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NUMERICAL SIMULATION OF FORCED CONVECTION NEAR A HEATED PLATE

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SUMMARY

The two-dimensional inviscid incompressible flow at high Reynolds number over an isothermally heated flat semi-infinite plate with a square leading edge is simulated numerically using a discrete-vortex model. Both the instantaneous and mean thermal fields are predicted using an alternating direction implicit finite difference approximation of the energy equation. The mean local heat transfer rate is found to be a global maximum near the point of mean reattachment, in line with observation. The mean flow field does not reveal the large velocity fluctuations that occur near reattachment. It is the instantaneous flow field that gives insight into the large-scale vortex formation and the strong vortical influence on the flow and heat transfer rates. Instantaneously, local maxima of heat transfer rates occur just downstream of a vortex, and local minima are found immediately trailing the vortices and in the separation bubble. A second case is considered in which the elemental vortices that represent the separating shear layer near the leading edge undergo lateral oscillation resulting in a reduced reattachment length. This results in an increase in both the global heat transfer rate and the mean rate at the mean point of reattachment.

1. INTRODUCTION

The high Reynolds number flow over the square leading edge of a long flat plate is characterized by the separation of a shear layer at each corner, its reattachment downstream, and the shedding of vortices that are convected along the plate surface. This type of flow has been modelled numerically using the discrete vortex method by a number of authors (e.g. Nagano et al., 1982, Kiya et al., 1982). These models have been found to satisfactorily predict many of the mean characteristics of the flow in addition to providing useful insight into its instantaneous features.

The present paper is concerned with using the discrete-vortex model as a basis for investigating the extended problem in which the forced convection takes place over a heated plate. Motivation for this research has been provided by experiments that show reduction of the mean reattachment length by sound stimulation can increase the Nusselt number on the plate by up to 40% in the mean and 100% locally at reattachment (Cooper et al., 1983). It is also observed, for sufficiently long plates, that the local heat transfer coefficient begins to decrease with reducing reattachment length some distance downstream of reattachment. It has been suspected that the large-scale vortex dynamics are important in determining these observed features. The precise means by which they do so has been difficult to infer from experimental results, which are collected in an averaged form. The present model is intended to additionally predict the instantaneous characteristics of the flow and the thermal field, thereby providing an insight into the mechanism of augmentation of heat transfer.

2. MATHEMATICAL DESCRIPTION OF MODEL

The model under consideration consists of the flow past a rigid two-dimensional heated flat plate with a square leading edge. The sides of the semi-infinite plate are aligned parallel to an approaching flow of uniform velocity and temperature. The fluid is assumed to be inviscid, incompressible and irrotational everywhere except at points where a simple inviscid line vortex is located. The flow is assumed to separate at the corners of the leading edge. Attention is focussed on the behaviour of the separated shear layer on the upper surface, taking as negligible the influence in this region of the vorticity generated at the lower leading edge corner.

2.1 Discrete-Vortex Model

2.1.1 Velocity potential

The separated shear layer is approximated by an array of line vortices. The local fluid velocity, which is determined kinematically from the vorticity field and the irrotational field, then determines the inviscid motion of these elemental vortices. In order to satisfy the condition of zero flow normal to the solid boundary, the plane is mapped conformally into one where the boundary is a half-plane and the boundary condition can be satisfied by including mirror-image vortices. The following Schwarz-Christoffel conformal transformation is used

$$z = \frac{H}{\pi}\left[(\lambda^2 - 1)^{\frac{1}{2}} - \mathrm{arcosh}\lambda\right] + iH,$$

where H is the semi-thickness of the plate.

The complex velocity potential Φ at position λ is then given by

$$\Phi = V_o \frac{H}{\pi}\lambda + \sum_{j=1}^{N} \frac{iG_j}{2\pi}\left[\log(\lambda - \lambda_j) - \log(\lambda - \lambda_j^*)\right],$$

where the first term on the right is due to the irrotational flow and the second term is due to the flow induced by the N vortices. Here, V_o is the velocity at upstream infinity, G_j is the strength of the j-th vortex and * denotes a complex conjugate.

