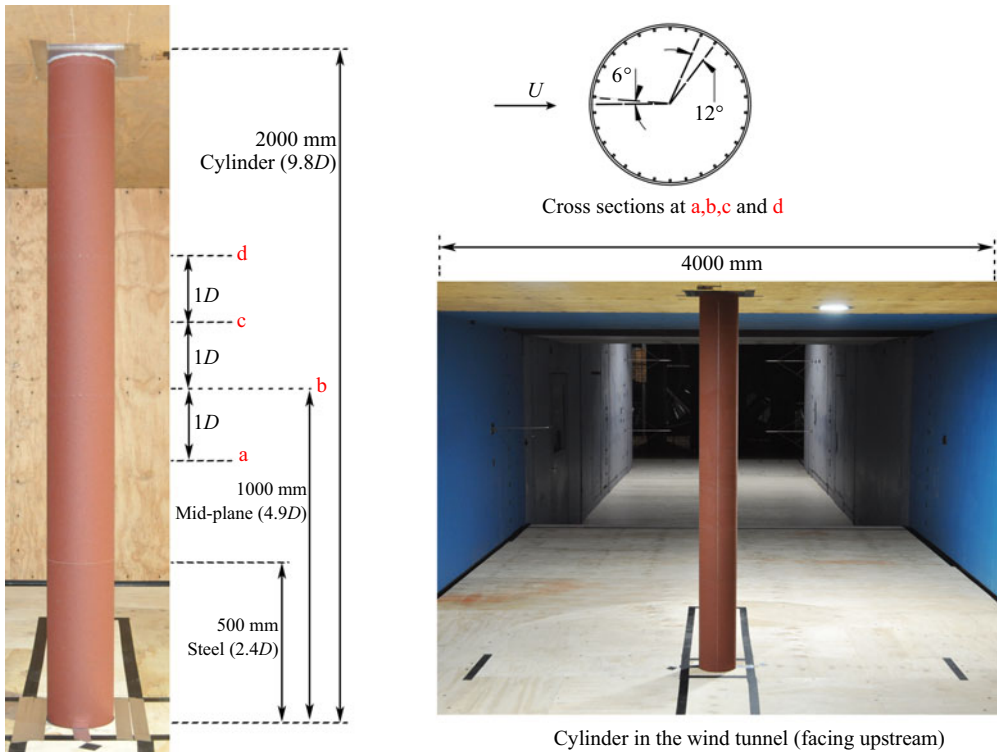


Figure 2. Schematic of the set-up showing a cylinder with sandpaper attached, its dimensions and measurement planes on the left, pressure tap locations at the top right and placement of the cylinder in the working section of the tunnel at the lower right.

confidence interval (*CI*) of a parameter is computed using

$$CI(x) = t(n - 1)_{\alpha/2} \times \frac{\sigma_x}{\sqrt{n}}, \quad (2.1)$$

where  $t(n - 1)_{\alpha/2}$  is the critical  $t$ -value for a significance level of  $\alpha$  and  $n - 1$  degrees of freedom. Here,  $n$  is the number of tests,  $x$  is the parameter of interest and  $\sigma_x$  is the standard deviation of the parameter across the repeat tests. For the current investigation  $\alpha$  is chosen to be 0.10, thus yielding a two-sided 90 % confidence interval. Thus, if the same tests and processing were to repeat a number of times, the confidence intervals present the bounds inside which the sample mean lies 90 % of the time.

Instantaneous coefficients of pressure drag and lift were obtained by integrating the circumferential pressure distributions and the corresponding mean and standard deviations were then obtained from these instantaneous coefficients. All the data presented here is the mean of the four spanwise measurement planes except in figure 8 for which instantaneous pressure distributions from the mid-plane are presented. Within the postcritical regime, variation in the coefficient of drag measured from different cross sections was within 2 % of the spanwise mean for all roughnesses tested and this variation reduced with decreasing roughness.

In an attempt to reduce the error in estimates of the drag coefficient,  $C_D$ , the minimum pressure coefficient,  $C_{P_m}$ , and the base pressure coefficient,  $C_{P_b}$ , due to linear interpolation of the discrete pressure distribution, the mean pressure distribution along



each cross section was interpolated using *modified Akima interpolation* (Akima 1970) before obtaining these parameters. This method was chosen over the traditional linear, cubic or spline interpolation since it minimises potential overshoots near the separation region. The effectiveness of this interpolation method was tested using continuous distributions obtained from computational fluid dynamics (CFD) and also from finer surface pressure measurements from the literature (Achenbach 1971; Güven *et al.* 1980; Ribeiro 1991; Cheung & Melbourne 1995; van Hinsberg 2015). Amongst the different circumferential pressure distribution profiles tested, the maximum deviation in estimates between the interpolated  $C_D$ ,  $C_{P_m}$  and continuous  $C_D$ ,  $C_{P_m}$  are 0.8 % and 2 %, respectively, while the difference between the linear  $C_D$ ,  $C_{P_m}$  and continuous  $C_D$ ,  $C_{P_m}$  estimates are 1 % and 4 % respectively.

A TSI 1201 platinum hot film was used with a 1750 constant temperature anemometer to measure the magnitude of the instantaneous planar velocity and, hence, estimate the predominantly streamwise velocity distribution in the immediate wake. This allowed a quantification of the shear layer thickness and frequency content in the separating shear layers. A 50- $\mu\text{m}$ -wide, 1-mm-long single-axis film with its axis parallel to the axis of the cylinder was mounted on a traverse system for this purpose. Velocity calibrations were performed to obtain a fifth-order polynomial to convert from voltage measurements of the film to the flow velocity. The upper frequency limit,  $f_c$  (−3 dB) for this arrangement is determined through a 1 kHz square wave input as 12.5 kHz (based on Freymuth 1977; Brunn 1995). Data from this hot film was acquired at 50 kHz for a period of at least 60 s (~1200 shedding cycles at the lowest speed of measurement). Spectra for the separated shear layer were obtained through velocity time series lasting 240 s (figure 17) and are plotted only to 6 kHz and the shear layer frequencies discussed were <300 Hz, well below  $f_c$ .

The blockage ratio of the set-up was only 5 % and, hence, the influence of the closeness of wind-tunnel walls is expected to be small. Nevertheless, mean values of  $C_D$ ,  $C_P$  and  $Re$  are corrected for blockage according to the procedure described in Roshko (1961) (based on Allen & Vincenti 1944) in order to provide easier comparison with the literature. (The accuracy of the blockage correction for the mean coefficients of drag and pressure has been previously verified by Farrell *et al.* 1977.) The fluctuating properties such as  $\sigma_{C_p}$  and  $\sigma_{C_L}$  have not been corrected for blockage since the applicability of blockage corrections to these parameters has not yet been established and the corrections are likely small. The maximum difference between the raw and the blockage corrected coefficients of drag is 3 %.

### 3. Results and discussion

#### 3.1. Drag and lift coefficients

Figure 3 plots, for a number of studies, the relationship of both the mean drag coefficient,  $C_D$ , and the fluctuating lift coefficient,  $\sigma_{C_L}$ , with Reynolds number. For clarity, the results of only the smoothest and the roughest cylinder in the current study are given in this figure. Figure 3(a) includes  $C_D$  measurements from the sectional pressure measurements of van Hinsberg (2015) while those obtained from their force measurements are excluded. Within the postcritical regime, the degree of agreement in  $C_D$  amongst the works is high, and perhaps surprising given the differences in the aspect and blockage ratios of their set-ups (see table 1). A reduction in aspect ratio is expected to decrease drag since the fluid feeding from the edge of the cylinder increases the base pressure (Basu 1985). Considering the

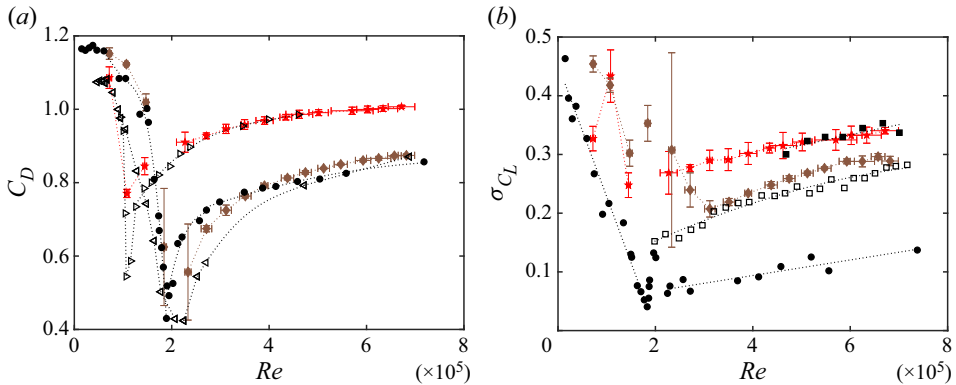


Figure 3. A comparison of (a) the mean drag coefficient and (b) the fluctuating lift coefficient in this study with earlier works. Confidence intervals were calculated as described in § 2.2:  $\cdots \triangle \cdots$  Achenbach (1971),  $k_s/D = 1.1 \times 10^{-3}$ ;  $\cdots \bullet \cdots$  van Hinsberg (2015),  $k_s/D = 1.2 \times 10^{-3}$ ;  $\cdots \triangleright \cdots$  Güven *et al.* (1980),  $k_s/D = 3.11 \times 10^{-3}$ ;  $\cdots \square \cdots$  Eaddy (2019),  $k_s/D = 1.2 \times 10^{-3}$ ,  $k/D = 0.8 \times 10^{-3}$ , aspect ratio 9;  $\cdots \blacksquare \cdots$  Eaddy (2019),  $k_s/D = 1.2 \times 10^{-3}$ ,  $k/D = 0.8 \times 10^{-3}$ , aspect ratio 5.7;  $\cdots \blacklozenge \cdots$  (brown) current,  $k_s/D = 1.1 \times 10^{-3}$ ;  $\cdots \star \cdots$  (red) current,  $k_s/D = 3 \times 10^{-3}$ .

large difference in the blockage ratios amongst the different set-ups (from 5% to 18%); this agreement is indicative that blockage corrections work well for the predictions of  $C_D$ .

Figure 3(b) gives the variation of the fluctuating coefficient of lift with Reynolds number. The scatter in measurements of  $\sigma_{C_L}$  across different investigations is comparatively larger than that of  $C_D$  owing to its higher sensitivity to aspect ratio, blockage ratio, mode of generation of roughness and method of measurement (Norberg 2003). In particular, the influence of aspect ratio and blockage is also evident in the difference in  $\sigma_{C_L}$  obtained from two different set-ups of the same roughness in Eaddy (2019). While their larger aspect ratio ( $AR = 9$ ) results for similar roughness agree well with the current study,  $\sigma_{C_L}$  values from the smaller aspect ratio ( $AR = 5.7$ ) are considerably higher.

On the other hand,  $\sigma_{C_L}$  reported in van Hinsberg (2015) is much lower than that for the current study at the same Reynolds number ( $\sim 0.1$  compared with  $\sim 0.25$ ) despite both set-ups having similar aspect ratio and relative roughness. A possible cause of this discrepancy is the method of generating and representing roughness. While van Hinsberg (2015) used plasmatic metal coating to generate surface roughness, the current work uses sandpaper. The roughness of a sandpaper is expected to be more randomly distributed than that obtained from machining the surface. The agreement in the mean  $C_D$  indicates that the influence of the roughness on the broad flow features is similar in the two investigations. The relatively large disagreement in  $\sigma_{C_L}$  could, in part, be due to the distribution of roughness and the resulting variation across different methods of generation.

In addition to the mode of roughness generation, the technique used for determining  $\sigma_{C_L}$  in the two studies is also different. van Hinsberg (2015) measured the instantaneous forces on the entire cylinder using a force balance to determine the fluctuating lift coefficient. In contrast, for the current study, the time-varying lift coefficient and its standard deviation are obtained through the integration of instantaneous pressure coefficients at a single cross section of the cylinder. This suggests that the spanwise correlation of fluctuating force components plays an important role; this will be examined in more detail in § 3.5. Moreover, preliminary experiments during the initial phase of the current study revealed high sensitivity of  $\sigma_{C_L}$  to the presence of a gap between the cylinder and wind-tunnel walls that is required in order to measure forces using external balances. Kacker, Pennington &

## Influence of surface roughness on postcritical flow

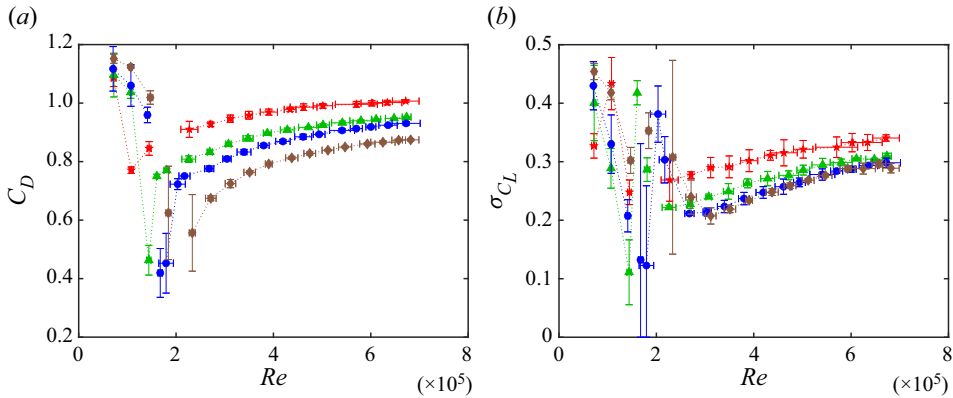


Figure 4. (a) Mean coefficient of drag,  $C_D$ , and (b) fluctuating coefficient of lift,  $\sigma_{C_L}$  for different roughness. Confidence intervals were calculated as described in § 2.2:  $\cdot \cdot \star \cdot \cdot$  (red),  $k_s/D = 3 \times 10^{-3}$ ;  $\cdot \cdot \blacktriangle \cdot \cdot$  (green),  $k_s/D = 1.9 \times 10^{-3}$ ;  $\cdot \cdot \bullet \cdot \cdot$  (blue),  $k_s/D = 1.4 \times 10^{-3}$ ; and  $\cdot \cdot \blacklozenge \cdot \cdot$  (brown),  $k_s/D = 1.1 \times 10^{-3}$ .

Hill (1974) reported that an airgap between the active and dummy cylinder as small as 0.1 mm could result in lift force measurements 5–10 times smaller than the actual value. Jones, Cincotta & Walker (1969) also found that gaps of 0.7 mm could have significant effects on the flow configuration over a smooth cylinder. More information about the influence of this gap and differences between lift coefficients measured through total force and cross-sectional pressure methods for smooth cylinders is given in Norberg (2003).

Limited and widely varying data amongst different studies and increased sensitivity of the lift to the parameters of flow set-up makes comparisons of  $\sigma_{C_L}$  for rough cylinders particularly difficult. Scarcity of information about  $\sigma_{C_L}$  for different roughnesses in the postcritical regime was a major motivation behind this work and the results presented here aim to address this knowledge gap.

Figure 4 gives the variation of the mean coefficient of drag and the fluctuating lift coefficient with Reynolds number for different roughnesses. Flow parameters in the critical/supercritical regime where the cylinder flow experiences combinations of *laminar separation*, *transition to turbulence in the shear layer*, *turbulent reattachment*, *intermittent turbulent separation* and *asymmetric shedding* are very sensitive to disturbances in the flow and, hence, the confidence intervals are larger. This region of high sensitivity extends up to  $Re \sim 2.3 \times 10^5$  for the smoothest roughness tested and up to  $Re \sim 1 \times 10^5$  for the roughest tested in the current work. Owing to the higher sensitivity of the lift to the pressure distribution near separation, these confidence intervals for the measurements of  $\sigma_{C_L}$  are larger than those of  $C_D$ . Tests for the same roughness and similar  $Re$  are found to intermittently contain the LSB for different proportions of time resulting in a wide range of  $\sigma_{C_L}$ , thus making it difficult to infer representative trends through the critical regime. The reader is referred to Cadot *et al.* (2015) for more information regarding the flow configuration and the stability of LSB through the critical regime. Note that § 3.4 explores the intermittency of the LSB and the influence of roughness in further detail. Repeatability is improved beyond the critical regime and the observed variations with Reynolds number are relatively smooth.

