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# The shedding of vorticity from a smooth surface

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**Abstract.** This work is concerned with the development of a numerical model for laminar flow separation from a smooth boundary. The concept of irreversible vorticity generation is used to formulate an algorithm to predict both the location of separation and the shedding rate of vorticity. Results of flow separation from a circular cylinder are presented. The calculated vortex sheets are visualized as streak lines in the wake of the cylinder. Streamline patterns are constructed from these calculations showing location of separation and wake structure.

## 1. Introduction

The representation of isolated rotational flow regions by either vortex sheets or vortices can be used to model viscous phenomena such as the flow of wakes, as described in the work by Faltinsen and Pettersen (1982), and the structure of large scale turbulence as in the work by Delcourt and Brown (1979). In the case of the flow separation from a smooth cylinder, Sarpkaya and Shoaff (1979) make use of the boundary layer solution by Pohlhausen (reported by Schlicting 1968) to locate the point where flow separation from the surface of the cylinder occurs. Although Pohlhausen's solution is based on a quasi-steady flow situation, Sarpkaya's model successively predicts lift, drag and Strouhal number in good agreement with experimental results. Many works on vortex shedding models have demonstrated the success of this technique. For example: The flow around complex structures by Pettersen and Faltinsen (1983) and the studies on fluctuating flow fields by Hourigan, Thompson Welsh and Stokes (1985).

The emphasis of this paper is on a numerical model for flow separation from a smooth surface. A feature in potential flow modelling, which includes the shedding of vorticity, is to introduce the characteristic of irreversible flow. If vorticity is being shed by a body when it is travelling in a certain direction, the reversal of the travel will not return the initial state – the shedding of vorticity is a one-way process. In other words, the direction of shedding is irreversible.

## 2. Irreversible shedding of vorticity

Consider the representation of a thin laminar boundary layer by a vortex sheet in two dimensional flow. A kinematic condition exists which requires that the linear density of vorticity on the sheet be equal to the local free stream velocity immediately outside the boundary layer. The sign of the vorticity depends on the direction of the free stream and the orientation of the boundary layer in relation to the surface. Unless flow separation occurs, the vorticity is convected downstream parallel to the surface.

This kinematic condition implies that if the free stream velocity increases in the direction of the flow, the vorticity on the sheet downstream increases correspondingly. The contributions to this increase come from the additional vorticity generated at the surface, and the vorticity convected from upstream. In this way, on a surface with a free stream velocity gradient positive in the direction of the flow, the vortex sheet which represents this boundary layer will appear not to be convected in the Eulerian frame of reference.

When the local free stream gradient is negative, the vorticity convected from upstream has to be negated by the vorticity of opposite sign generated at the surface in order to satisfy the kinematic condition. As opposed the low Reynolds number case, these two groups of opposite sign vorticity remain separated from each other. It is evident that they will appear in the region of negative free stream gradient. The upstream boundary of their occurrence is at the location where the velocity of the local free stream is the maximum.

The present work only models primary separation. The vorticity downstream of the location of maximum free stream velocity is represented by a free vortex sheet. Initially this vortex sheet represents the boundary layer on the cylinder. The convection of this free vortex sheet is computed and flow separation can be observed as the vortex sheet is convected away from the surface. It must be noted that the location at which the free vortex sheet is attached to the cylinder will migrate along the cylinder according to the occurrence of the maximum free stream velocity.

## 3. Numerical model and observations

A model for the shedding of vorticity from a circular cylinder has been constructed under the assumption of no destruction of vorticity. A vortex sheet is placed at a small distance from the cylinder to represent a laminar boundary layer. The free vortex sheet will begin immediately downstream from the location of maximum velocity on the free slip surface of the cylinder. The linear vorticity density, which enters the free vortex sheet, is equal to the maximum velocity on the surface. This location is defined as the shedding point of vortex sheet. The numerical technique for the computation of the convection of the vortex sheet is given by Soh (1983).