The velocity field in the physical plane is given by $d\Phi/dz$ at points other than vortex centres. The limiting net velocity at the centre of a vortex is obtained by subtracting in the image plane the potential due to the vortex. The velocity in the physical plane at the k-th vortex centre then contains the Routh correction term (e.g. Clements, 1973).

Other details of the model are essentially the same as those provided by Kiya et al. (1982), except that vorticity reduction along the plate is introduced differently, as follows.

2.1.2 Vorticity generation and the Kutta condition

A determination is required of the strength of each elemental vortex, which represents a segment of the vortex sheet shed from the leading edge. The rate of creation of vorticity at a separation point is normally obtained from the kinematic condition $dG/dt = 1/2\, V_s^2$, where V_s is the velocity at the outer edge of the shear layer. In this model, the velocity V_s is determined at a fixed point at a distance 0.01H in front of, and parallel to, the top surface of the plate. This point is chosen to represent the position of the outer edge of a shear layer separating from the front face. The rate of generation of vorticity is equal to the relative tangential acceleration of the fluid and the boundary without taking viscosity into account. In the case of a fixed plate, the vorticity generating mechanism therefore involves only the tangential pressure gradient within the fluid (Morton, 1984). The large pressure drop around the leading edge thus results in substantial vorticity production. The distance from the plate used to determine V_s is selected to give a vorticity generation rate consistent with the pressure drop around the leading edge.

The Kutta condition requires that the velocity is zero at the point in the transformed plane corresponding to the leading edge corner. This condition, together with the assumption that the boundary layer separates tangentially to the body surface, uniquely determines the nascent vortex position.

2.1.3 Surface pressure gradients and vorticity reduction

In addition to the pressure drop at the leading edge, there is a significant recovery in the mean tangential pressure gradient near reattachment of the separated shear layer. This leads to production of vorticity of sense opposite to that generated at the leading edge. The effect of the entrainment of this new vorticity into the large scale vortical structures is to reduce their overall circulation. This is accounted for in the present model by reducing the strengths of elemental vortices that recirculate upstream as a result of passing close to the plate surface. The amount of vorticity reduction is fixed so that the mean circulation passing further downstream is consistent with the mean tangential pressure recovery along the surface.

2.2 Incorporation of Heat Transfer

The dimensionless energy equation to be solved is given by

$$\frac{\partial T}{\partial t} + \underline{u}\cdot\underline{\nabla}T = Pe^{-1}\,\nabla^2 T ,$$

where T is the local fluid temperature, $\underline{u}$ is the local fluid velocity and Pe is the Peclet number. The normalizing quantities are the difference in temperature between the wall and the incident fluid, the fluid velocity at upstream infinity and the plate semi-width. Here, the fluid velocity is determined through the Biot-Savart integral which describes the motion induced by vorticity in the flow. In the present case, the vorticity distribution is determined through solution of the vorticity equation by the discrete vortex model, as described above.

The boundary conditions are that the temperature is constant along the top surface of the plate and a different constant in the uniform flow at upstream infinity.

2.2.1 Finite-difference procedure

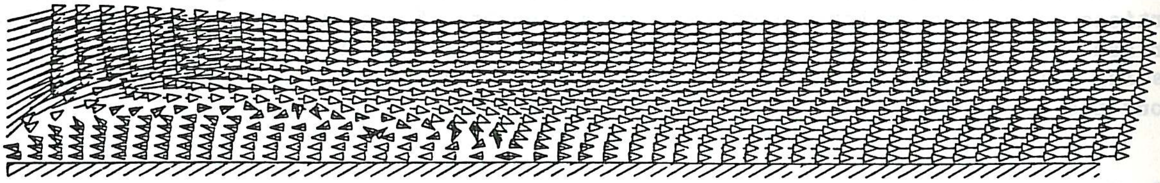
A non-conservative finite-difference approximation to the energy equation is used. The diffusion and convection terms are replaced by second-order central difference approximations. The scheme is then solved using the Peaceman-Rachford ADI algorithm. The advancement of the energy equation solution for each time step of advancement of the discrete vortices is performed by alternating between rows and columns of the finite difference mesh system at the corresponding half time-steps.