An increase in roughness leads to a reduction in the critical Reynolds number, the Reynolds number at which  $C_D$  is minimum. Roughness particles promote the transition to turbulence near the surface of the cylinder and, hence, an increase in roughness leads

to an earlier transition in the boundary layer, i.e. transition to turbulence immediately after separation, and the LSB occurs at a lower Reynolds number for larger roughness. From the current tests, the critical Reynolds number of the roughest surface tested ( $k_s/D = 3 \times 10^{-3}$ ) is  $\sim 1 \times 10^5$  while that of the smoothest surface tested ( $k_s/D = 1.1 \times 10^{-3}$ ) is  $\sim 2 \times 10^5$ . Beyond the critical Reynolds number, the flow configuration changes from stable reattachment of the separation bubble on both sides of the cylinder to asymmetric reattachment and eventually to no reattachment on the two sides of the cylinder (Cadot *et al.* 2015). This results in an increase in  $C_D$  as Reynolds number increases in the supercritical regime. Increasing roughness causes the LSB to become more unstable, thereby leading to a lower range of  $Re$  where an intermittent LSB occurs and, hence, a smaller supercritical regime. By destabilising the LSB, roughness promotes coherent vortex shedding over the span of the cylinder and, hence, increases the forces on the overall span of the cylinder since the flow separates earlier when the LSB is absent.

At postcritical Reynolds numbers, the boundary layer in the adverse pressure gradient region is turbulent and hence, separates later than the laminar boundary layer of subcritical regime. Due to this delayed separation, the postcritical  $C_D$  is lower than that of the subcritical  $C_D$  for a given roughness. On the other hand, an increase in surface roughness causes an earlier separation of turbulent boundary layers in adverse pressure gradients due to an increased momentum deficit near the wall (Song & Eaton 2002; Aubertine, Eaton & Song 2004). Thus, at a fixed postcritical  $Re$ ,  $C_D$  increases with an increase in roughness. The mean coefficients of drag  $C_D$  are found to be  $\sim 0.87$  and  $\sim 1.01$ , respectively, for the smoothest and roughest cylinders at the highest Reynolds number of the current tests ( $6.8 \times 10^5$ ).

Similar to  $C_D$ , the postcritical  $\sigma_{C_L}$  increases with increasing roughness. Power spectra of the velocity fluctuations given in figure 16 reveal an increased energy content at the shedding frequency for increased roughness. This indicates that an increase in roughness results in larger fluctuations and hence a larger  $\sigma_{C_L}$ . At  $Re \sim 6.7 \times 10^5$ ,  $\sigma_{C_L}$  is 0.28 and 0.34, respectively, for the smoothest and roughest cylinder tested.

### 3.2. Circumferential pressure distributions

Mean and standard deviation circumferential pressure distributions presented in this study are the average of distributions over the four spanwise locations. To better highlight variations, these spanwise averages are presented only for half of each circumference. These half-circumference pressure distributions are the average of the upper and lower halves of the cylinder. Across all the postcritical Reynolds numbers ( $Re \geq 3 \times 10^5$ ) and roughnesses tested, the maximum deviation,  $|\overline{C_P}(\theta) - \overline{C_P}(360 - \theta)|$  is 0.1 and this is seen in the large gradient region of  $0^\circ \leq \theta \leq 90^\circ$ . Over  $100^\circ \leq \theta \leq 180^\circ$ , this maximum reduces to  $< 0.01$ . The mean deviation across all the tests is less than 0.08 for  $0^\circ \leq \theta \leq 180^\circ$ .

Figure 5 gives the circumferential mean pressure coefficient of cylinders of different roughnesses at postcritical Reynolds numbers. For a given roughness, the coefficient of pressure over the upstream region of the cylinder ( $\sim (300^\circ - 0^\circ - 60^\circ)$ ) does not exhibit any noticeable change with Reynolds number, generally following the potential flow solution. The observable changes with increasing Reynolds number are a decrease in the coefficient of pressure in the downstream/base region ( $\sim (100^\circ - 180^\circ - 260^\circ)$ ), and an increase in the coefficient of pressure in the acceleration region ( $\sim (75^\circ - 90^\circ)$ ).

To examine these changes in further detail, three additional variables are introduced. Base pressure,  $C_{P_b}$ , is the mean coefficient of pressure over the downstream side of

## Influence of surface roughness on postcritical flow

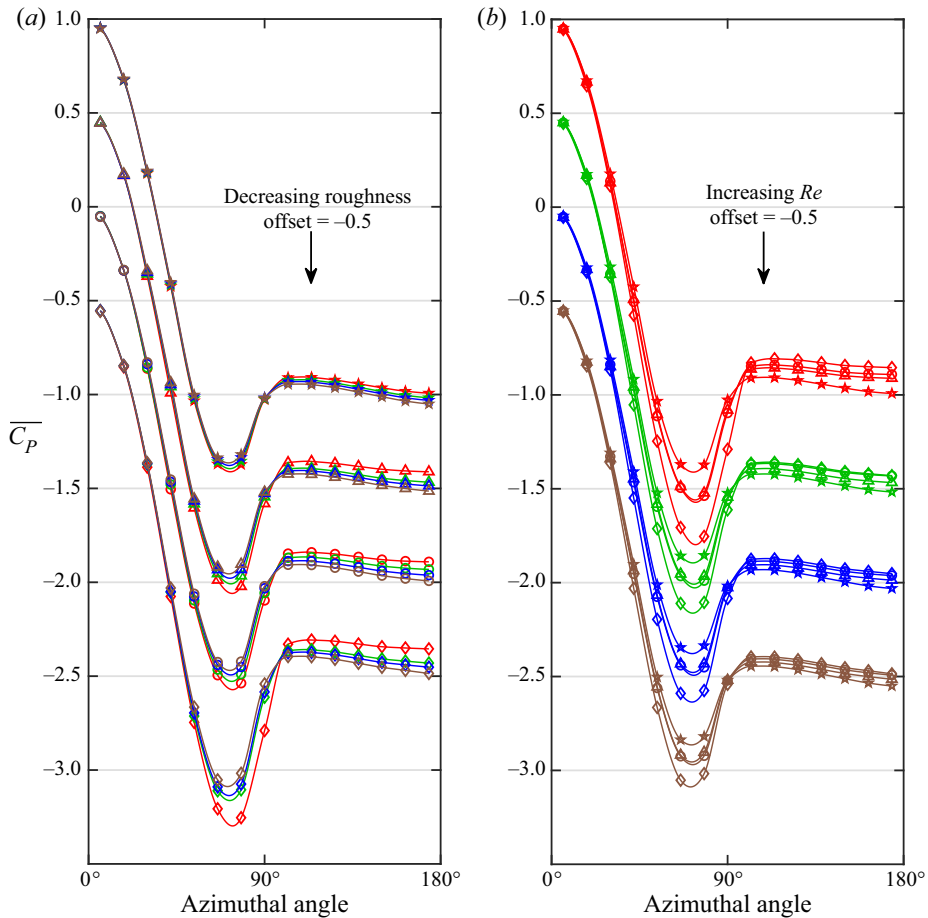


Figure 5. Circumferential distributions of mean coefficient of pressure:  $\star$ ,  $k_s/D = 3 \times 10^{-3}$ ;  $\Delta$ ,  $k_s/D = 1.9 \times 10^{-3}$ ;  $\circ$ ,  $k_s/D = 1.4 \times 10^{-3}$ ;  $\diamond$ ,  $k_s/D = 1.1 \times 10^{-3}$ ; red solid line,  $Re \sim 3 \times 10^5$ ; green solid line,  $Re \sim 4 \times 10^5$ ; blue solid line,  $Re \sim 5 \times 10^5$ ; and brown solid line,  $Re \sim 6.7 \times 10^5$ . (a) Effect of Reynolds number for different roughness. (b) Effect of roughness for different Reynolds number. Note that successive families of curves have been offset in the pressure coefficient by  $-0.5$ .

the cylinder. Minimum pressure,  $C_{P_m}$ , is the minimum coefficient of pressure on the cross-sectional surface. Wake angle,  $\theta_w$ , is the angle at which a linear regression fit of the coefficient of pressure profile from  $C_{P_m}$  to  $C_{P_b}$  intersects  $C_{P_b}$ . Both  $C_{P_m}$  and  $\theta_w$  given are the mean of the values found on each half of the cylinder. Furthermore, all three variables presented are the corresponding spanwise averages over a sample of four cross sections. The definitions of these variables for a representative  $C_P$  distribution are given in the figure 6(a).

While  $C_{P_b}$  forms the prominent contribution to the coefficient of drag,  $C_{P_m}$  is indicative of the degree of deceleration of flow velocity near the surface of the cylinder, i.e. the lower the  $C_{P_m}$  the higher the velocity outside the boundary layer. In addition,  $\theta_w$  is a nominal estimate of the separation angle obtained from the circumferential mean  $C_P$  distribution. This estimate is smaller than the actual separation angle given by  $\theta_s$  by  $\sim 5^\circ$  (Güven *et al.* 1980); indeed, this correlates well with the  $\theta_s$  measurements of Achenbach (1971) as seen in figure 6(b).

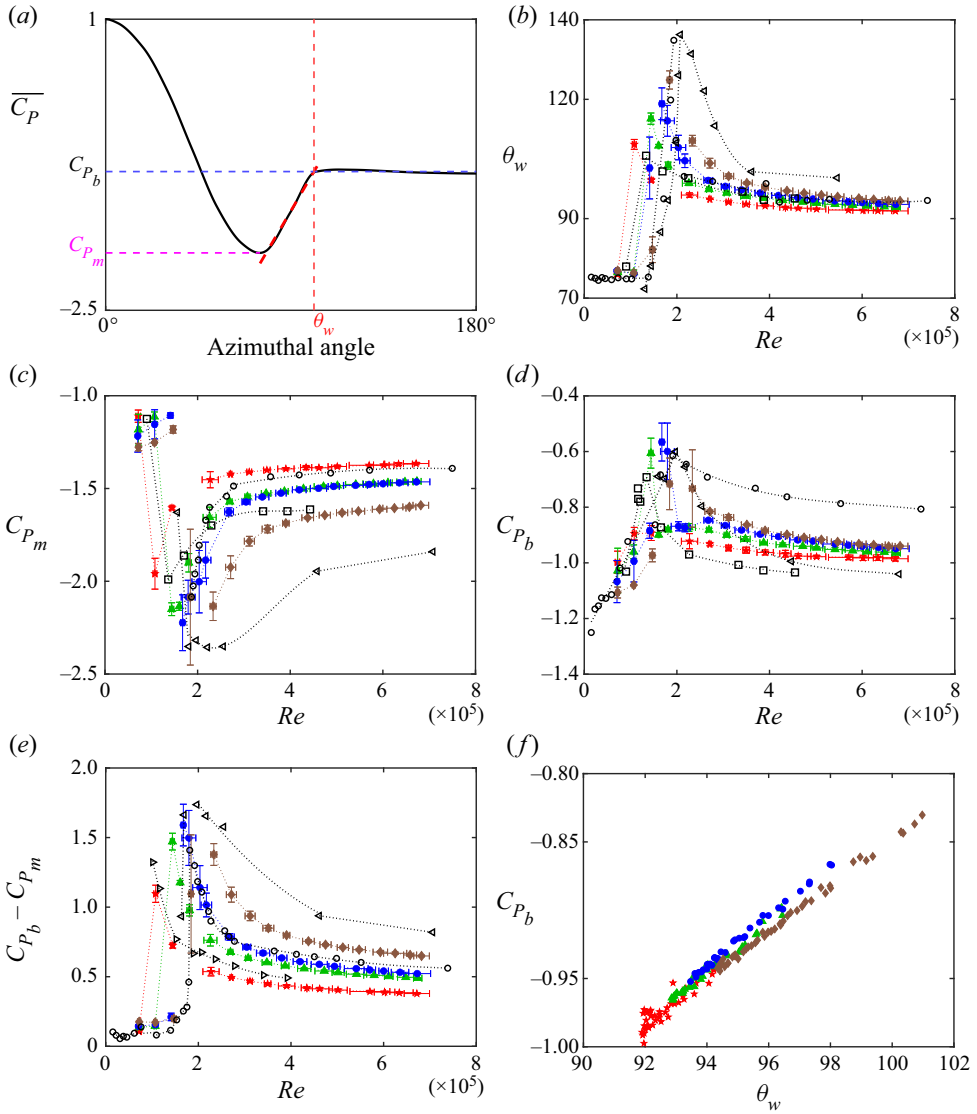


Figure 6. (a) Parameters of coefficient of pressure, (b) variation of wake angle, (c) variation of coefficient of minimum pressure, (d) variation of coefficient of base pressure, (e) pressure rise to separation with Reynolds number and (f) base pressure vs separation angle. Confidence intervals were calculated as described in § 2.2 Current:  $\cdot \cdot \star \cdot \cdot$  (red)  $k_s/D = 3 \times 10^{-3}$ ;  $\cdot \cdot \blacktriangle \cdot \cdot$  (green)  $k_s/D = 1.9 \times 10^{-3}$ ;  $\cdot \cdot \bullet \cdot \cdot$  (blue)  $k_s/D = 1.4 \times 10^{-3}$ ;  $\cdot \cdot \blacklozenge \cdot \cdot$  (brown)  $k_s/D = 1.1 \times 10^{-3}$ ;  $\cdot \cdot \circ \cdot \cdot$  van Hinsberg (2015)  $k_s/D = 1.2 \times 10^{-3}$ ;  $\cdot \cdot \triangleleft \cdot \cdot$  Achenbach (1971),  $k_s/D = 1.1 \times 10^{-3}$ ;  $\cdot \cdot \triangleright \cdot \cdot$  Güven *et al.* (1980),  $k_s/D = 3.11 \times 10^{-3}$ ;  $\cdot \cdot \square \cdot \cdot$  Güven *et al.* (1980),  $k_s/D = 2.5 \times 10^{-3}$ .

On the surface of a smooth cylinder, the laminar boundary layer in the subcritical flow separates earliest, near  $\sim 80^\circ$  on a smooth cylinder, followed by a turbulent boundary layer, between  $\sim 100$ – $110^\circ$  on a smooth cylinder and a boundary layer with a LSB in the critical regime separates more downstream, between  $\sim 120$ – $140^\circ$  on a smooth cylinder than the latter (Basu 1985). A similar trend is observed in the measurements of  $\theta_w$  against Reynolds number from the current results. At subcritical Reynolds numbers,  $\theta_w$  is  $\sim 75^\circ$

for all the roughnesses tested and an increase in Reynolds number results in an increasing  $\theta_w$  until it reaches a maximum ( $\sim 108^\circ$  and  $\sim 125^\circ$  for the roughest and the smoothest cylinder, respectively). A further increase leads to a decrease in the wake angle and at large postcritical Reynolds numbers the wake angle approaches an asymptotic value of  $\sim 95^\circ$ . This indicates that the separation angle also reaches an asymptotic value of  $\sim 100^\circ$  in agreement with Achenbach (1971).

Similar to the wake angle, the base pressure coefficient increases and then decreases with increasing Reynolds number. The peak in the base pressure occurs in the critical regime and coincides with the peak in the separation angle and also the minimum drag coefficient. As the Reynolds number is increased further, the change in base pressure decreases until eventually approaching an asymptotic value of  $\sim -0.95$ . The convergence of  $C_{P_b}$  with roughness at large Reynolds numbers is slightly wider than that of  $\theta_w$ . Changes in the minimum pressure,  $C_{P_m}$  are in the opposite direction, i.e. an increase in Reynolds number causes a decrease and then an increase in  $C_{P_m}$ , before eventually approaching an asymptotic value. Unlike  $\theta_w$  and  $C_{P_b}$ , the asymptotic value in the minimum pressure at the highest Reynolds numbers tested is a function of roughness, with larger roughness leading to a higher  $C_{P_m}$ , i.e. larger deceleration of the flow.

