MISLEADING DYE VISUALIZATION NEAR A 3D STAGNATION POINT WITH APPLICATIONS TO THE VORTEX BREAKDOWN BUBBLE

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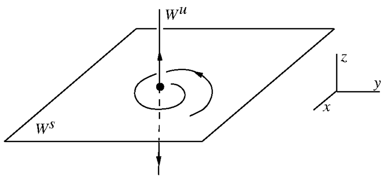
Summary An analytical model, based on the Fokker-Planck equation, is constructed of the dye visualization expected near a three-dimensional stagnation point in a swirling fluid flow. The model is found to predict dye traces that oscillate in density and position in the meridional plane in which swirling flows are typically visualized. Predictions based on the model for the steady vortex breakdown bubble are compared with computational fluid dynamics predictions and experimental observations. It is shown that even for a perfectly axisymmetric flow and breakdown bubble, the combined effect of dye diffusion and the inevitable small errors in the dye injection position lead to the false perception of an open bubble structure with folds near the lower stagnation point. The asymmetries in the predicted flow structures can be remarkably similar to those observed in flow observations and computational predictions with geometric asymmetries of the rig. Thus, when interpreting dye visualization patterns in steady flow, even if axisymmetric flow can be achieved, it is important to take into account the relative diffusivity of the dye and the accuracy of its injection.

INTRODUCTION

It is well known that the use of tracers such as dye in flow visualisation can lead to misleading observations in the case of *unsteady* flows. Although the fallacies associated with streaklines in unsteady flows have been well documented, misleading visualization involving *steady* flows are less common; it is generally assumed that they provide an accurate depiction of the flow structures. In steady flows, stagnation points are known to lead to chaotic regions in which the trajectories of tracing particles separate exponentially fast. The local flow close to a stagnation point is well understood, and depends essentially on the eigenvalues of the linearization of the velocity field. Complex flow patterns arise when the global structure of the velocity field includes a mechanism for re-injecting fluid particles which move away from a hyperbolic stagnation point back into a neighbourhood of the point [2]. Here we demonstrate how, through a combination of mass diffusion of the tracer fluid and the error in the tracer injection location, not only can "asymmetry" and "open" structure appear for a steady purely axisymmetric flow involving vortex breakdown, but the structures can be appear remarkably similar to those observed experimentally [5, 3, 4].

AN ANALYTICAL MODEL FOR DYE DISTRIBUTION NEAR A STAGNATION POINT

To explore the distribution of dye at the downstream part of a steady vortex breakdown bubble, we study the flow in the vicinity of the stagnation point that constitutes the end-point of the bubble on the cylinder axis. We define an xyz coordinate system with origin at the stagnation point and the z-axis along the cylinder axis and consider the linearization $\mathbf{v}(\mathbf{r}) = \mathbf{A}\mathbf{r}$, where $\mathbf{A}$ is the Jacobian matrix of the full velocity field. Assuming the flow has axial symmetry, we have



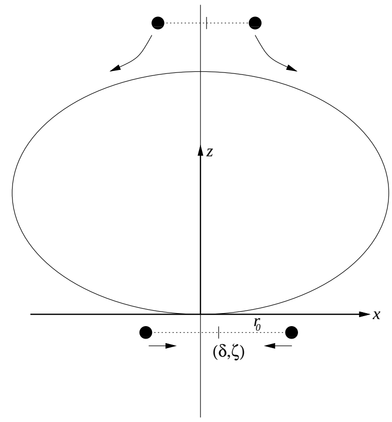
$$\mathbf{A} = \begin{pmatrix} -\alpha & \beta & 0 \\ -\beta & -\alpha & 0 \\ 0 & 0 & 2\alpha \end{pmatrix}, \tag{1}$$

where $\alpha = -\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$, $\beta = \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are derivatives evaluated at the stagnation point. With $\alpha > 0$, the xy plane is the stable manifold W^s of the stagnation point, consisting of streamlines which spiral around the axis with angular velocity β (swirl) and are attracted exponentially with a rate α to the stagnation point. The surface W^s is the linear approximation of the lower part of the bubble surface. Streamlines starting off W^s will also perform a spiralling motion, but will be repelled from W^s along the unstable manifold W^u, the z-axis. The probability density for a particle is governed by the Fokker-Planck equation

$$\frac{\partial p}{\partial t} = -\left[\frac{\partial}{\partial x}((-\alpha x + \beta y)p) + \frac{\partial}{\partial y}((-\beta x - \alpha y)p) + \frac{\partial}{\partial z}(2\alpha z p)\right] + \frac{\epsilon^2}{2}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right). \tag{2}$$