The origin of the finite-difference mesh was placed at a distance 0.2H downstream of the plate corner to avoid the singularity in potential at that point. A 50 x 30 mesh system representing an area 20H x 2.5H above the plate was used in each calculation.

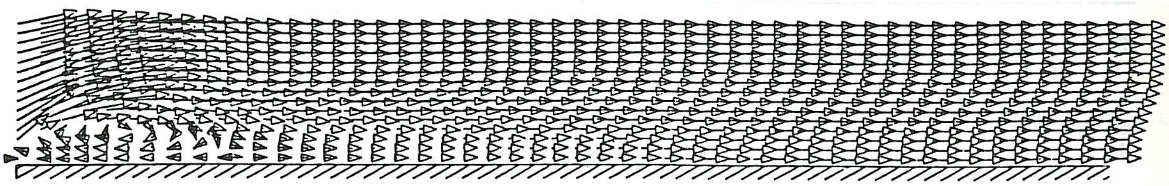
3. RESULTS AND DISCUSSION

The heat transfer characteristics for two different cases are considered: in each case, the heat and flow parameters have the same values with the exception that a pseudo-sound field is applied in one case. That is, a periodic lateral stimulation of the separating shear layer, represented by the elemental vortices, is employed. The oscillation had amplitude $H/4\pi$ and frequency $0.2(U/H)$ and was restricted to the first five elemental vortices in the shear layer. This crudely approximates the effect of a transverse sound field. It is found in this case that the frequency of vortex shedding from the separation bubble is locked to the frequency of shear layer oscillation. This is precisely the result obtained experimentally using sound as the perturbing mechanism (Parker and Welsh, 1983).

The Peclet number used was 20. This is sufficiently large relative to unity to make convection the dominant mechanism of heat transfer on a global scale, but small enough that satisfactory solutions can be obtained using a limited number of finite difference mesh points.



a.



b.

FIGURE 1. MEAN VELOCITY VECTOR FIELDS FOR a. NO STIMULATION and b. STIMULATION

Figures 1.a and 1.b show the local mean velocity vectors for the flows without and with shear-layer stimulation. The perturbation reduces the mean reattachment length from approximately 9 to less than 4 plate semi-widths. In each case, a thin shear layer originating close to the leading edge separates an outer region of high velocity from the inner bubble region containing recirculating flow of low velocity. This picture coincides closely with the classic separation bubble derived from averaged experimental data. The non-zero normal gradient of the velocity along the plate surface indicates the flow of vorticity shed from the leading edge. There was found to be agreement between the positions of maximum normal velocity gradient and maximum normal thermal gradient, which are located near reattachment. That is, the position of maximum vorticity is associated with that of highest heat transfer rate, in the mean. The time-averaged isotherm profiles for the two cases are shown in Figures 2.a and 2.b. However, details of large-scale vortex formation, their shedding from the bubble and their local influence on the heat transfer rate are left unrevealed by the mean results.