For the smoothest cylinder tested ( $k_s/D = 1.1 \times 10^{-3}$ ), the changes in  $\theta_w$ ,  $C_{P_b}$  and  $C_{P_m}$  when the Reynolds number is increased from  $Re \sim 4 \times 10^5$  to  $Re \sim 6 \times 10^5$  are approximately 4%, 6% and 5%, respectively. The corresponding changes for the roughest cylinder tested ( $k_s/D = 3 \times 10^{-3}$ ) are 1%, 3% and 2%, respectively. At the largest Reynolds number tested ( $Re \sim 6 \times 10^5$ ), the changes in these quantities, ( $\theta_w$ ,  $C_{P_b}$  and  $C_{P_m}$ ) from the smoothest cylinder to the roughest cylinder tested are 3%, 5% and 14%, respectively. These observations indicate that while the flow near and beyond separation becomes nearly independent of roughness at large Reynolds numbers, flow in the deceleration region and particularly near the minimum pressure is still dependent on the degree of surface roughness, even though it is relatively independent of the Reynolds number.

Amongst different investigations, the degree of scatter in the pressure rise to separation  $C_{P_b} - C_{P_m}$  is slightly larger than that in  $C_D$  and  $C_{P_b}$  (note the different y-scales in figure 6c,e). This is in contrast to the proposal of Güven *et al.* (1980) and Farrell *et al.* (1977) that the pressure rise to separation is less sensitive to the effects of aspect ratio and blockage than  $C_{P_b}$  and  $C_{P_m}$ . The present rough cylinder data show that the base pressure results among different studies have a smaller scatter followed by the minimum pressure. The deviation in the pressure rise to separation is a consequence of that in minimum pressure. In this regard, the use of spanwise variation of minimum pressure to ascertain the effects of blockage and aspect ratio in the postcritical region is recommended over the base pressure distribution that has been used before (Stansby 1974; Fox & West 1990).

Blockage corrections to pressure distributions (Allen & Vincenti 1944; Roshko 1961) do not include the effect of wake blockage on pressure gradients (Farrell *et al.* 1977) and is a possible cause of the larger scatter in the pressure distribution parameters from different studies when compared with that of  $C_D$ . Another limitation in the application of blockage corrections is that the corrected  $C_P$  distributions do not directly correspond to the corrected  $C_D$  values since the corrections are independent. In this context, one should exercise care in applying the corrections to predict the pressure distributions free of wind-tunnel influences, since an agreement in the blockage corrected  $C_D$  does not imply agreement in the blockage corrected  $C_P$ . Moreover, corrections to  $\sigma_{C_L}$  are unavailable since the influence of blockage on the fluctuating parameters is still uncertain.

This emphasises the importance of data obtained from a high-aspect-ratio and low-blockage set-up similar to that used in the current work.

Since wake angle and base pressure appear to be in good correlation, the coefficient of base pressure,  $C_{P_b}$ , is plotted against the wake angle  $\theta_w$  in the postcritical regime ( $Re \geq 3 \times 10^5$ ) in figure 6(f). The data plotted are obtained from tests of different roughnesses, i.e. different minimum pressure (see figure 6c) and, thus, a different pressure distribution before separation. Hence, a good correlation across different roughnesses indicates that the base pressure is a strong function of the separation angle and only a weak function of the flow before separation in the postcritical regime. From figure 6(b), in postcritical flows, an increase in roughness at a constant Reynolds number or an increase in Reynolds number for a fixed roughness lead to a decrease in the separation angle. Figure 6(f) indicates that the change in base pressure coefficient arising due to a change in the separation angle is similar for all roughnesses tested. Since the base pressure coefficient forms the major contribution to the coefficient of drag, this results in the drag coefficient being similar in postcritical flows regardless of whether the separation angle is a result of increasing roughness or increasing Reynolds number. This forms the basis for collapse of  $C_D$  when plotted against roughness Reynolds number,  $Re_{k_s}$ , and this is further discussed in § 3.7.

### 3.3. Fluctuating pressure coefficient

Figure 7 gives the distribution of the fluctuating pressure coefficient on the cylinder surface at different Reynolds numbers and roughnesses. The two significant trends with increasing Reynolds number post the critical regime are an increase in  $\sigma_{C_p}$  over the circumference (more evident in the downstream region); and an upstream movement of the angle at which  $\sigma_{C_p}$  is maximum. A peak in  $\sigma_{C_p}$  is an indication of boundary-layer separation in the vicinity and since separation moves upstream with increasing Reynolds number, the location of maximum  $\sigma_{C_p}$  also moves upstream. The smaller local maximum near  $\sim 160^\circ$  might indicate shear-layer roll up closer to the surface of the cylinder near that region. This smaller peak is less significant at lower Reynolds numbers (near  $Re \sim 3 \times 10^5$ ) especially for the smoother cylinders due to weaker vortex shedding at those Reynolds numbers. For a given roughness, an increase in velocity leads to an increase in the turbulent kinetic energy near the surface, causing an overall increase of  $\sigma_{C_p}$  as the Reynolds number is increased. Moreover, earlier separation is also expected to cause larger fluctuations in the downstream region of the cylinder.

Figure 7(b) gives the variation of  $\sigma_{C_p}$  with roughness for various Reynolds numbers. For a given Reynolds number,  $\sigma_{C_p}$  increases over the circumference of the cylinder with an increase in roughness. In addition, the peak in  $\sigma_{C_p}$  increases and moves upstream as roughness is increased for a fixed Reynolds number. This change is similar to that seen when Reynolds number is increased for a fixed roughness.

### 3.4. Power spectra of lift and Strouhal number of vortex shedding

Figure 8(a) gives the power spectral density (PSD) of fluctuations in the lift coefficient at a Reynolds number of  $6.7 \times 10^5$  for different roughnesses. A similar distribution with a significant peak is seen in the majority of the tests (all at postcritical Reynolds numbers) indicating the presence of strong periodic vortex shedding. Since the lift force is induced by the alternating vortex shedding, the second half of the lift cycle must mirror the first half, which means only odd harmonics can contribute to a periodic signal. This can be seen in the presence of the third harmonic throughout the postcritical regime.



### Influence of surface roughness on postcritical flow

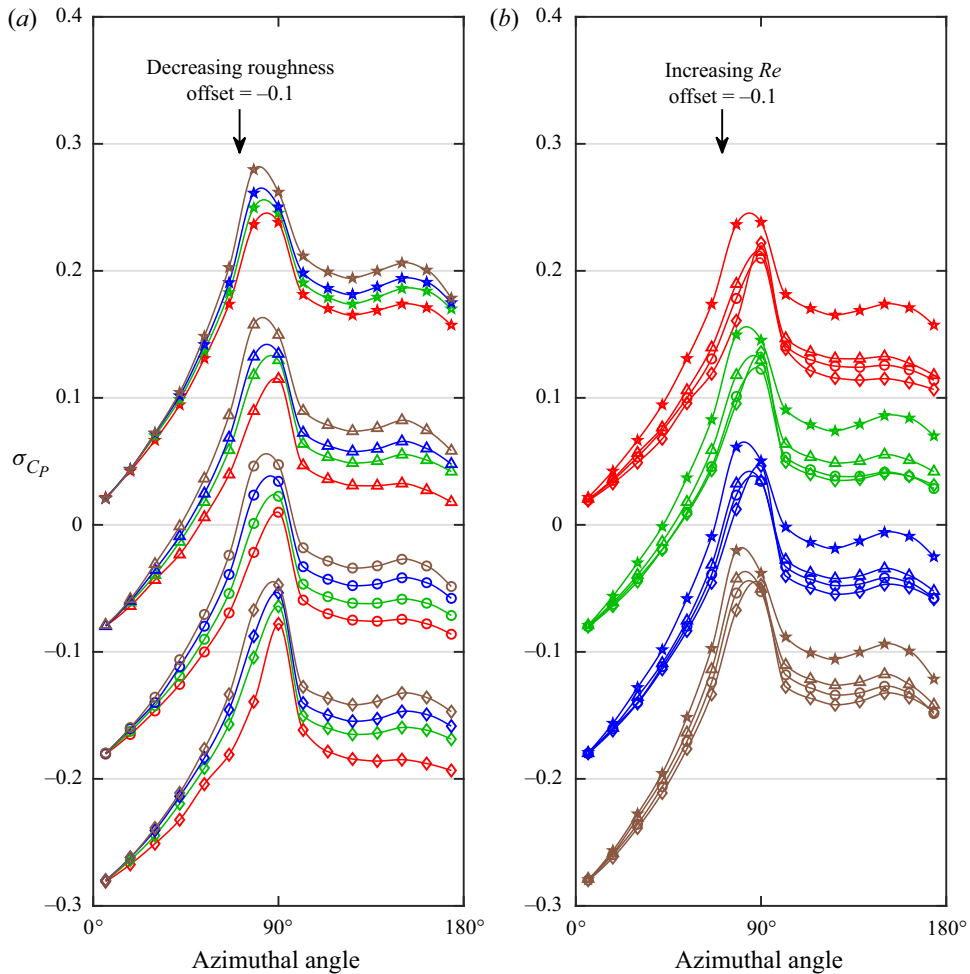


Figure 7. Circumferential distributions of the fluctuating coefficient of pressure:  $\star$ ,  $k_s/D = 3 \times 10^{-3}$ ;  $\Delta$ ,  $k_s/D = 1.9 \times 10^{-3}$ ;  $\circ$ ,  $k_s/D = 1.4 \times 10^{-3}$ ;  $\diamond$ ,  $k_s/D = 1.1 \times 10^{-3}$ ; red solid line,  $\sim 3 \times 10^5$ ; green solid line,  $\sim 4 \times 10^5$ ; blue solid line,  $\sim 5 \times 10^5$ ; and brown solid line,  $\sim 6.7 \times 10^5$ . (a) Effect of Reynolds number for different roughness. (b) Effect of roughness for different Reynolds number. Note that successive sequences of curves have been offset in pressure coefficient by  $-0.1$ .

Achenbach & Heinecke (1981) found that energy of lift fluctuations is distributed in a narrow band of frequencies for rough cylinders through the critical regime, while on the other hand Szechenyi (1975) and Batham (1973) found a broadband spectra without an obvious dominant frequency near the critical Reynolds number. A clear shedding frequency was found in the lift spectra for all tests except for  $k_s/D = 1.4 \times 10^{-3}$  and  $k_s/D = 1.1 \times 10^{-3}$  at Reynolds numbers near  $Re \sim 2 \times 10^5$ . As an example, gives time histories of the coefficients of pressure, lift and drag (plotted against a non-dimensional convective time scale,  $tU_\infty/D$ , also referred to as  $t^*$  henceforth) observed for  $k_s/D = 1.4 \times 10^{-3}$  at a Reynolds number of  $1.64 \times 10^5$  are given in figures 8(b), 8(c) and 8(d), respectively.

For  $0 < t^* \leq 1200$ , a stable LSB exists on both sides of the cylinder, which can be inferred from the lower mean  $C_D$  and mean  $C_L$  of  $\sim 0$ . The presence of the LSB delays

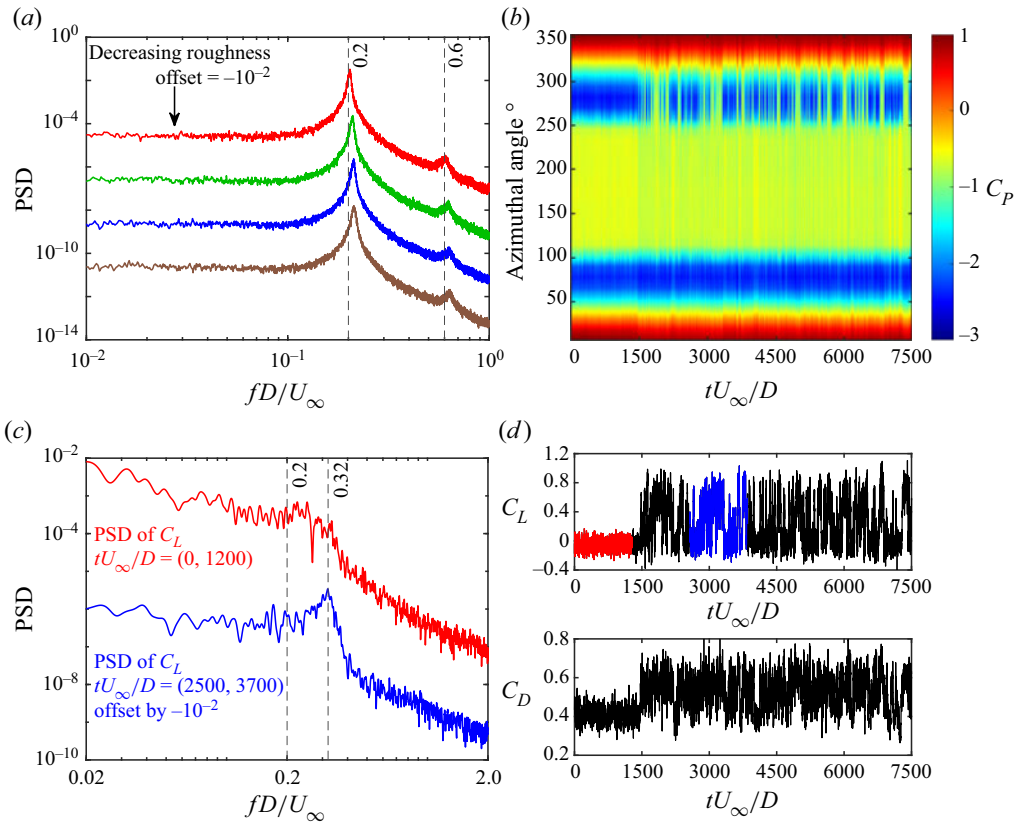


Figure 8. (a) Frequency spectra of the lift coefficient for different roughnesses at Reynolds number of  $6.7 \times 10^5$ : red,  $k_s/D = 3 \times 10^{-3}$ ; green,  $k_s/D = 1.9 \times 10^{-3}$ ; blue,  $k_s/D = 1.4 \times 10^{-3}$ ; brown,  $k_s/D = 1.1 \times 10^{-3}$ . (b) Coefficient of pressure vs time on  $k_s/D = 1.4 \times 10^{-3}$  at Reynolds number of  $1.64 \times 10^5$ . (c) Spectra of  $C_L$  for different time intervals. (d) Time history of  $C_L$  and  $C_D$  of that test.

separation, increases the base pressure and, hence, decreases  $C_D$ . For  $t^* > 1200$ , the LSB near  $(\theta \sim (300^\circ - 250^\circ))$  becomes unstable and intermittent leading to increased  $C_D$  and a non-zero mean  $C_L$ . The onset of instability is found to occur over the entire span but since the LSB becomes intermittent, the separation line in the spanwise direction at a given instant of time is expected to be jagged and, hence, vortex shedding is incoherent (Bearman 1984). This is also seen in the frequency spectra of the sectional lift coefficient from two different periods,  $0 < t^* \leq 1200$  and  $2500 < t^* \leq 3700$ . A dominant frequency is present in the spectrum when the LSB is stable on both sides of the cylinder, while the spectrum resulting from an asymmetric intermittent LSB is broad range. The spectrum for the entire duration of the test ( $t^* = 7500$ ,  $t = 120$  s) is broadband in nature and, hence, these Reynolds numbers were excluded from figure 10 despite there being strong periodicity over a short period. In addition, the Strouhal number pertaining to the wake flow when there is a dominant frequency or a narrow band in the energy spectrum while in the ‘two-bubble’ state is higher than the Strouhal numbers in both subcritical and postcritical regimes and is  $\sim 0.32$ . Bearman (1969) also found a higher frequency in the ‘two-bubble’ state.