First we consider symmetrical flow around a cylinder. Experimentally this situation occurs when a long splitter plate is attached to the horizontal diameter of the cylinder. The cylinder

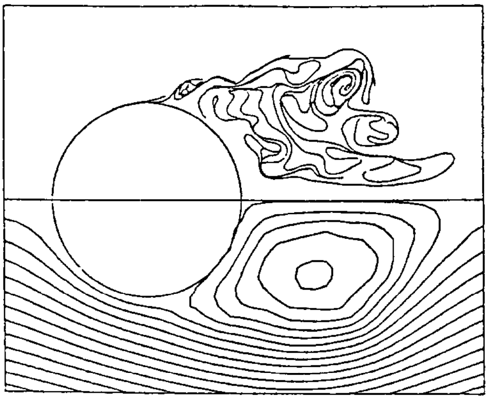

Fig. 1. Vortex sheet and streamline patterns for symmetrical shedding of vorticity by a cylinder at $t = 12.00$.

with unit radius is travelling with unit velocity in the negative $x$-axis. The plot of the vortex sheet shed by the cylinders and the streamlines are shown in fig. 1. The vortex sheet folds into a strips pattern with very close spacing between stripes. The way the vortex sheet back-tracks seems to reflect the flow visualization study reported by Chong, Lim and Perry (1980). The streamline pattern shows the wake in the form of a large bubble behind the cylinder. The