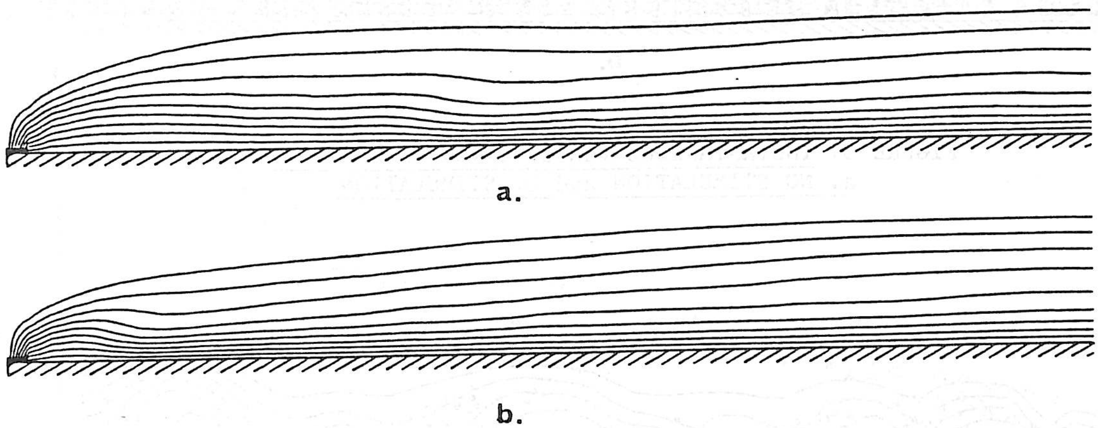


FIGURE 2. MEAN ISOTHERMS FOR a. NO STIMULATION AND b. STIMULATION. PECLET NUMBER IS 20. THE ISOTHERM VALUES REPRESENT DIFFERENCES OF 10% IN TEMPERATURE

Plots of local instantaneous velocity vectors, characteristic of the flows without and with stimulation, are shown in Figures 3.a and 3.b, respectively. The dynamic nature of the separation bubble and flow downstream of reattachment is now evident. It is true that the flow inside the bubble close to the leading edge is not dissimilar to the mean case, although some early-stage shear-layer roll-up is apparent. However, an entirely different picture emerges near reattachment. Here, large-scale vortical structures are manifest and significant normal velocities and velocity gradients appear near the plate surface. The 'smoothed out' appearance of the mean flow evidently masks the rapidly swirling nature of the instantaneous flow near reattachment. The higher heat transfer rate in this region appears to result from the action of this highly fluctuating flow, in which colder fluid is moved quickly towards and away from the heated plate surface.

The intense local influence of the vortices shed from the separation bubble on the heat transfer is apparent from the instantaneous isotherms plotted in Figures 4.a (no stimulation) and 4.b (stimulation). It is interesting to note that local maximum heat transfer rates obtain just downstream of the shed vortices and local minimum values are found close by upstream. Why this should be so becomes clear when the instantaneous velocity vectors near the large vortical structures are considered (see Figures 3.a and 3.b). The flow near the plate just upstream of a vortex is fairly uniform - this laminar-like condition providing higher thermal resistivity. In contrast, the fluid impacting on the plate just downstream of the vortex has come from colder regions further away from the plate. This results in the larger thermal gradients there, as shown by the isotherm plots in Figure 4.