This intermittent LSB is absent for the two rougher cylinders ( $k_s/D = 1.9 \times 10^{-3}$  and  $k_s/D = 3 \times 10^{-3}$ ) in the current tests. Thus, the prevalence of an intermittent LSB reduces as the roughness is increased, i.e. the supercritical regime becomes smaller as the

## Influence of surface roughness on postcritical flow

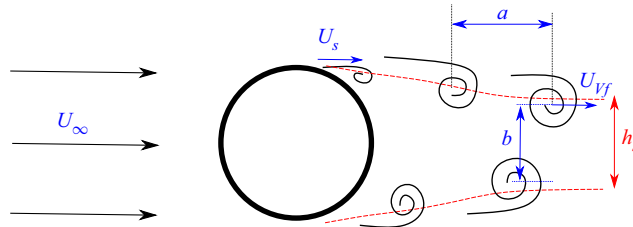


Figure 9. The length and velocity scales used in the calculation of universal Strouhal numbers,  $St_R$  and  $St_B$ ; red dashed line, mean shear layers.

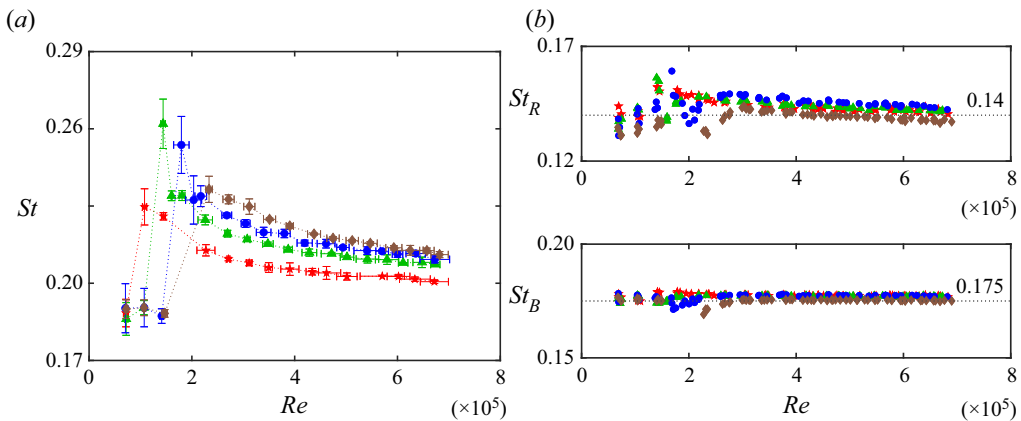


Figure 10. (a) Strouhal number and (b) the Roshko number (top),  $St_R$ , and the Bearman number (bottom),  $St_B$ , for different roughnesses. Confidence intervals were calculated as described in § 2.2. Plots:  $\cdot \cdot \star \cdot \cdot$  (red),  $k_s/D = 3 \times 10^{-3}$ ;  $\cdot \cdot \blacktriangle \cdot \cdot$  (green),  $k_s/D = 1.9 \times 10^{-3}$ ;  $\cdot \cdot \bullet \cdot \cdot$  (blue),  $k_s/D = 1.4 \times 10^{-3}$ ;  $\cdot \cdot \blacklozenge \cdot \cdot$  (brown),  $k_s/D = 1.1 \times 10^{-3}$ .

roughness is increased. In other words, surface roughness is found to promote coherent shedding in the Reynolds number space (in agreement with Achenbach & Heinecke 1981).

The Strouhal number of the vortex shedding initially increases and then decreases with the Reynolds number, similar to the changes in wake angle and base pressure. This variation of Strouhal number with Reynolds number for different roughnesses is given in figure 10(a). For a given roughness, the Strouhal number decreases with an increasing Reynolds number in the postcritical flow, indicating that the wake width increases (Roshko 1961). This increase is a consequence of the upstream movement of the separation angle and results in reduced base pressure and, thus, increased drag. An increase in roughness at a given postcritical Reynolds number also increases wake width and, hence, decreases the Strouhal number. Current data also confirm the prediction of Achenbach & Heinecke (1981) that the postcritical Strouhal numbers for rough cylinders are smaller than those for smooth cylinders. In line with the asymptotic behaviour of separation angle, base pressure and coefficient of drag, the Strouhal number eventually approaches asymptotic values at large Reynolds numbers ( $\sim 0.2$  for the roughest surface tested and  $\sim 0.21$  for the smoothest).

Two proposed *universal Strouhal numbers*: the *Roshko number*,  $St_R$ , (Roshko 1954, 1961) and the *Bearman number*,  $St_B$ , (Bearman 1967) were computed for the current tests and the results are presented in figure 9. While the Roshko number uses wake width

and velocity near the separation point, the Bearman number makes use of lateral vortex separation and velocity near the separation point as the characteristic length and velocity scales, respectively. The Roshko number (Roshko 1954) is calculated as

$$St_R = \frac{fh_s}{U_s}, \quad (3.1)$$

where  $f$  is the frequency of shedding,  $U_s$  is the velocity at the edge of the boundary layer near the separation, and  $h_s$  is the lateral (cross-stream) spacing between the shear layers when they become parallel. Here  $U_s$  can be estimated through Bernoulli's equation as

$$U_s = U_\infty(1 - C_{P_b})^{0.5}. \quad (3.2)$$

From simple momentum considerations:

$$-C_{P_b}h_s = C_D D. \quad (3.3)$$

On the other hand, the Bearman number (Bearman 1967) is

$$St_B = \frac{fb}{U_s}, \quad (3.4)$$

where  $b$  is the lateral spacing between the centres of vortices of the same sign. Here  $f$  is the frequency of shedding,  $a$  is the longitudinal spacing between vortex centres,  $U_{Vb}$  is the velocity of vortex centres relative to the body (stationary cylinder in this investigation) and  $U_{Vf}$  is the velocity of vortex centres relative to the freestream ( $U_\infty = U_{Vf} + U_{Vb}$ ). From (3.4),

$$\left. \begin{aligned} St_B &= \frac{fba}{aU_\infty(1 - C_{P_b})^{0.5}}, \\ fa &= U_{Vb}, \\ St_B &= \frac{b U_{Vb}}{a U_\infty (1 - C_{P_b})^{0.5}} \cdot 1 \end{aligned} \right\} \quad (3.5)$$

Kronauer's stability criterion states that the vortex street aligns itself such that the vortex-street drag coefficient,  $C_{D_s}$ , is minimum with respect to the aspect ratio,  $b/a$ . This criterion can be simplified to

$$\frac{b U_{Vb}}{a U_\infty} = F(C_D St). \quad (3.6)$$

where  $F$  is a function resulting from the stability criterion. Further information on the evaluation of the Bearman number is given in Appendix A.2.

Thus, the Roshko and the Bearman numbers for a given Reynolds number can be estimated using the coefficient of drag,  $C_D$ , the Strouhal number of shedding,  $St$ , and the base pressure coefficient,  $C_{P_b}$ . In these calculations of  $St_R$  and  $St_B$ , measured values (and not blockage-corrected ones) of  $C_D$ ,  $St$  and  $C_{P_b}$  are used, since the influence of blockage and blockage corrections on the vortex street is uncertain (Bearman 1967). The variation of  $St_R$  and  $St_B$  with Reynolds number for various roughnesses is given in figure 10(b).

Over the range of Reynolds numbers and roughnesses tested,  $St_B$  has less scatter than  $St_R$ . The values of  $St_R$  and  $St_B$  were found to be  $\sim 0.14$  and  $\sim 0.17$  in excellent agreement with those found by Adachi (1997) at similar Reynolds numbers on rough cylinders. The scatter in both  $St_R$  and  $St_B$  increases in the critical regime owing to uncertainties

of measurements. Moreover, a slight downward trend in  $St_R$  is found with increasing Reynolds number suggesting that perhaps  $St_R$  is not truly universal. On the other hand, the variation in the postcritical regime is still small over the Reynolds number range considered but there is a noticeable spread with roughness. It is of note that the tests in the supercritical regime where periodic shedding is sporadic are omitted in these calculations.

### 3.5. *Spanwise correlation of the lift*

The one-sided, axial correlation length of lift  $\Lambda$  is determined following the procedure outlined in § 2.1 of Norberg (2003) and the results are given in figure 11. A sample calculation for the correlation length is given in Appendix A. Owing to the sparse spanwise distribution of pressure measurements, there is some scatter in the calculated axial correlation length. Despite the scatter, it is clear that  $\Lambda$  does not vary significantly with roughness or Reynolds number in the postcritical regime. Broadly,  $\Lambda$  takes the value of  $\sim 3D$  in the subcritical regime and drops to  $\sim 1.5D$  in the critical regime indicating a drop in coherence due to the intermittent separation. In the postcritical regime,  $\Lambda$  is  $\sim 4D$  for all the roughnesses tested. These results are in excellent agreement with Buresti (1981) where correlation lengths were found using hot-wire measurements. This agreement is surprising given the difference in aspect ratios between the two set-ups and the method of measurement. As mentioned previously, correlation lengths of  $\sim 3.2D$  and  $\sim 3.9D$  were found at particular postcritical  $Re$  by Batham (1973) ( $AR = 6.6$ ), Ribeiro (1991) ( $AR = 6$ ), respectively, and the current results are in good agreement with them. However, Eddy (2019) reported correlation lengths of  $\sim 2.5$ – $3.4D$  for an aspect ratio of 9 and  $6D$  for an aspect ratio of 5.7. The turbulence intensity of the free stream in Eddy (2019) was 4.4 %, and this could be the cause of the slightly lower correlation lengths for the larger aspect ratio set-up. It is interesting that an  $AR$  of 5.7 in Eddy (2019), which is similar to that in Ribeiro (1991) and Batham (1973), resulted in a much higher correlation length. An increased blockage and vibration in their low aspect ratio set-up could also contribute to this discrepancy. The difference in fluctuating lift coefficient between the two set-ups supports this hypothesis.

A correlation length of  $\sim 4D$  thus appears to be the best estimate for postcritical flows over rough cylinders. This length is significantly larger than the  $\Lambda \sim 1.5D$  for smooth cylinders at similar Reynolds numbers (Batham 1973; King 1977; Blackburn & Melbourne 1996), indicating that the roughness increases spanwise uniformity of the flow in the Reynolds number range of  $Re = 3 \times 10^5$  to  $6.5 \times 10^5$ . Schewe (1983) showed that the transition to the postcritical regime for smooth cylinders begins at  $Re = 2.5 \times 10^6$ . Hence, it is likely that the smooth cylinder results of correlation lengths at  $3 \times 10^5 \leq Re \leq 6.5 \times 10^5$  were obtained in flow regimes that contain LSBs, thus explaining the lower correlation lengths. While the difference in axial correlation length of lift between smooth and rough cylinders at the same Reynolds number is significant, all roughnesses tested showed similar correlation lengths implying that the magnitude of the roughness has little effect on the extent of spanwise correlation in the range of Reynolds numbers tested.

### 3.6. *On behaviour of the flow beyond $Re \sim O(10^6)$*

The largest Reynolds number tested in the current work is  $6.7 \times 10^5$  and all of the variables measured, i.e.  $C_D$ ,  $\sigma_{C_L}$ ,  $C_{P_b}$ ,  $C_{P_m}$ ,  $\theta_w$ ,  $St$  and  $\Lambda$ , appear to have reached or are close to their asymptotic values with respect to increasing Reynolds number. This observation is in agreement with those of Güven *et al.* (1980), Buresti (1981) and Nakamura & Tomonari (1982) up to the largest  $Re$  tested in those works. Shih *et al.* (1993) and Adachi (1997)

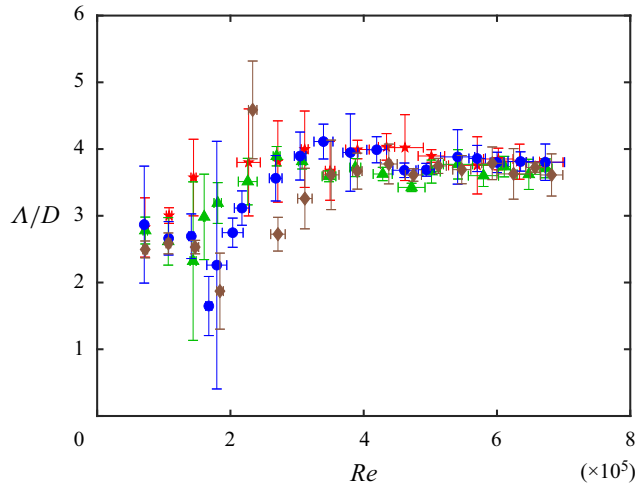


Figure 11. Axial correlation length of lift for different roughness and Reynolds numbers. Confidence intervals were calculated as described in § 2.2. Plots:  $\star$  (red),  $k_s/D = 3 \times 10^{-3}$ ;  $\blacktriangle$  (green),  $k_s/D = 1.9 \times 10^{-3}$ ;  $\bullet$  (blue),  $k_s/D = 1.4 \times 10^{-3}$ ;  $\blacklozenge$  (brown),  $k_s/D = 1.1 \times 10^{-3}$ .

investigated rough cylinders at Reynolds numbers up to  $O(10^7)$  and observed no evidence of the drag curve turning over as the Reynolds number was increased further towards the upper limit of those experiments.

On the other hand, the uncorrected drag coefficient,  $C_D$  in Achenbach (1971) for  $k_s/D = 9 \times 10^{-3}$  decreased from  $\sim 1.3$  at  $Re \sim 3 \times 10^6$  to  $\sim 1.2$  at  $Re \sim 3 \times 10^7$ . The magnitude of decrease in  $C_D$  was smaller in the blockage corrected drag coefficients,  $\sim 1.04$  at  $Re \sim 3 \times 10^6$  to  $\sim 1$  at  $Re \sim 3 \times 10^7$ . In addition, van Hinsberg (2015) found that for larger Reynolds numbers,  $C_D$ ,  $\sigma_{C_L}$ ,  $St$  and to a minor extent  $C_{P_b}$  and  $C_{P_m}$  reached a peak before decreasing slightly beyond  $Re \sim 10^6$ . However, these trends were found to occur only with the data obtained from the flow generated through the larger total pressures (i.e. higher Mach numbers).

For a fixed diameter, larger Reynolds numbers result in larger Mach numbers, thus increasing the effect of compressibility of the working fluid. Jones *et al.* (1969) investigated Reynolds numbers from  $0.4 \times 10^6$  to  $14 \times 10^6$  on smooth cylinders at Mach numbers ranging from 0.1 to 0.6.  $C_D$  vs  $Re$  for Mach numbers above  $M > 0.2$  showed the drag reaching a peak prior to remaining almost constant or decreasing slightly. Moreover, this behaviour was exacerbated on increasing Mach number. Thus, further investigations as  $Re \rightarrow O(10^7)$  over a range of Mach numbers are recommended to understand the influence of compressibility on different flow variables.

