in a medium. The appropriate form of $D_{\mu\upsilon}(k)$ for a realistic electron distribution in the present context can be included simply by inserting it in (7).

The same procedure as described above can be used to calculate electron-proton scattering cross-sections. These calculations are in progress and will be presented in subsequent papers.

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Rapidly Rotating Core-Collapse Models

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Abstract: Very few (if any at all) three dimensional models of the final evolutionary stages of a star's life have appeared in the literature. Such models may be important if the stellar core maintains sufficient rotational energy during it's lifetime so that when collapse finally occurs, the increase of rotational energy to gravitational energy, may lead to a non-axisymmetric instability.

A sequence of core collapse models with decreasing rotation rate is considered. These models were calculated using a three dimensional, post-Newtonian, hydrodynamical, numerical code. The results show that for high precollapse rotational energies the core can become unstable resulting in the formation of what resemble 'spiral arms'. Unfortunately, because of limits on computer time, the calculations had to be discontinued shortly after this development occurred.

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At present little is known on the role that rotation plays during the dynamical collapse phase at the end of a massive (i.e. 8 $M_{\odot} < M_{ZAMS} < 70\ M_{\odot}$) star's life. During this final stage the core collapses through approximately 2 orders of magnitude in radius. This causes α, the ratio of rotational to gravitational potential energy, to increase so that angular momentum will be conserved. If the precollapse value is large enough then the subsequent increase will lead to important dynamical effects.

An indication of what can occur comes from the classical study of uniformly-rotating homogeneous (i.e. Maclaurin) ellipsoids (see Tassoul 1978 for a review). This work indicates that the outcome of the collapse is likely to be determined by how large α becomes. For $\alpha < 0.14$ the body is stable. However for $0.14 < \alpha < 0.27$ a Maclaurin spheroid is unstable to non-axisymmetric perturbations and will evolve towards a more energetically stable form such as a triaxial (e.g. Jacobi) ellipsoid through the action of viscous or gravitational radiation damping. This new form may also be unstable. Further for $\alpha > 0.27$ a Maclaurin ellipsoid is dynamically unstable. Unfortunately this theory only predicts when certain modes become unstable and not the final state. In this regard there have been very few attempts to numerically model the evolution of rotationally unstable bodies (but see Durisen and Tohline 1981, for the evolution of a rapidly ($\alpha = 0.33$) rotating polytrope).

Importantly these classical results seem to be applicable to more general rotation laws and density distributions such as differentially rotating polytropes (Bodenheimer and Ostriker 1973).

What happens during the final dynamical collapse stage will depend crucially on just how much angular momentum has been exchanged between the core and the envelope during the preceding evolution. (If one assumes that no angular momentum is exchanged between the core and the envelope throughout the entire evolution then, given typical ZAMS rotation rates, the core will reach a critical angular velocity well before reaching the pulsar stage.) Endal and Sofia (1978) used an axisymmetric numerical model to evolve rotating 7 and 10 solar mass stars from pre-main sequence until their hydrogen exhausted cores had grown to approximately 1 $M_{\odot}$. At that stage the timestep was too small to continue the calculations. However a rough extrapolation of their results lead them to conclude that such stars would become unstable to non-axisymmetric perturbations prior to explosive carbon burning. They concluded that the core may fragment leading to a very complicated subsequent evolution.

However Hardorp (1974) and also Greenstein *et al.* (1977) take the view that there is more or less a continuous exchange of angular momentum throughout the entire evolution. Their main evidence for this comes from the comparatively slow rotation rates of pulsars and white dwarfs. For instance, one can estimate the rotation rate of the Crab pulsar at birth from the present rate of decrease in angular velocity. The value arrived at is only one tenth of that necessary to cause equatorial mass shedding. Therefore these authors think that considerable angular momentum exchange occurs during normal stellar evolution.

Reconciling the two different points of view is difficult.

Perhaps magnetic fields play a role redistributing angular momentum during and after the final collapse. In any case it seems worthwhile to examine the possible outcomes of a dynamical core collapse as a function of the initial rotation state.

The Numerical Method

The numerical model uses Smoothed Particle Hydrodynamics (Gingold and Monaghan 1982) modified to include post-Newtonian terms. Since the presentation at the 1983 conference (Thompson 1983) there have been two main changes. A new form of the equation of state (Lamb *et al.* 1978) is now being used. This is based on the discovery that the entropy per particle will stay close to 1 k throughout the entire collapse (Bethe *et al.* 1979). A key conclusion is that this will prevent neutrons from dripping out of the nuclei at 10^{11} g/cm^3, since this would require a significant increase in the entropy. This prevents the adiabatic index from dropping sharply at this point and means that the gravitational and pressure forces will be in closer equilibrium than was previously thought.