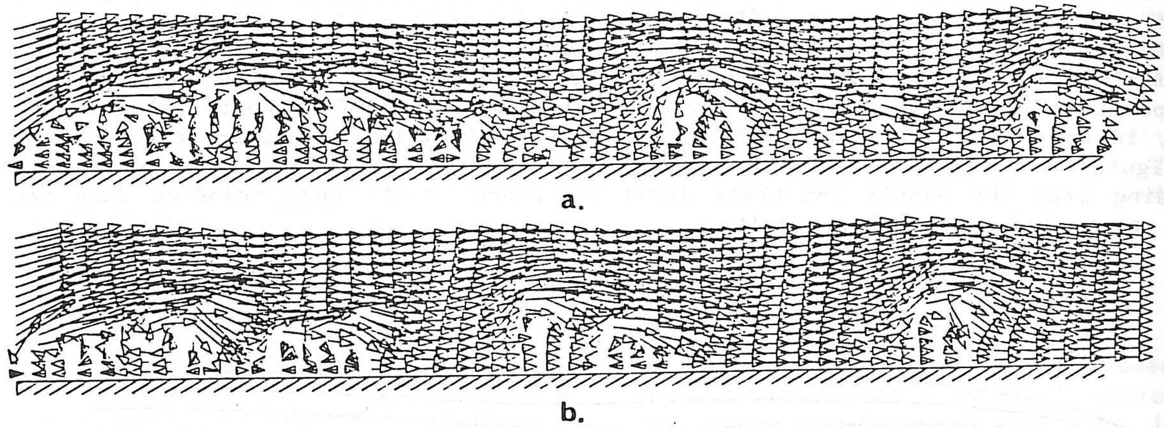


FIGURE 3. INSTANTANEOUS VELOCITY VECTOR FIELDS FOR a. NO STIMULATION and b. STIMULATION

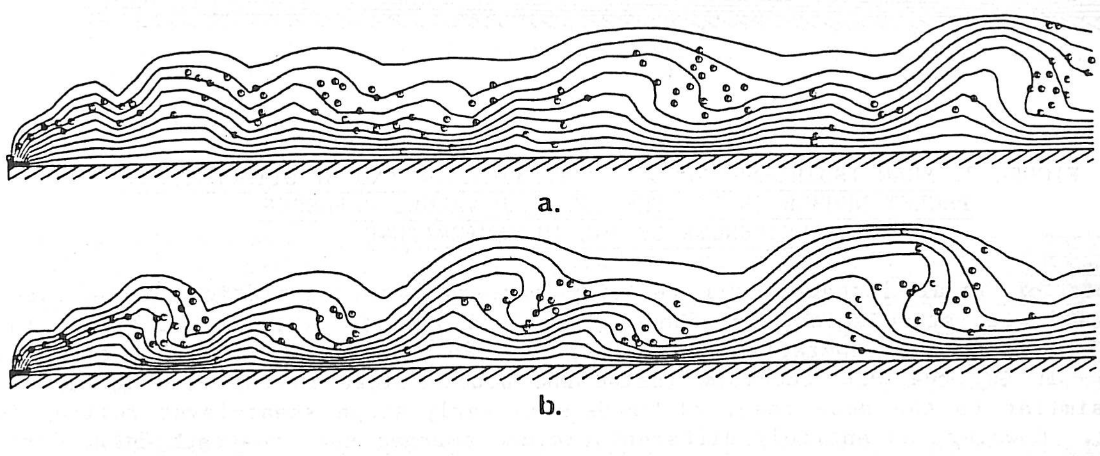


FIGURE 4. INSTANTANEOUS ISOTHERMS FOR a. NO STIMULATION AND b. STIMULATION. PECLET NUMBER AND ISOTHERM VALUES AS FOR FIGURE 2

The local mean Nusselt number along the plate for the two cases is shown in Figure 5. The effect of applying shear layer stimulation is the movement of the point of peak heat transfer, along with the mean reattachment position, towards the leading edge. The maximum local Nusselt number is increased by about 27% and the average Nusselt number is greater by about 10%. However, the local Nusselt number in the stimulated case falls below that of no-stimulation past 8 semi-plate widths. There appears to be a limiting effect on the ability of the vortices to transport heated fluid away from the surface and mix it with distant colder fluid. These trends are in line with those observed experimentally by Cooper et al. (1983).

The increase in the maximum heat transfer rate as the mean bubble length is reduced is also in line with experimental results (Cooper et al., 1983). In the experiments of Ota and Kon (1974) and those of McCormick et al. (1984), it is found that the mean local Nusselt number along the plate scales as the two-thirds power of the Reynolds number. In what is essentially inviscid flow, the Reynolds number does not appear in the momentum or vorticity equations. It is only implicit in the normalized energy equation; the Peclet number being the product of the Prandtl and Reynolds numbers. It is hypothesized then that in these forced convective flows over flat plates in which the velocity field attains a limiting form at high Reynolds number, the distribution of $Nu/Pe^{2/3}$ will in fact be independent of the Peclet number. This is supported by the near-coincidence of the scaled Nusselt number at reattachment in the present unperturbed case with that obtained experimentally by McCormick et al. (1984) - a value close to 0.16 is obtained in each case in spite of a difference in Peclet number of several orders of magnitude. It is therefore possible to compare predictions of the model with experimental results.