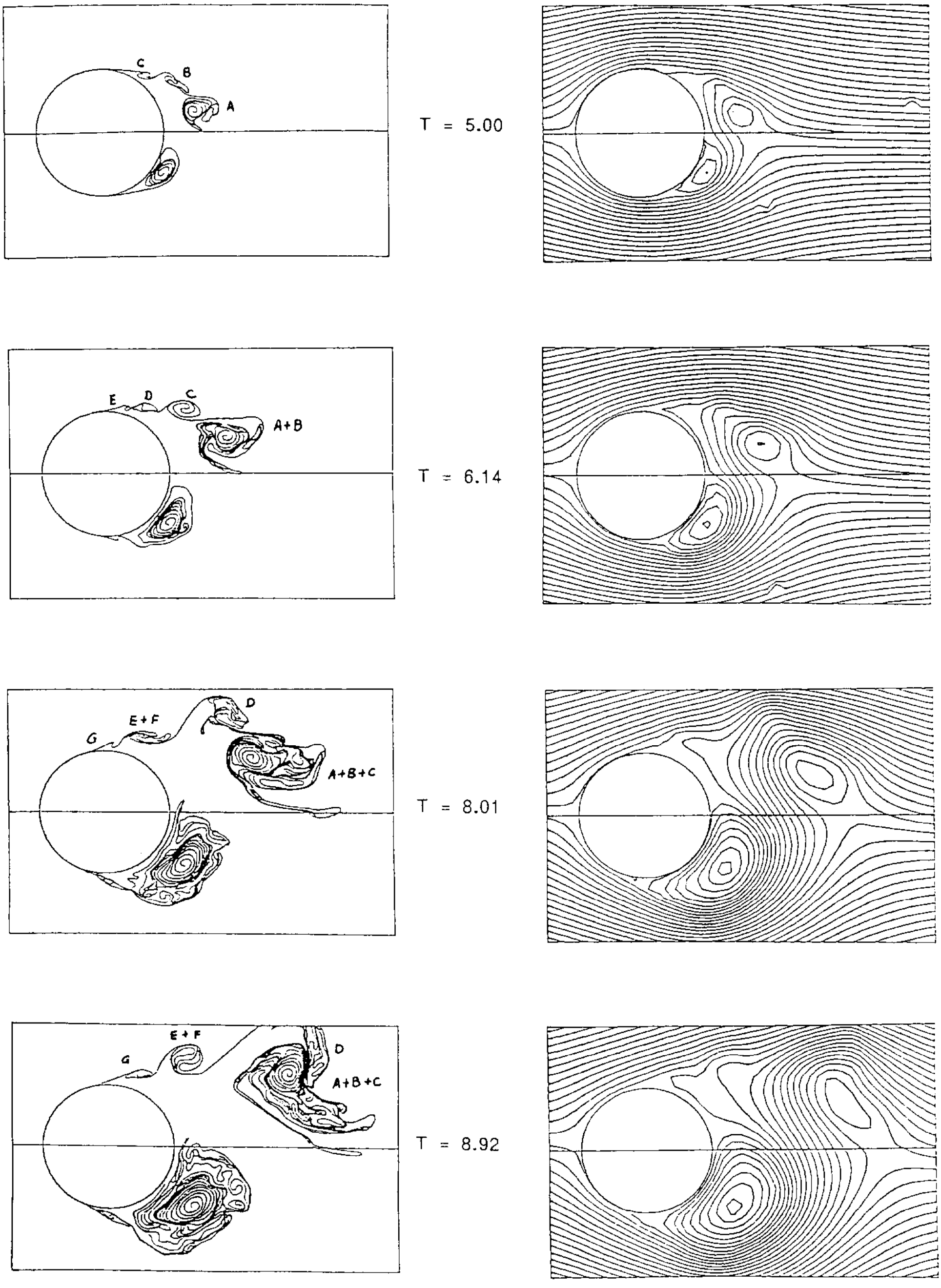

Fig. 2. Vortex sheet and streamline for asymmetrical shedding of vorticity. A sequence of time frames.

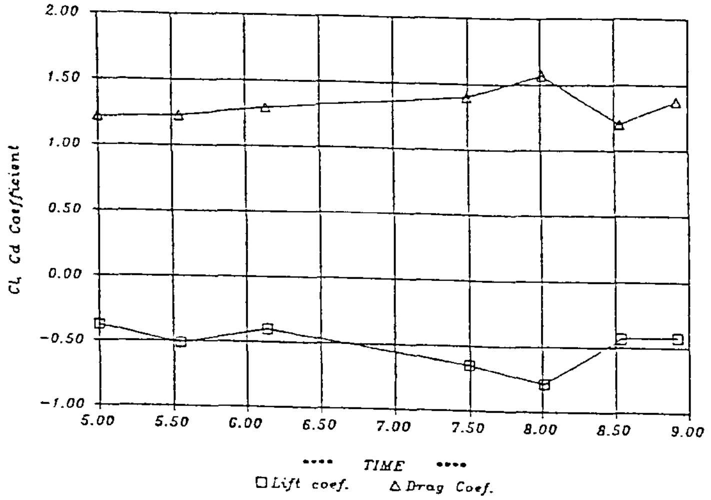

Fig. 3. Lift and drag coefficients plotted aganist time for asymmetical flow.

shedding point of the vortex sheet begins at a point 90 ° from the $x$-axis and has reached 110 ° at 12 units of time. The drag coefficient fluctuated in the range of 0.79 to 0.93.

In order to achieve asymmetric flow in the wake, a portion of the lower vortex sheet has been removed at time 4.3. Thereafter, the vortex sheet developed two large spirals. Fig. 2 shows four time frames of the vortex sheet movements and the corresponding streamline patterns. At time 5.00, the upper vortex sheet consists of a large spiral ‘A’ and two small spirals ‘B’ and ‘C’. At subsequent times, more small spirals are generated. By the end of 8.92 units of time, the upper cylinder has shed seven spirals (‘A’ to ‘G’). However, at this time spirals ‘B’, ‘C’ and ‘D’ are captured by ‘A’ and roll up into one eddy which consists of at least three spirals. Concurrent to this capturing process, spirals ‘E’ and ‘F’ are amalgamated at time 8.01. This process of capturing small spirals of vortex sheet by the larger spiral seems to be a very effective mechanism by which a large eddy can absorb the vorticity shed from the cylinder. The same capturing process is also found in the lower vortex sheet.