The other important change was to the form of the artificial viscosity used in the numerical scheme. This new artificial viscosity was described at last years conference (Pongracic *et al.* 1984). It is important in that it allows the calculations to proceed further than previously.

The Model

Only stars massive enough to evolve through the various nuclear burning stages to possess an iron core are considered. The current theory predicts that stars with ZAMS masses in the range 15-70 $M_\odot$ will be able to evolve fairly smoothly through all the nuclear burning stages to form an iron core. In addition it is believed that at least some stars with masses in the range 8-15 $M_\odot$ will also manage to eventually evolve through to this stage although their actual evolutionary paths may be considerably more complicated. (See Trimble 1982, 1983, for a review of pre-supernova evolution.)

Since iron is the most stable element no further exothermic reactions are possible. Once the core mass increases to M_{ch}, the Chandrasekhar limiting mass, electron degeneracy pressure can no longer provide support against gravity and collapse is inevitable.

This numerical model considers the inner core alone. The initial model for the collapse studies uses a 1.1 $M_\odot$ core, which is just in excess of M_{ch} for the equation of state used. The model core is artificially supported against collapse by raising the pressure by 10% above its true value. To initiate the collapse the pressure is suddenly reduced to its correct value.

Since the rotation state is unknown at the pre-collapse stage several forms for the initial rotation law were tried. This rotation was imposed on the core prior to starting the collapse. For most of the cases examined the initial value of α was very small initially (<0.01) so that one would expect that the pre-collapse model would not be too far from equilibrium.

For most of the cases examined the main effect of rotation is to cause some flattening of the core during the collapse. However for $\alpha_{initial} > 0.002$ it appears that the rotational energy builds up enough to lead to a dynamical instability. Consider a differentially rotating core with an angular momentum of 8.2×10^{47} erg-sec. (This rotation is almost sufficient to prevent the core from collapsing.) In this case α reaches a value of approximately 0.22 during the collaspe. Initially it was approximately 0.01. The numerical results indicate that the core will become dynamically unstable near maximum density. Figure 1 shows rotation plane density contours approximately 1 second after initiating the collapse. At this stage the core has become slightly triaxial. One pattern rotation period later structures resembling spiral arms form (Figure 2). Finally Figure 3 shows the core after a further three quarters of a rotation period. Matter in the spiral arms has been shed into an equatorial ring which expands outwards from the core taking with it a considerable proportion of the angular momentum. Unfortunately this was the last model calculated because of restriction on computer time.

The remaining central blob of matter is still triaxial at the end of the calculations and will presumably radiate energy and angular momentum through the emission of gravitational

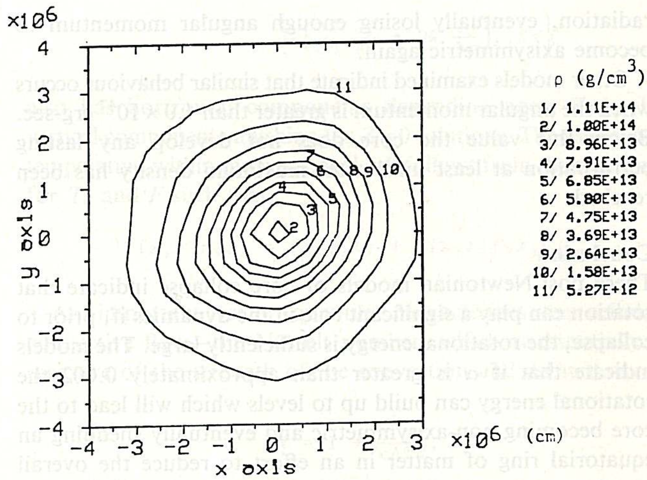

Figure 1—Rotation plane density contours approximately one second after the start of the collapse. The contours are quite distorted at this stage. The core is no longer axisymmetric.