### 3.7. The roughness Reynolds number and curve collapse

In postcritical flows, the influence of an increase in roughness is similar to that of an increase in Reynolds number, i.e. the minimum pressure,  $C_{P_m}$ , increases while base pressure,  $C_{P_b}$ , and wake angle,  $\theta_w$ , decrease. From figure 6, it is also clear that  $C_{P_b}$  and  $\theta_w$  approach asymptotic values with increasing roughness similar to the variation with increasing Reynolds number. The peak in the fluctuating pressure distribution moves upstream and  $\sigma_{C_p}$  in the downstream region of the cylinder increases with increasing roughness, and these changes too are qualitatively similar to those that occur with

## Influence of surface roughness on postcritical flow

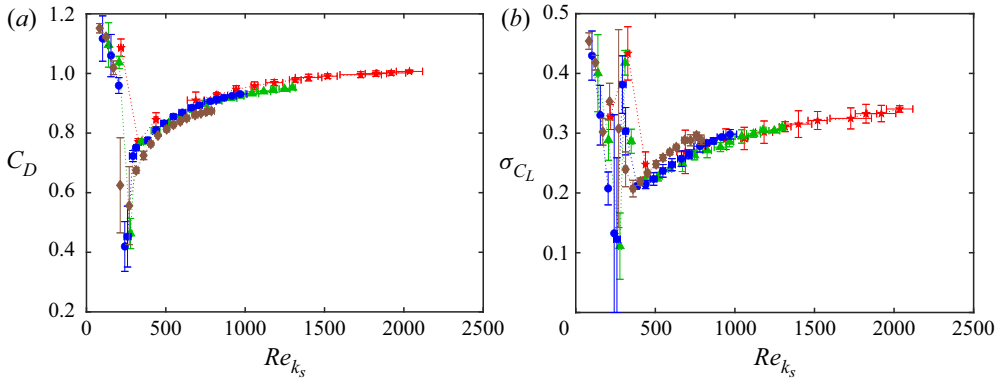


Figure 12. (a) Mean drag coefficient and (b) fluctuating lift coefficient as a function of Roughness Reynolds number:  $\cdots \star \cdots$  (red),  $k_s/D = 3 \times 10^{-3}$ ;  $\cdots \blacktriangle \cdots$  (green),  $k_s/D = 1.9 \times 10^{-3}$ ;  $\cdots \bullet \cdots$  (blue),  $k_s/D = 1.4 \times 10^{-3}$ ;  $\cdots \blacklozenge \cdots$  (brown),  $k_s/D = 1.1 \times 10^{-3}$ .

increasing Reynolds number. On the other hand, axial correlation length is  $\sim 4D$  in the postcritical regime for all roughnesses tested and, hence, the magnitude of spanwise correlation of forces is the same for all roughnesses tested in postcritical flow.

The variation in all the above-mentioned properties is similar whether it is a result of a change in Reynolds number or a change in roughness. This, combined with the difficulty of achieving postcritical Reynolds numbers while maintaining acceptable blockage and aspect ratio in most wind tunnels provides the support for the use of roughness to simulate the effect of postcritical flow proposed by Szechenyi (1975).

Figure 12 gives the variation of bulk properties, mean drag and fluctuating lift as functions of Roughness Reynolds number,  $Re_{k_s} = (\rho U_\infty k_s) / \mu = Re(k_s/D)$ . In postcritical flows and for a given Roughness Reynolds number, the maximum deviation from the mean  $C_D$  among different roughnesses tested is only  $\sim 3\%$ . While the corresponding difference in the fluctuating coefficient of lift is higher (within 10%), this is of the order of the experimental error in predictions of  $\sigma_{C_L}$  as seen in figure 12(b).

Figure 13 gives the mean and fluctuating pressure distribution from different roughnesses at varying Reynolds numbers but at the same Roughness Reynolds number of 720. It is evident that the differences in mean  $C_P$  are only in the acceleration region and  $C_P$  nearby and postseparation is similar for all roughnesses. In the fluctuating pressure distribution, the peak of  $\sigma_{C_P}$  occurs near the same azimuthal angle. However, quantitative differences in  $\sigma_{C_P}$  in the downstream region are larger than those in the mean  $C_P$  and these contribute towards the differences in  $\sigma_{C_L}$ .

Mean base pressure,  $C_{P_b}$ , wake angle,  $\theta_w$ , the total contribution of base  $C_P$  to the drag coefficient,  $C_{D_{base}}$ , and the contribution of upstream  $C_P$  to the drag coefficient,  $C_{D_{rest}}$ , Strouhal number,  $St$ , and the axial correlation length,  $\Lambda$ , are plotted against Roughness Reynolds number in figure 14. The wake angle, which, as described previously is representative of the separation angle in the postcritical regime, is fairly constant at a given Roughness Reynolds number irrespective of the roughness used to achieve it. As can be seen in figure 6(f),  $C_{P_b}$  is a strong function of  $\theta_w$ . Consequently, the base pressure and the contribution of the pressure distribution in the base region to drag is constant for a given Roughness Reynolds number. The contribution of the upstream region of drag is a function of roughness since flow in the acceleration region and  $C_{P_m}$  is a function of roughness even at large Reynolds numbers. However, this contribution is minor when

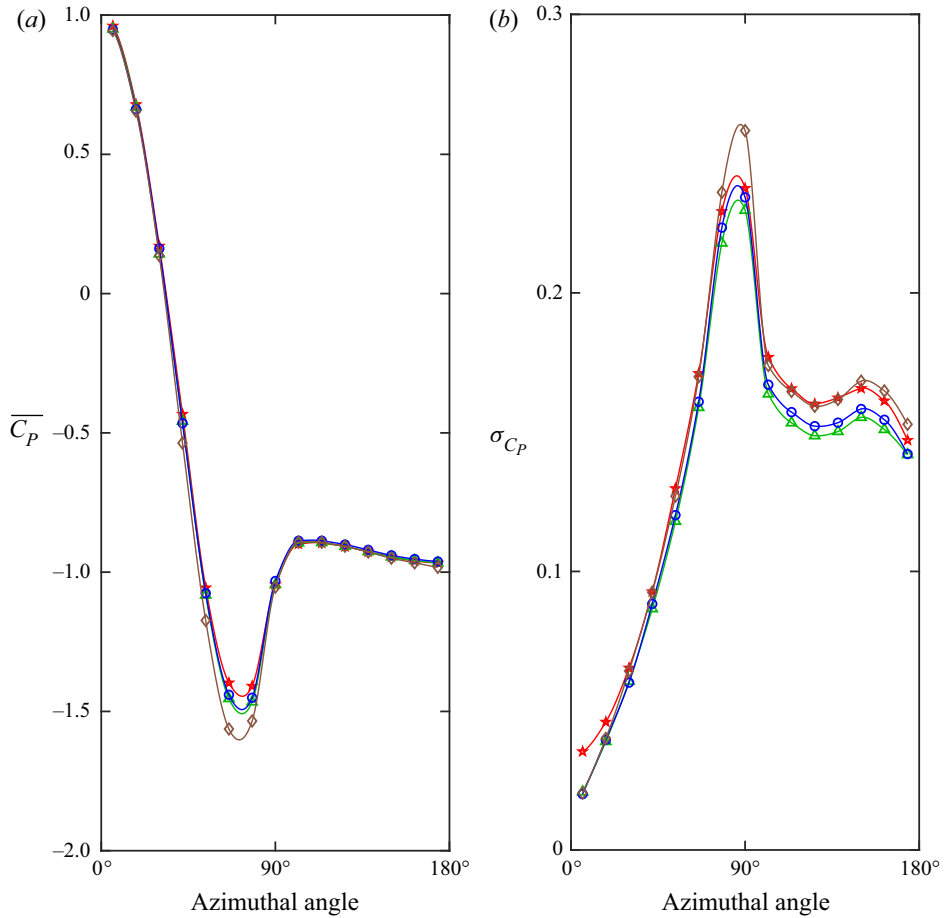


Figure 13. Circumferential distributions of (a) mean pressure and (b) fluctuating pressure coefficients at Roughness Reynolds number of 720:  $\star$  (red),  $k_s/D = 3 \times 10^{-3}$ ;  $\triangle$  (green),  $k_s/D = 1.9 \times 10^{-3}$ ;  $\circ$  (blue),  $k_s/D = 1.4 \times 10^{-3}$ ;  $\diamond$  (brown),  $k_s/D = 1.1 \times 10^{-3}$ .

compared with the contribution of base pressure, leading to only a minor deviation in the total drag coefficient.

Since the separation angle, drag coefficient and base pressure collapse with Roughness Reynolds number, it follows that wake parameters also collapse with roughness Reynolds number thereby leading to a collapse of the Strouhal number. Strouhal number found from spectral peaks of the lift fluctuations is plotted against roughness Reynolds number in figure 14(e).

To summarise, the mean drag coefficient and the Strouhal number collapse well when plotted against roughness Reynolds number, since wake ( $\sim$  separation) angle and consequently, base pressure and wake width collapse with roughness Reynolds number. On the other hand, the minimum pressure and the fluctuating lift coefficient are more sensitive to local flow perturbations and, hence, different roughness levels and Reynolds numbers lead to differences in  $\sigma_{C_L}$ .

However, the asymptotic values of the base pressure, and the separation angle in postcritical flows for smooth cylinders (from Güven *et al.* 1980) are different from those of rough cylinders in the current tests. A collapse of the drag coefficient with roughness



*Influence of surface roughness on postcritical flow*

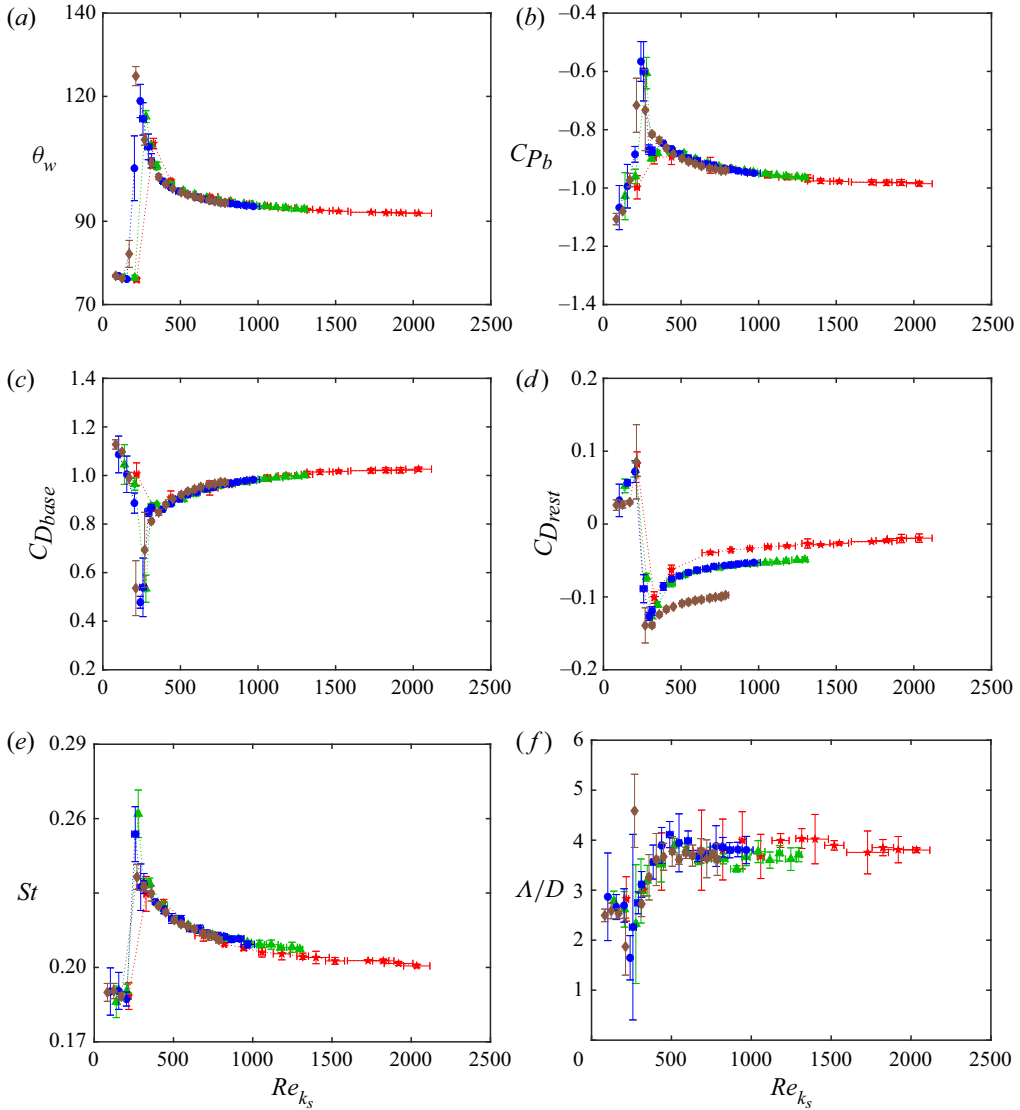


Figure 14. Variation of (a) wake angle, (b) base pressure coefficient, (c) base pressure contribution to drag, (d) upstream pressure contribution to drag, (e) Strouhal number and (f) axial correlation length of lift with roughness Reynolds number:  $\cdot \cdot \star \cdot \cdot$  (red),  $k_s/D = 3 \times 10^{-3}$ ;  $\cdot \cdot \blacktriangle \cdot \cdot$  (green),  $k_s/D = 1.9 \times 10^{-3}$ ;  $\cdot \cdot \bullet \cdot \cdot$  (blue),  $k_s/D = 1.4 \times 10^{-3}$ ;  $\cdot \cdot \blacklozenge \cdot \cdot$  (brown),  $k_s/D = 1.1 \times 10^{-3}$ .

Reynolds number that includes data from smooth cylinders is hence expected to have a larger deviation than found by Güven *et al.* (1980). Moreover, axial correlation lengths for smooth cylinders in postcritical flows are unknown and the similarity among spanwise flow over smooth and rough cylinders in postcritical flows is still uncertain.

Thus, the use of roughness Reynolds number is recommended in instances for which postcritical flow of a slightly rough cylinder is simulated using a rougher cylinder at a lower Reynolds number. Since the deviation in the collapse of  $C_D$  through roughness Reynolds number is only from the minimum pressure region, this collapse would work well if the

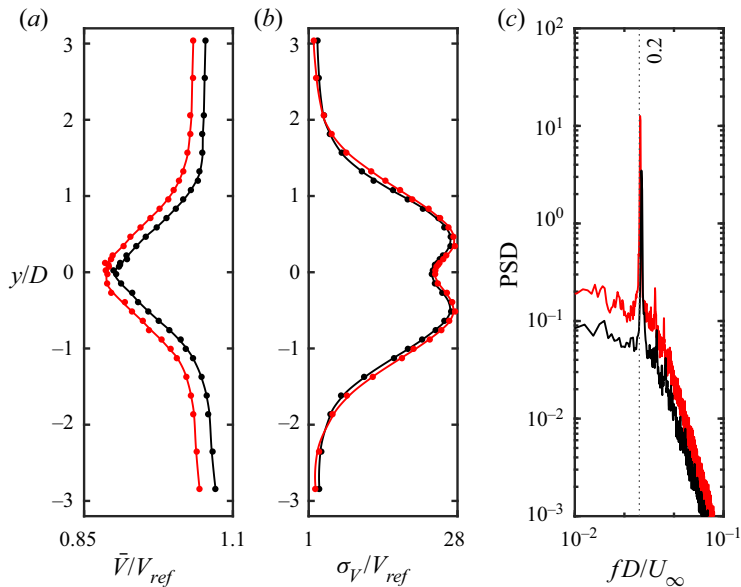


Figure 15. Mean, fluctuating in-plane velocity distribution and spectra of fluctuations at point of maximum  $\sigma_V$  in the wake of the cylinder of roughness,  $k_s/D = 3 \times 10^{-3}$ : black,  $Re \sim 1.9 \times 10^5$ ; red,  $Re \sim 3.8 \times 10^5$ .

roughnesses tested have similar characteristics and cause similar deceleration in the flow. This explains the collapse of  $C_D$  observed in Szechenyi (1975) (since all the roughnesses were generated from glass beads) and the current work, and the deviations observed when different roughnesses were taken into account in Güven *et al.* (1980). The comparisons in Güven *et al.* (1980) were also from flow set-ups of different geometrical properties and investigations that attempt the curve collapse while incorporating different kinds of roughness elements in the same set-up are required to further identify the limitations to the curve collapse using roughness Reynolds number.