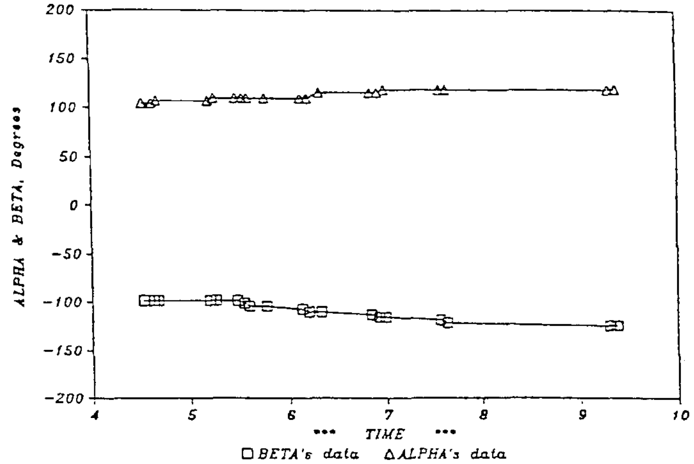

Fig. 4. Movements of shedding point. $\alpha$ = inclincation of upper shedding point; $\beta$ = inclincation of lower shedding point.

It is evident that the vortex sheet for asymmetrical flow (fig. 2) shows roll up structures which implicate large scale turbulence. In contrast, the pattern of the vortex sheet for symmetrical flow (fig. 1) appears to be random in nature. If one were to regard this vortex sheet as a streak line, then the presence of diffusion will smear this random-like streak line into a homogeneous cloud. This has been observed in smoke tunnel studies.

The plot of vortex sheets in fig. 2 is incomplete as it only shows certain streak line patterns. The influence of vorticity can be represented diagrammatically by the streamline plots. Here the influence of small vortex sheet spiral is insignificant. The two large vortex sheet spirals appear as two sets of closed-looped streamlines. It must be noted that the centers of these loops do not necessarily coincide with the core of the spiral.

The drag coefficient varies in the range from 1.22 to 1.54. In the time interval from $t = 5.00$ to 8.92, the lift coefficient and the drag coefficient are found to be in the range $-0.40$ to $-0.76$ and 1.22 to 1.54, respectively. These values are plotted against time in figure 3. The movement of the shedding points of vorticity are expressed in angles inclined from the positive $x$-axis, which is the down stream direction. These are shown in fig. 4.

## 4. Concluding remarks

The idea of representing a laminar boundary layer by a vortex sheet has brought out an algorithm for determining the shedding rate and location of shed vorticity. Some features of the initial state of asymmetrical flow are illustrated showing the process of capturing of small vortex sheet spirals by a larger spiral. The comparison between symmetrical and asymmetrical flow shows that the vortex sheet technique of flow simulation is capable of representing both roll-up and random structures in the flow.

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## References

Chong, M.S., Lim, T.T. and Perry, A.E. (1980) The vortex shedding process behind a circular cylinder, The Seventh Australasian Hydraulics and Fluid Mechanics Conference, The Institution of Engineers, Australia, August 330–333.

Delcourt, B.A.G. and Brown, G.L. (1979) The evolution and emerging structure of a vortex sheet in an inviscid and viscous fluid modelled by a point vortex method. Second Symposium on Turbulent Shear Flows, July, Imperial College, London, England.

Faltinsen, O.F. and Pettersen, B. (1982) Vortex shedding around two dimensional bodies at high Reynolds number, The Fourteenth Symposium on Naval Hydrodynamics, Office of Naval Research, USA.

Hourigan, K., Thompson, M.C., Welsh, M.C. and Stokes, A.N. (1985) Discrete vortex model of flow over a semi-infinite plate in a fluctuating field., Proc. Fourth International Conference on Numerical Methods in Laminar and Turbulent Flow, The University of Wales, Swansea, Wales.

Pettersen, B. and Faltinsen, O. (1983) Separated flow around circular cylinders, International Workshop on Ship and Platform Motions, ed. Yeung, R.W., October, University of California, Berkeley.