The Schmidt number is the ratio of the diffusivities of the fluid and the dye, i.e., $Sc = \frac{\nu}{\epsilon^2/2}$. The probability density of a single dye particle is given by the solutions $p(\mathbf{r}, t)$ to Eq. (2). To obtain the total dye density, this must be integrated over all initial conditions and all release times. Assuming that dye which is originally released slightly off-axis above the breakdown bubble reach the lower part of the bubble at slightly displaced circles $\mathbf{r}_c = (r_0 \cos\theta + \delta, r_0 \sin\theta, \zeta), \theta \in [0, 2\pi[$, an asymptotic expansion for the dye stationary density $\rho(x, y, z)$ can be obtained. For a line along the lower part of the bubble one obtains for $\rho = \rho(x, 0, 0)$, for $x \neq 0$ and ϵ, δ, ζ small

$$\rho = \frac{1}{\epsilon}\sqrt{\frac{2}{\pi\alpha}}\frac{1}{\sqrt{r_0^4 - x^4}}\left[1 + \frac{\text{sign}(x)}{r_0}\left\{\frac{r_0^4 + x^4}{r_0^4 - x^4}\cos\left[\frac{\beta}{\alpha}\ln\left(\frac{|x|}{r_0}\right)\right] + \frac{\beta}{\alpha}\sin\left[\frac{\beta}{\alpha}\ln\left(\frac{|x|}{r_0}\right)\right]\right\}\delta - \frac{2r_0^4\alpha}{r_0^4 - x^4}\left(\frac{\zeta}{\epsilon}\right)^2\right] \tag{3}$$



If particles are released symmetrically with respect to the cylinder axis, i.e., $\delta = 0$, the equilibrium density also is symmetric. However, if $\delta \neq 0$, a pattern of high and low dye densities occurs. If x/r_0 is not too large such that

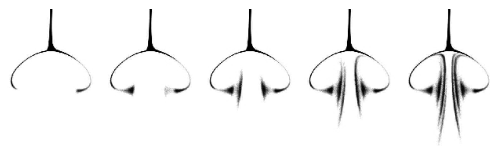
$$\frac{r_0^4 + x^4}{r_0^4 - x^4} \approx 1, \tag{4}$$

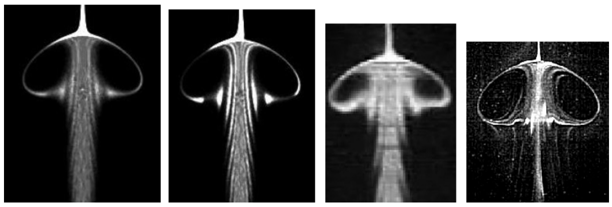
it is easy to see there is a sequence of local maxima $x_n \to 0$ of ρ which form a geometric progression,

$$x_{n+1} = \exp\left(-2\pi\frac{\alpha}{\beta}\right)x_n, \tag{5}$$

and, correspondingly, a sequence of minima in-between the maxima with the same distribution.

NUMERICAL STUDIES OF DYE TRACES AND EXPERIMENTAL COMPARISONS





The flow field was found numerically for a Reynolds number of 1850 and an aspect $h/D = 1.75$. A recirculation bubble is found on the central axis, similar in general shape to that observed experimentally except that it has a closed form and is constrained to be purely axisymmetric. To the left, top row, is shown successive stages of tracer particles approaching the lower stagnation point of the bubble. The generation of the density variation predicted analytically is clearly seen.
Simulation with offset = $0.00125D$ (left) and $0.00375D$ (second left) are shown in the second row. Dye visualisations from [5] and [4] (right and second right) show the apparently open bubble. Note the similarity of the numerically found dye structures and those seen experimentally.

The theoretical result (3) predicts an infinite number of oscillations in the dye density as the stagnation point is approached. This is feature introduced by the asymptotics used to derive the solution, which breaks down for the very long flow times encountered by dye which comest very close to the stagnation point. However, the geometric progression (5) is expected to hold for the outermost folds. This is confirmed from a numerical determination of ρ shown in the graph to the left. From this one estimates from (5) that $\alpha/\beta = 0.105$ with a standard deviation of 0.0072. From the velocity field we find directly $\alpha/\beta = 0.091$, in reasonable agreement.

CONCLUSIONS

Experimental observations of asymmetric and open vortex breakdown bubbles can be reasonably reproduced by computational simulation even in a purely axisymmetric flow field. An analytical model, based on the Fokker-Planck equation, is found to predict well the density variations of the dye visualization expected near the downstream three-dimensional stagnation point of the vortex breakdown bubble. Imperfections in rigs have been shown previously to lead to asymmetrical and open vortex breakdown patterns, and may well be the dominant influence in many cases [1]. However, the flow structures perceived using dye visualization for flows that contain an hyperbolic point are strongly influenced by the relative diffusivity of the tracing medium and inevitable inaccuracy in its injection. The main conclusion from the current study is that even if a perfect experimental rig could be constructed, visualization techniques using dye will likely result in the observation of asymmetrical and open vortex breakdown bubbles. This conclusion can be logically extended to other flows with hyperbolic critical points which will cause substantial amplification of perturbations as dye advects towards the stagnation point.

References

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