Szechenyi (1975) proposed three thresholds in the roughness simulation i.e.  $Re_\delta \sim 200$  as the lower limit for the return of coherent vortex shedding,  $Re_\delta \sim 1000$  as the lower limit after which the coefficients of drag, lift and the Strouhal number remain fairly constant, and a relative roughness ratio of  $\delta/D = 2.2 \times 10^{-3}$  (where  $\delta$  is the diameter of the glass beads used to generate roughness) as the upper limit after which the coherence in vortex shedding breaks down. For the roughnesses tested in the current study, the lower limit of  $Re_{k_s}$  at which the coherent shedding reestablishes intermittently is  $\sim 300$  while the steady nature (i.e. less uncertainty in the estimates) of the fluctuating lift coefficient, Strouhal number and the axial correlation length of lift are established at  $\sim 500$ . As mentioned previously, increase in roughness in the current experiments stabilises vortex shedding rather than disrupting it and coherent vortex shedding is seen at all Reynolds numbers for relative roughness,  $k_s/D = 3.1 \times 10^{-3}$ . Moreover, Buresti (1981) found coherent shedding with relative roughness as high as  $k/D = 12 \times 10^{-3}$ . This discrepancy among the observations is probably due to the use of glass beads in Szechenyi (1975) to generate roughness while sandpaper was used in Buresti (1981) and the current experiments.

### 3.8. Intermediate wake measurements

Mean and fluctuating velocity profiles and the PSD of velocity at 3 diameters downstream of the 40 grit cylinder are given in figure 15. These are measured using a single-axis

### Influence of surface roughness on postcritical flow

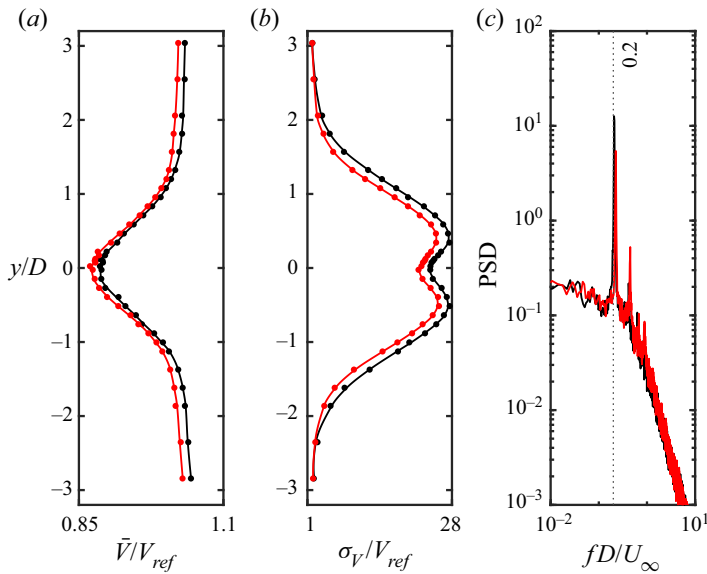


Figure 16. Mean, fluctuating in-plane velocity distribution and spectra of fluctuations at point of maximum  $\sigma_V$  in the wake at,  $Re \sim 3.8 \times 10^5$ : black,  $k_s/D = 3 \times 10^{-3}$ ; red,  $k_s/D = 1.1 \times 10^{-3}$ .

hot-film anemometer with its axis parallel to the axis of the cylinder and, hence, the hot wire is sensitive to the total in-plane velocity. This total in-plane velocity is denoted by  $V$  while  $y$  is the cross-stream distance from the centre of the cylinder.

For a fixed roughness, the mean velocity in the wake decreases with an increase in Reynolds number and this increase in the velocity deficit leads to higher drag. Fluctuations in the velocity increase with increasing Reynolds number, albeit at a lower rate than the mean velocity. Power spectra of the velocity fluctuations (at the point at which fluctuating velocity is maximum) show that energy content over the entire frequency range is higher for the larger Reynolds number tested. Similar to the observation from  $C_L$  spectra, Strouhal number of the dominant frequency decreases slightly as the Reynolds number increases in the postcritical regime.

Velocity profiles and spectra downstream of two different roughnesses but at the same Reynolds number are given in figure 16. The mean velocity deficit is similar for the two roughnesses while the wake width is slightly smaller for the lower roughness. Consistent with the fluctuating lift coefficient,  $\sigma_{C_L}$ , and the fluctuating coefficient of pressure,  $\sigma_{C_p}$ , the fluctuating velocity shows a clear increase with increasing roughness, indicating an increase in the turbulent kinetic energy. While the energy density over the broad frequency range is similar for both roughnesses, the dominant shedding frequency has more energy for the larger roughness, possibly because this roughness is well into the postcritical regime. This shows that at a given Reynolds number, energy fluctuations are larger for the larger roughness and that the increased energy is concentrated predominantly at the shedding frequency.

Wake profiles for the cylinders with relative roughness  $k_s/D = 1.9 \times 10^{-3}$  and  $k_s/D = 1.1 \times 10^{-3}$  through the near and intermediate wake are given in Appendix A.

### 3.9. Wake frequencies

Figure 17 shows the PSD of velocity fluctuations at different downstream locations in the wake for two roughnesses,  $k_s/D = 1.9 \times 10^{-3}$  and  $k_s/D = 1.1 \times 10^{-3}$ , at  $Re_{k_s} \sim 550$ . At

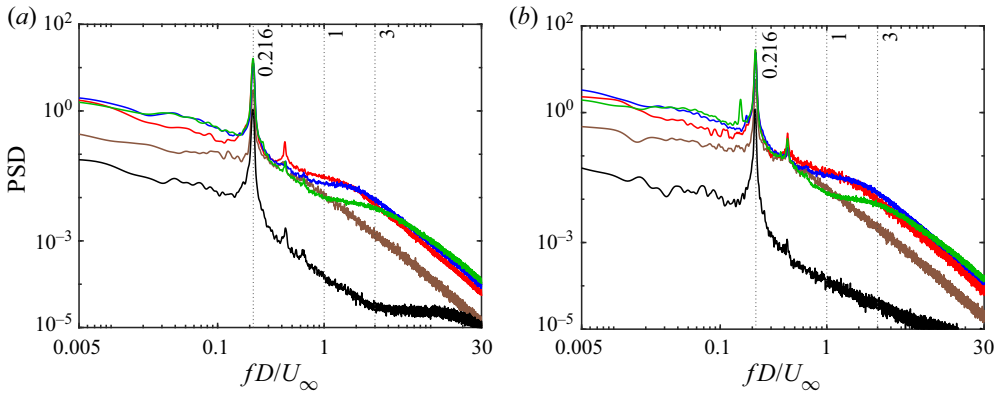


Figure 17. Frequency spectra of velocity fluctuations in the wake: (a)  $k_s/D = 1.9 \times 10^{-3}$  at  $Re = 2.9 \times 10^5$  and  $Re_{k_s} \approx 550$  and (b)  $k_s/D = 1.1 \times 10^{-3}$  at  $Re = 4.7 \times 10^5$  and  $Re_{k_s} \approx 510$ . Spectra obtained at the location of the cross-stream maximum velocity fluctuation at distance downstream from the centre of the cylinder: black,  $0D$ ; green,  $0.25D$ ; blue,  $0.5D$ ; red,  $1D$ ; brown,  $6D$ .

each of these locations, a spectrum is obtained from the time history of in-plane velocity at a point where  $\sigma_V/V_{ref}$  is maximum in the cross-stream profile. The energy in fluctuations before separation is lower than that postseparation over the entire frequency range. Spectra in the shear layer (at  $0.25D$ ,  $0.5D$  and  $1D$ ) have the highest energy among the measurement range and, moreover, a plateau in the PSD is found at higher frequencies of  $fD/U_\infty \sim 1-3$ . These frequencies also had a higher spatial amplification factor,  $\alpha$ , Khor, Sheridan & Hourigan (2011) confirming that these are the frequencies at which shear-layer instabilities grow fastest. Furthermore, as one moves downstream, the frequency in the shear layers reduces slightly indicating an increase in shear-layer thickness. A similar plateau is found at postcritical Reynolds numbers for smooth cylinders in Lehmkühl *et al.* (2014) and was considered a signature of Kelvin–Helmholtz frequencies,  $f_{KH}$ , in the shear layer. While shear-layer frequencies in the smooth cylinder follow the power law suggested by Prasad & Williamson (1997) ( $f_{KH}/f_V \propto Re^{0.67}$ ),  $f_{KH}$  in the current results is an order of magnitude lower for the same Reynolds number. For instance, at a Reynolds number of  $3.8 \times 10^5$ , the power law for smooth cylinders predicts  $f_{KH}D/U_\infty \sim 30$  while the current results show the broadband (non-dimensional) frequency between 1 and 3.

To further investigate this deviation, the momentum thickness of the shear layer,  $\Theta$  was computed according to the procedure outlined in Khor *et al.* (2011) and the results are tabulated in table 3. Since the hot-film anemometer measures the total in-plane velocity, current values of momentum thickness are expected to be only approximate. The key takeaway however, is that the momentum thickness of the shear layers for rough cylinders is an order of magnitude higher than that of the predicted smooth-cylinder momentum thickness,  $\sim 0.003$  at  $0.5D$  according to the correlation given in Khor *et al.* (2011) at the same Reynolds number. Since the boundary-layer thickness is larger on rough surfaces than on smooth surfaces for a given postcritical Reynolds number, separated shear layers are also expected to be thicker.

The empirical power-law correlations of  $f_{KH}$  and  $\Theta/D$  vs  $Re$  for smooth cylinder flows do not contain data from the postcritical regime where shear layers are expected to be turbulent and, hence, thicker than laminar shear layers. Moreover, as the Reynolds number increases, the energy content of the turbulent shear layer is expected to be distributed over a wider range of frequencies thereby making it difficult to discern an exact value for  $f_{KH}$ .

Relative roughness $k_s/D$	$Re$	$\Theta/D$ at $(x/D = 0.25)$	$\Theta/D$ at $(x/D = 0.5)$
$1.4 \times 10^{-3}$	$2.9 \times 10^5$	0.0092	0.0212
$1.4 \times 10^{-3}$	$3.8 \times 10^5$	0.0112	0.0239
$1.1 \times 10^{-3}$	$3.8 \times 10^5$	0.0068	0.0203
$1.1 \times 10^{-3}$	$4.7 \times 10^5$	0.0092	0.0224

Table 3. Momentum thickness ( $\Theta/D$ ) of the shear layer at different streamwise locations.

However, current predictions of momentum thickness agree remarkably well with those of shear-layer frequencies when the inverse proportionality between the frequency and shear-layer thickness is taken into account, i.e. while the frequency peak of shear layers is an order of magnitude smaller than that obtained from the smooth cylinder correlations, the momentum thickness is an order of magnitude larger.

#### 4. Conclusions

This study has drawn new insights into the aerodynamic behaviour of roughened cylinders at postcritical Reynolds numbers and elucidates prior inconsistencies in the literature for this flow. We have presented a new consistent, reliable and comprehensive data set that minimises the confounding effects of blockage and aspect ratios, and the variability they can induce. The motivation for this study was the variable and sometimes contradictory conclusions drawn from a number of previous studies on rough cylinders at high Reynolds numbers, making it difficult to use prior literature findings for reliable prediction and application.

This work consists of wind tunnel experiments covering the precritical to the postcritical Reynolds number range based on cylinders of aspect ratio,  $AR = 9.8$ , positioned in a wind tunnel with a blockage ratio of 5%. We quantify the key parameters of the flow such as the drag and lift coefficients, circumferential pressure distributions and their corresponding spectra, and the axial correlation length of lift for relative roughness ratios,  $1.1 \times 10^{-3} \leq k_s/D \leq 3 \times 10^{-3}$ , and Reynolds numbers,  $0.5 \times 10^5 \leq Re \leq 6.7 \times 10^5$ . Analysis of these results leads to several important findings.

- (i) Corrections to the influence of wind tunnel walls work well for the mean drag coefficient,  $C_D$ . The blockage corrected drag coefficients from the current work are in good agreement with those from Achenbach & Heinecke (1981), Güven *et al.* (1980) and van Hinsberg (2015) despite widely varying blockage. On the other hand, there is a non-negligible scatter in parameters like  $C_{P_b}$  and  $C_{P_m}$  across different investigations. This difference brings into question the applicability of blockage corrections to  $C_P$ , especially at large blockages ( $> 15\%$ ) found in Güven *et al.* (1980) and Achenbach & Heinecke (1981).
- (ii) Moreover, Eaddy (2019) found a large ( $\sim 30\%$ ) increase in  $\sigma_{C_L}$  for an increase in aspect ratio from 5.7 to 9 and blockage from 8% to 12%. The blockage corrections to the fluctuating coefficient have not been verified yet thereby necessitating the use of a low-blockage and high-aspect-ratio set-up in order to get accurate measurements of the fluctuating coefficient of lift.
- (iii) We present, for the first time, the fluctuating lift coefficient,  $\sigma_{C_L}$  for a range of surface roughnesses in the postcritical flows measured through a low-blockage and high-aspect-ratio set-up. Despite having similar geometrical parameters (and similar

$C_D$ ), current results of  $\sigma_{C_L}$  vary significantly from those of van Hinsberg (2015) for a similar relative roughness. Possible causes of the discrepancy are the differences in the method of generation of surface roughness, the measurement technique along with the influence of gaps between the tunnel walls and the cylinder present in the set-up of van Hinsberg (2015).

- (iv) This study reveals a strong correlation between the separation angle (quantified through wake angle,  $\theta_w$ ) and the base pressure (quantified through  $C_{P_b}$ ) of the cylinder in the postcritical regime that is consistent across the range of roughnesses tested. The wake angle,  $\theta_w$ , and, consequently, the base pressure,  $C_{P_b}$ , Strouhal number,  $St$ , and the coefficient of drag,  $C_D$ , decrease with increasing Reynolds number for a given roughness in the postcritical regime. On the other hand, minimum pressure,  $C_{P_m}$ , and the fluctuating coefficient of lift,  $\sigma_{C_L}$ , increase with increasing Reynolds number. These quantities eventually approach asymptotic values at large Reynolds numbers. Amongst these quantities, only the asymptotic value of  $C_{P_m}$  strongly depends on the degree of roughness and is not well described by only the roughness Reynolds number. Measurements by Jones *et al.* (1969) in a pressurised wind tunnel suggest that at very high Reynolds numbers the drag coefficient could reach a maximum and then decrease slightly as the Reynolds number is increased further. This might be caused by compressibility effects since the Mach number was not negligible in those experiments. Further studies at very large Reynolds numbers ( $\sim 10^7$ ) and different Mach numbers are recommended to understand the influence of compressibility better.
- (v) Time histories and frequency spectra of the lift signal revealed that intermittency of the LSB disturbs coherent vortex shedding and no dominant frequency exists in such a flow. This supercritical flow is found to occur over a smaller range of Reynolds number as roughness increases, i.e. coherent (spanwise) and periodic vortex shedding exists over a larger Reynolds number space for higher roughness. A distinct frequency in shedding is found at all the Reynolds numbers tested for a cylinder with relative surface roughness  $k_s/D \geq 1.9 \times 10^{-3}$  but not for  $k_s/D \leq 1.4 \times 10^{-3}$  at Reynolds numbers near  $Re \sim 2 \times 10^5$ .
- (vi) This study reports the axial correlation length of lift for a range of roughnesses at Reynolds numbers  $Re \leq 6.7 \times 10^5$ . In excellent agreement with the broad estimates of Buresti (1981), the axial correlation length,  $\Lambda$ , at  $Re \geq 3 \times 10^5$  is  $\sim 4D$  for all roughnesses tested. This length is significantly higher than that for smooth cylinders ( $\sim 1.5D$ ) at similar Reynolds numbers (King 1977) indicating an earlier reestablishment of coherent shedding with increasing roughness.
- (vii) This work also quantifies the velocity distributions in the wake of rough cylinders in postcritical flows. We show that the flow velocity at 3 diameters downstream of the cylinder decreases as the Reynolds number or roughness is increased. For a given Reynolds number, the velocity in the wake recovers faster in the streamwise direction for the lower roughness.
- (viii) Momentum thickness of the shear layer for rough cylinders is found to be  $\Theta/D \sim 0.02$  at  $0.5D$  downstream of the cylinder. This is an order of magnitude higher than that of empirically predicted momentum thickness for smooth cylinders at the same Reynolds numbers. Consequently, the frequencies in the shear layer postseparation for the rough cylinder are found at  $f_{KH}D/U_\infty \sim 2-3$ , an order of magnitude lower than the smooth cylinder predictions.
- (ix) Flow properties such as  $C_D$ ,  $\sigma_{C_L}$  and  $C_{P_b}$  are plotted against the roughness Reynolds number,  $Re_{k_s}$ , for different roughnesses, the conclusions are as follows.