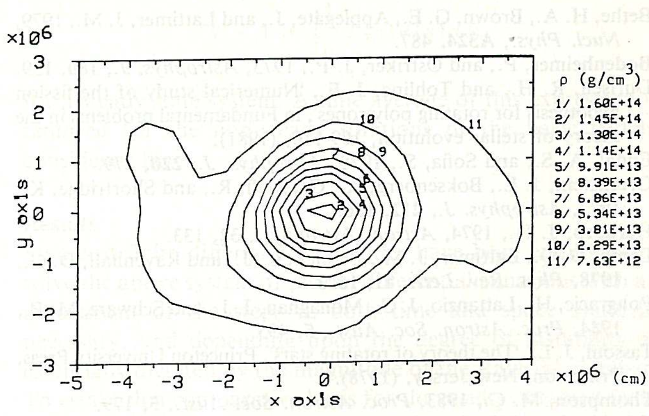

Figure 2—After a further pattern rotation period the triaxial perturbation has developed into structures resembling 'spiral arms'.

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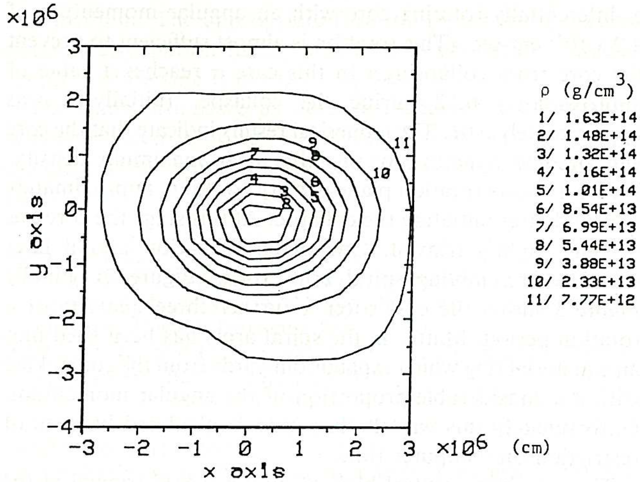

Figure 3 — The final model calculated (approximately three quarters of a rotation period after the model shown in Figure 2). By this time the matter in the arms has been shed into an equatorial ring (not shown) but the core remains triaxial.

radiation, eventually losing enough angular momentum to become axisymmetric again.

Other models examined indicate that similar behaviour occurs when the angular momentum is greater than 4.0×10^{47} erg-sec. Below this value the core does not develop any lasting perturbation at least until after maximum density has been reached.

Conclusion

These post-Newtonian models of core collapse indicate that rotation can play a significant role in the dynamics if, prior to collapse, the rotational energy is sufficiently large. The models indicate that if α is greater than approximately 0.002 the rotational energy can build up to levels which will lead to the core becoming non-axisymmetric and eventually shedding an equatorial ring of matter in an effort to reduce the overall angular momentum. The loss of axisymmetry would lead to a large flux of gravitational radiation.

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The Role of Kinetic Boundary Conditions in Generating Type II Solutions for Rayleigh-Benard Convection

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Introduction

A new family of solutions for stationary convection (Murphy and Lopez 1984) has been established which exists within the astrophysical range of parameter values — large Rayleigh number and low Prandtl number. These single mode Type II solutions, which have a non-zero component of vertical vorticity, apparently do not exist at higher Prandtl numbers and are characterized by a lower vertical velocity and heat flux, when compared to the equivalent single mode Type I solutions for Rayleigh — Benard convection with zero vertical vorticity. In turn the vertical component of vorticity associated with Type II solutions is responsible for modifying the horizontal components of the velocity field to establish cyclonic or swirling type solutions within the hexagonal convection cell.

So far numerical time-dependent investigations have established the existence of these Type II solutions only in the case when the stress-free boundary conditions hold — the ones usually considered appropriate in astrophysical situations. The question now arises as to whether or not these Type II solutions, which persist in the absence of any external effects associated with rotation or an applied magnetic field, are just a manifestation of the choice of free boundary conditions. If this should be the case, and in spite of their helical structure and other significant physical features, their validity in any astrophyical applications would at least be in doubt.

The principle objective of this study is to now establish what effect the choice of boundary conditions has on the generation and growth of a vertical component of vorticity, and also ascertain if the astrophysical choice of free-free boundary conditions represents the most efficient in terms of convective heat transport. Accordingly, time integrations of the governing equations, including the vertical vorticity terms, for the four possible combinations of boundary conditions have been undertaken and the results then examined on a comparative basis with particular emphasis on the magnitude of the total heat transfer and magnitude of the vertical vorticity at the respective boundaries.

Equations

When the single mode approximation is employed the time dependent non-linear equations for Rayleigh — Benard convection in a Boussinesq layer of fluid, which is heated uniformly from below, are given by (Lopez and Murphy 1983):