## *Influence of surface roughness on postcritical flow*

- (a) Across the range of surface roughness tested, the wake angle follows a consistent trend with the roughness Reynolds number. The strong correlation between the wake angle and base pressure leads to a collapse of base pressure. This further results in a collapse of the Strouhal number and the drag coefficient with the roughness Reynolds number.
- (b) The deviation in the mean drag coefficient,  $C_D$ , among different roughnesses is within 3% of the mean  $C_D$  for the range of roughness Reynolds number considered. The major contribution for the  $C_D$  collapse is from the agreement of base pressure between different roughnesses at the same roughness Reynolds number (indicated by the collapse of  $C_{P_b}$  with  $Re_{k_s}$ ), while the deviation is due to differences in the degree of deceleration before separation caused by different roughness (represented by  $C_{P_m}$ ).
- (c) The correlation of  $\sigma_{C_L}$  with  $Re_{k_s}$  is more consistent (deviation within  $\sim 10\%$  of the mean of different roughness) than that reported by Szechenyi (1975) and Eaddy (2019). Geometrical differences in the set-ups used by those authors are the likely cause of this discrepancy. In particular, it appears that the low cylinder aspect ratio and high blockage ratio play an important role.
- (d) The similarity parameter to collapse variables in the current study is  $Re_{k_s} = Re(k_s/D)$ , which is different from the  $Re(k/D)^{0.6}$  dependence proposed by Nakamura & Tomonari (1982). Nakamura & Tomonari (1982) used polystyrene particles to generate the distributed roughness and adopted the roughness parameter,  $k$ , as the size of roughness particles, while the current study makes use of sandpaper and uses the equivalent sand-grain roughness,  $k_s$ . This difference could be the cause of the difference between the two similarity parameters, but this aspect should be investigated further.
- (e) Reynolds numbers of large engineered structures or even their components are often difficult to achieve in standard wind tunnels while maintaining acceptable blockage and aspect ratios. Current results support the argument that, with the knowledge of surface roughness and Reynolds numbers for real-life structures of interest, a similar flow scenario can be achieved in wind tunnels at a much lower Reynolds number provided that roughness is tuned to match the Roughness Reynolds number (within the range of  $Re \lesssim 7 \times 10^5$  and  $1.1 \times 10^{-3} \leq k_s/D \leq 3 \times 10^{-3}$ ).

**Funding.** This research was supported by the Australian Government through the Australian Research Council's Linkage Project program through project LP180100234.

**Declaration of interests.** The authors report no conflicts of interest.

### **Author ORCIDs.**

 Anil Pasam <https://orcid.org/0009-0007-4001-3817>;

 Mark C. Thompson <https://orcid.org/0000-0003-3473-2325>.

## **Appendix A**

### *A.1. Transfer function used for pressure measurements*

Figure 18 gives the transfer function calculated based on the length of the tube used to measure the pressure signal (from Bergh & Tijdeman 1965). This transfer function is applied to all the pressure measurements in the current investigation.

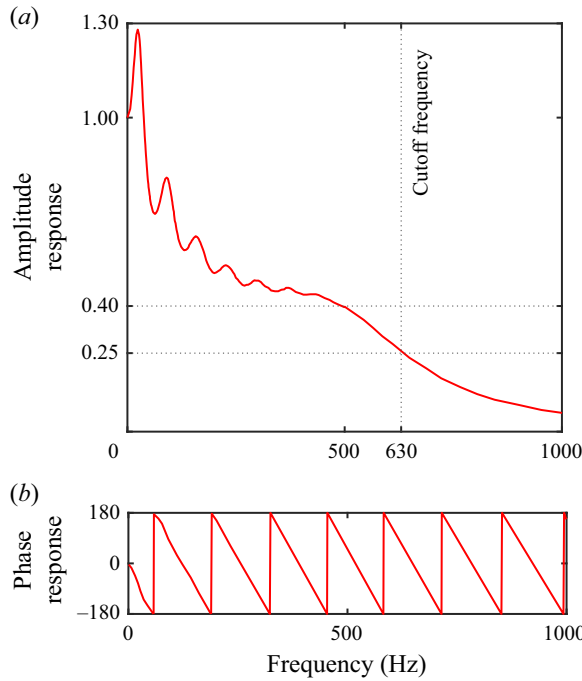


Figure 18. Transfer function applied to the pressure measurements.

A.2. Evaluating axial correlation length of lift

Figure 19 gives the coefficient of correlation between the fluctuations of lift coefficients measured at different spanwise locations. The correlation coefficient is defined as

$$R(s) = \frac{\Sigma \left( C_L(0) - \overline{C_L(0)} \right) \left( C_L(s) - \overline{C_L(s)} \right)}{\sqrt{\Sigma \left( C_L(0) - \overline{C_L(0)} \right)^2} \sqrt{\Sigma \left( C_L(s) - \overline{C_L(s)} \right)^2}}, \quad (A1)$$

where  $\Sigma$  indicates the summation over all the time steps in the duration of the test,  $\overline{C_L}$  indicates the time average lift coefficient in that test,  $C_L$  is the instantaneous lift coefficient at each time step,  $C_L(0)$  indicates the lift coefficient at the reference position (measurement plane at  $-1D$ ) and  $C_L(s)$  indicates the lift coefficient at the same instant measured at a spanwise separation of  $s$ .

A model function is fit over the discrete spanwise measurements in order to estimate the axial correlation length of lift. This function is given by (Norberg 2003)

$$R(s) = \alpha \exp\left(\frac{-s}{\Lambda_1}\right) + (1 - \alpha) \left(1 + \left(\frac{s}{C\Lambda_2}\right)^n\right)^{-1}, \quad (A2)$$

where  $C$  is  $\sin(\pi/n)/(\pi/n)$ .

For each of the tests, (A2) is used as a curve fit to the correlation functions by varying  $\alpha$ ,  $\Lambda_1$ ,  $\Lambda_2$  and  $n$ . The axial correlation length is then determined by

$$\Lambda = \alpha(\Lambda_1) + (1 - \alpha)(\Lambda_2). \quad (A3)$$



### Influence of surface roughness on postcritical flow

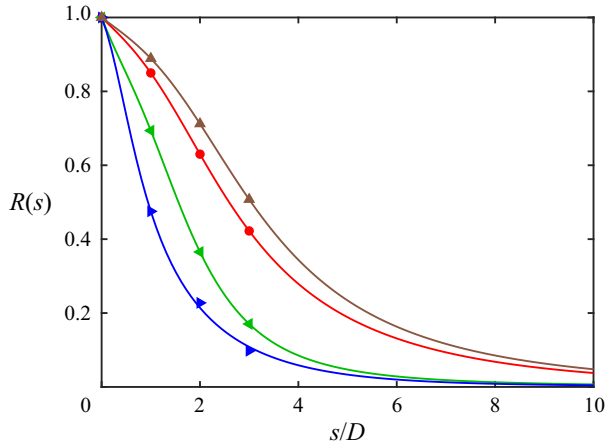


Figure 19. Correlation between the lift coefficients measured at different spanwise locations. Surface roughness,  $k_s/D = 1.4 \times 10^5$ . Reynolds numbers: red,  $Re \sim 0.7 \times 10^5$ ; green,  $Re \sim 1.64 \times 10^5$  (same test as figure 8); blue,  $Re \sim 1.68 \times 10^5$ ; brown,  $Re \sim 6.4 \times 10^5$ .

For instance, the correlation lengths for the distributions in the figure 19 are 3.4, 1.9, 1.4 and 3.8, respectively, for the range of Reynolds number provided. Figure 19 also highlights the uncertainty associated with the flow configuration in the supercritical regime.

#### A.3. Evaluating the Bearman number

From the potential flow model, the vortex drag coefficient,  $C_{D_s}$ , caused by two rows of staggered point vortices is given by (Milne-Thomson 1962; Bearman 1967)

$$C_{D_s} = \frac{4}{\pi} \left( \frac{U_{Vf}}{U_\infty} \right)^2 \left[ \coth^2 \frac{\pi b}{a} + \left( \frac{U_\infty}{U_{Vf}} - 2 \right) \frac{\pi b}{a} \coth \frac{\pi b}{a} \right], \quad (A4)$$

where

$$C_{D_s} = \frac{D_s}{0.5 \rho U_\infty^2 a}. \quad (A5)$$

Here,  $a$  and  $b$  are the longitudinal (streamwise) and lateral (cross-stream) separations of the vortices, and  $U_\infty = U_{Vb} + U_{Vf}$  is the freestream velocity, with  $U_{Vb}$  and  $U_{Vf}$  the velocity vortices relative to the body and the velocity of vortices relative to the freestream at the separation point, respectively. Finally,  $D_s$  is the drag due to the vortex wake.

From (A5),

$$C_{D_s} a = C_D D \Rightarrow C_{D_s} \frac{U_{Vb}}{f} = C_D D \Rightarrow C_{D_s} \frac{U_{Vb}}{U_\infty} = C_D \frac{fD}{U_\infty} \Rightarrow C_{D_s} \frac{U_{Vb}}{U_\infty} = C_D St. \quad (A6)$$

Again, noting that  $U_\infty = U_{Vb} + U_{Vf}$  gives

$$C_{D_s} \left( 1 - \frac{U_{Vf}}{U_\infty} \right) = C_D St. \quad (A7)$$

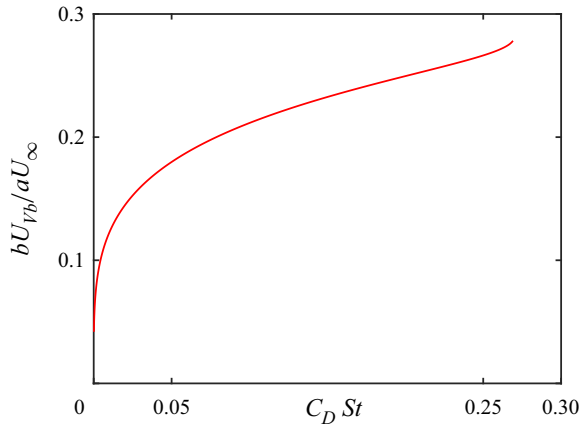


Figure 20. Relationship between  $bU_{Vb}/aU_{\infty}$  and  $C_D St$  evaluated from Kronauer’s stability criterion.

Kronauer’s stability criterion states that for a given  $U_{Vf}/U_{\infty}$ , the aspect ratio,  $b/a$ , of the resulting vortex street is the one that corresponds to the minimum drag coefficient,

$$\frac{\partial C_{D_s}}{\partial \left(\frac{b}{a}\right)} = 0. \tag{A8}$$

From (A4) and (A8),

$$2 \cosh \frac{\pi b}{a} = \left(\frac{U_{\infty}}{U_{Vf}} - 2\right) \sinh \frac{\pi b}{a} \left(\cosh \frac{\pi b}{a} \sinh \frac{\pi b}{a} - \frac{\pi b}{a}\right). \tag{A9}$$

To summarise, there are three unknown parameters so far,  $C_{D_s}$ ,  $U_{Vf}$  and  $(b/a)$ . Equations (A4), (A7) and (A9) can be used to solve for these unknowns. Given the product  $C_D St$ ,  $bU_{Vb}/aU_{\infty}$  can thus be estimated. The variation of  $bU_{Vb}/aU_{\infty}$  for a range of  $C_D St$  is given in figure 20 and this relationship forms (3.6).

$$\frac{b U_{Vb}}{a U_{\infty}} = F(C_D St). \tag{A10}$$

Thus, measurements of  $C_D$ ,  $St$  and  $C_{P_b}$  can be used to evaluate the Bearman number  $St_B$ ,

$$St_B = \frac{b U_{Vb}}{a U_{\infty}} \frac{1}{(1 - C_{P_b})^{0.5}}. \tag{A11}$$

#### A.4. Velocity profiles in the wake of rough cylinders

Figures 21(a) and 21(b) give the mean and fluctuating velocity profiles, respectively, for the  $k_s/D = 1.9 \times 10^{-3}$  cylinder near the cylinder surface. While mean velocity increases steeply through the shear layer, fluctuating velocity increases, reaches a peak and then decreases. Since the difference in Reynolds number is small, there appears to be no significant difference in the mean and fluctuating velocity profiles between the two Reynolds numbers. The shear layer is slightly wider for the higher Reynolds number indicating that there was longer growth and, hence, slightly earlier separation. This observation is supported by the trends observed in  $\theta_w$  previously.

*Influence of surface roughness on postcritical flow*

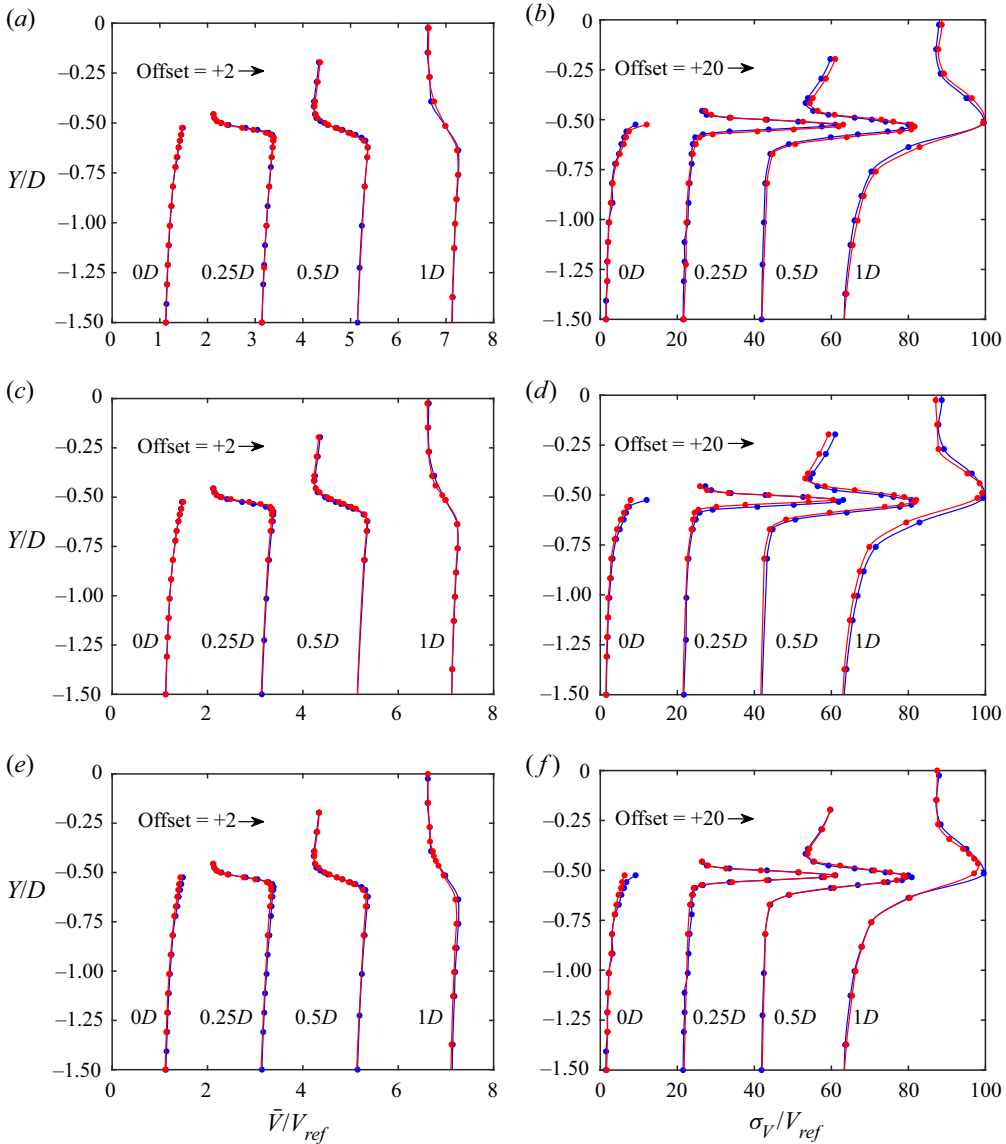


Figure 21. Mean and fluctuating velocity profiles downstream of the cylinder: (a,b)  $k_s/D = 1.9 \times 10^{-3}$  at (blue)  $Re = 2.9 \times 10^5$  and (red)  $Re = 3.8 \times 10^5$ ; (c,d)  $Re \sim 3.8 \times 10^5$  over (blue)  $k_s/D = 1.9 \times 10^{-3}$  and (red)  $k_s/D = 1.1 \times 10^{-3}$ ; and (e,f)  $Re_{k_s} \sim 530$  over (blue)  $k_s/D = 1.9 \times 10^{-3}$  and (red)  $k_s/D = 1.1 \times 10^{-3}$ .

Figures 21(c) and 21(d) give the mean and fluctuating profiles, respectively, at the same Reynolds number for varying roughness. For a given Reynolds number, an increase in roughness leads to a wider shear layer (ascertained from the fluctuating velocity profile) at the same position downstream of the cylinder, implying an earlier separation for larger roughness. Figures 21(e) and 21(f) give the mean and fluctuating profiles, respectively, at the same roughness Reynolds number for varying roughness. It is evident that the velocity profiles are very similar, further reinforcing that the separation angle and, hence, the shear layer and wake width close to the body of the cylinder postseparation is similar for different

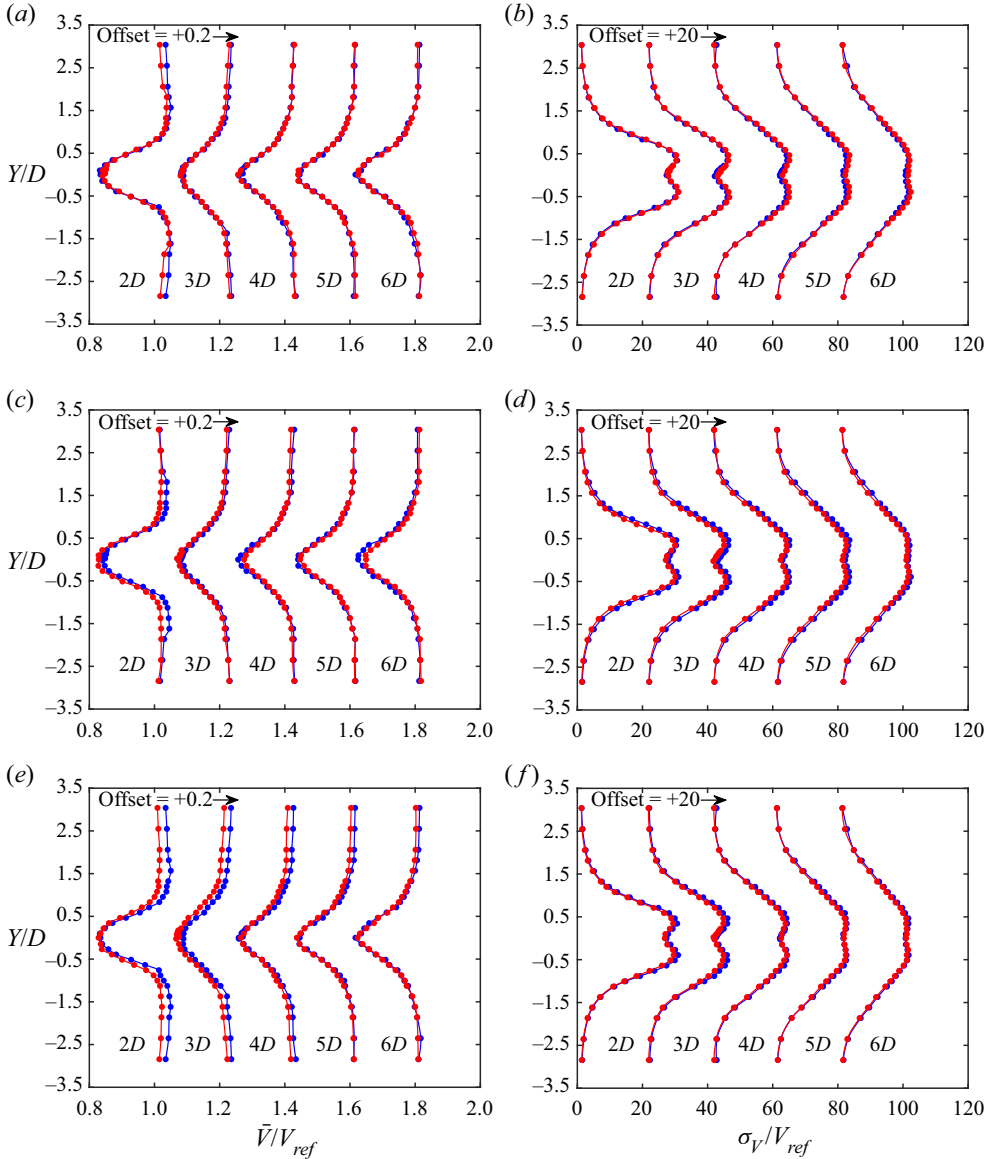


Figure 22. Mean and fluctuating velocity profiles downstream of the cylinder: (a,b)  $k_s/D = 1.9 \times 10^{-3}$  at (blue)  $Re = 2.9 \times 10^5$  and (red)  $Re = 3.8 \times 10^5$ ; (c,d)  $Re \sim 3.8 \times 10^5$  over (blue)  $k_s/D = 1.9 \times 10^{-3}$  and (red)  $k_s/D = 1.1 \times 10^{-3}$ ; and (e,f)  $Re_{k_s} \sim 530$  over (blue)  $k_s/D = 1.9 \times 10^{-3}$  and (red)  $k_s/D = 1.1 \times 10^{-3}$ .

roughness at the same roughness Reynolds number. This also agrees with the previous observation of the Strouhal number collapse with roughness Reynolds number.

Figure 22 gives the velocity distribution further downstream of the cylinder. As mentioned previously, owing to the limited range of Reynolds numbers tested for a given roughness, the differences in the wake between the two Reynolds numbers are almost insignificant. On the other hand, for a given Reynolds number, a decrease in roughness increases the velocity deficit and wake width close to the cylinder. This velocity deficit also diffuses faster for the smoother cylinder than that of the rougher cylinder indicating a

faster velocity recovery. Mean and fluctuating velocity profiles are similar for different roughnesses at the same roughness Reynolds number, especially beyond 3 diameters downstream of the cylinder.

REFERENCES

- ACHENBACH, E. 1971 Influence of surface roughness on the cross-flow around a circular cylinder. *J. Fluid Mech.* **46** (2), 321–335.
- ACHENBACH, E. & HEINECKE, E. 1981 On vortex shedding from smooth and rough cylinders in the range of Reynolds numbers  $6 \times 10^3$  to  $5 \times 10^6$ . *J. Fluid Mech.* **109**, 239–251.
- ADACHI, T. 1997 Effects of surface roughness on the universal Strouhal number over the wide Reynolds number range. *J. Wind Engng Ind. Aerodyn.* **69–71**, 399–412.
- AKIMA, H. 1970 A new method of interpolation and smooth curve fitting based on local procedures. *J. ACM* **17** (4), 589–602.
- ALLEN, H.J. & VINCENTI, W.G. 1944 Wall interference in a two-dimensional-flow wind tunnel, with consideration for the effect of compressibility. *NACA Tech. Rep.* 782.
- AUBERTINE, C.D., EATON, J.K. & SONG, S. 2004 Parameters controlling roughness effects in a separating boundary layer. *Intl J. Heat Fluid Flow* **25** (3), 444–450, turbulence and Shear Flow Phenomena (TSFP-3).
- BASU, R.I. 1985 Aerodynamic forces on structures of circular cross-section. Part I. Model-scale data obtained under two-dimensional conditions in low-turbulence streams. *J. Wind Engng Ind. Aerodyn.* **21** (3), 273–294.
- BATHAM, J.P. 1973 Pressure distributions on circular cylinders at critical Reynolds numbers. *J. Fluid Mech.* **57** (2), 209–228.
- BEARMAN, P.W. 1967 On vortex street wakes. *J. Fluid Mech.* **28** (4), 625–641.
- BEARMAN, P.W. 1969 On vortex shedding from a circular cylinder in the critical Reynolds number regime. *J. Fluid Mech.* **37** (3), 577–585.
- BEARMAN, P.W. 1984 Vortex shedding from oscillating bluff bodies. *Annu. Rev. Fluid Mech.* **16**, 195–222.
- BERGH, H. & TIJDEMAN, H. 1965 Theoretical and experimental results for the dynamic response of pressure measuring systems. *Tech. Rep.* NLR-TR F.238. Nationaal Lucht- en Ruimtevaartlaboratorium.
- BLACKBURN, H.M. & MELBOURNE, W.H. 1996 The effect of free-stream turbulence on sectional lift forces on a circular cylinder. *J. Fluid Mech.* **306**, 267–292.
- BRUNN, H.H. 1995 *Hot-Wire Anemometry*. Oxford University Press.
- BURESTI, G. 1981 The effect of surface roughness on the flow regime around circular cylinders. *J. Wind Engng Ind. Aerodyn.* **8** (1), 105–114.
- CADOT, O., DESAI, A., MITTAL, S., SAXENA, S. & CHANDRA, B. 2015 Statistics and dynamics of the boundary layer reattachments during the drag crisis transitions of a circular cylinder. *Phys. Fluids* **27** (1), 014101.
- CHAKROUN, W.M., RAHMAN, A.A.A. & QUADRI, M.M.A. 1997 The effect of surface roughness on flow around a circular cylinder. *Wind Engng* **21** (1), 1–12.
- CHEUNG, J. & MELBOURNE, W. 1995 Effects of surface roughness on a circular cylinder in supercritical turbulent flow. In *Twelfth Australasian Fluid Mechanics Conference*, pp. 25–28.
- DUARTE RIBEIRO, J. 1992 Fluctuating lift and its spanwise correlation on a circular cylinder in a smooth and in a turbulent flow: a critical review. *J. Wind Engng Ind. Aerodyn.* **40** (2), 179–198.
- EADDY, M. 2019 Lift forces on smooth and rough circular cylinders in low and high turbulence flows. PhD thesis, Monash University, Australia.
- FAGE, A. & WARSAP, J.H. 1929 The effects of turbulence and surface roughness on the drag of a circular cylinder. Reports and Memoranda No. 1283, Aeronautical Research Committee.
- FARELL, C., CARRASQUEL, S., GÜVEN, O. & PATEL, V.C. 1977 Effect of wind-tunnel walls on the flow past circular cylinders and cooling tower models. *Trans. ASME J. Fluids Engng* **99** (3), 470–479.
- FOX, T.A. & WEST, G.S. 1990 On the use of end plates with circular cylinders. *Exp. Fluids* **9** (4), 237–239.
- FREYMUTH, P. 1977 Frequency response and electronic testing for constant-temperature hot-wire anemometers. *J. Phys. E: Sci. Instrum.* **10** (7), 705.
- GÜVEN, O., FARELL, C. & PATEL, V.C. 1980 Surface-roughness effects on the mean flow past circular cylinders. *J. Fluid Mech.* **98** (4), 673–701.
- VAN HINSBERG, N.P. 2015 The Reynolds number dependency of the steady and unsteady loading on a slightly rough circular cylinder: from subcritical up to high transcritical flow state. *J. Fluids Struct.* **55**, 526–539.
- JONES, G.W., CINCOTTA, J.J. & WALKER, R.W. 1969 Aerodynamic forces on a stationary and oscillating circular cylinder at high Reynolds numbers. *NASA Tech. Rep.* R-300.
- KACKER, S.C., PENNINGTON, B. & HILL, R.S. 1974 Fluctuating lift coefficient for a circular cylinder in cross flows. *J. Mech. Engng Sci.* **16** (4), 215–224.

## A. Pasam and others

- KHOR, M., SHERIDAN, J. & HOURIGAN, K. 2011 Power-spectral density estimate of the Bloor–Gerrard instability in flows around circular cylinders. *Exp. Fluids* **50** (3), 527–534.
- KING, R. 1977 A review of vortex shedding research and its application. *Ocean Engng* **4** (3), 141–171.
- LEHMKUHL, O., RODRÍGUEZ, I., BORRELL, R., CHIVA, J. & OLIVA, A. 2014 Unsteady forces on a circular cylinder at critical Reynolds numbers. *Phys. Fluids* **26** (12), 125110.
- MILNE-THOMSON, L.M. 1962 *Theoretical Hydromechanics*. Macmillan.
- NAKAMURA, Y. & TOMONARI, Y. 1982 The effects of surface roughness on the flow past circular cylinders at high Reynolds numbers. *J. Fluid Mech.* **123**, 363–378.
- NIKURDASE, J. 1933 Stromungsgesetze in rauhen rohren. *Forsch. Ing. Wes.* **361**.
- NORBERG, C. 2003 Fluctuating lift on a circular cylinder: review and new measurements. *J. Fluids Struct.* **17** (1), 57–96.
- PRASAD, A. & WILLIAMSON, C.H.K. 1997 The instability of the shear layer separating from a bluff body. *J. Fluid Mech.* **333**, 375–402.
- RIBEIRO, J.L.D. 1991 Effects of surface-roughness on the 2-dimensional flow past circular cylinders 1. Mean forces and pressures. *J. Wind Engng Ind. Aerodyn.* **37** (3), 299–309.
- ROSHKO, A. 1954 On the drag and shedding frequency of two-dimensional bluff bodies. *Nat. Adv. Comm. Aero. Wash. Tech. Note*, no. TN-3169.
- ROSHKO, A. 1961 Experiments on the flow past a circular cylinder at very high Reynolds number. *J. Fluid Mech.* **10** (3), 345–356.
- SCHEWE, G. 1983 On the force fluctuations acting on a circular cylinder in crossflow from subcritical up to transcritical Reynolds numbers. *J. Fluid Mech.* **133**, 265–285.
- SHIH, W.C.L., WANG, C., COLES, D. & ROSHKO, A. 1993 Experiments on flow past rough circular cylinders at large Reynolds numbers. *J. Wind Engng Ind. Aerodyn.* **49** (1–3), 351–368.
- SONG, S. & EATON, J. 2002 The effects of wall roughness on the separated flow over a smoothly contoured ramp. *Exp. Fluids* **33** (1), 38–46.
- SPEIDEL, L. 1954 Einfluß der oberflächenrauigkeit auf die strömungsverluste in ebenen schaufelgittern. *Forsh. Geb. IngWes.* **20** (5), 129–140.
- STANSBY, P.K. 1974 The effects of end plates on the base pressure coefficient of a circular cylinder. *Aeronaut. J.* **78** (757), 36–37.
- SZECHENYI, E. 1975 Supercritical Reynolds number simulation for two-dimensional flow over circular cylinders. *J. Fluid Mech.* **70** (3), 529–542.
- THOMPSON, M.C. & HOURIGAN, K. 2005 The shear layer instability of a circular cylinder wake. *Phys. Fluids* **17** (2), 021702.
- ZAN, S.J. & MATSUDA, K. 2002 Steady and unsteady loading on a roughened circular cylinder at Reynolds numbers up to 900 000. *J. Wind Engng Ind. Aerodyn.* **90** (4), 567–581.