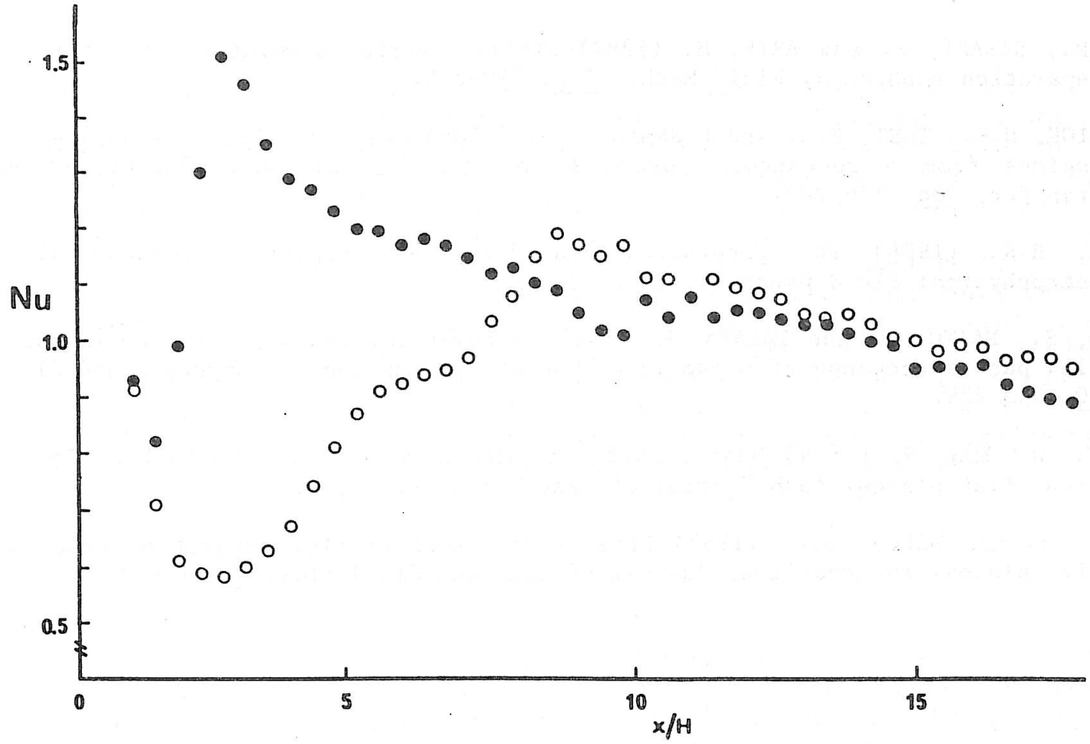


FIGURE 5. LOCAL MEAN NUSSELT NUMBER Nu VERSUS DISTANCE (x) ALONG PLATE SURFACE
O NO STIMULATION
● STIMULATION

4. CONCLUSIONS

A discrete-vortex model in conjunction with a finite difference scheme has been used to simulate the heat transfer in a two-dimensional separated flow over a flat heated plate. A much fuller understanding of the heat transfer characteristics of the flow is provided by considering the instantaneous flow. The mean flow masks the highly fluctuating nature of the separation bubble. The effect of an oscillation of the separating shear layer is to couple the frequency of large-scale vortex formation with the oscillation frequency. This reduces the mean reattachment length and increases the mean Nusselt number at reattachment. The point of mean reattachment is found close to the position of maximum mean local Nusselt number. Plots of the instantaneous velocity and thermal fields show that the vortices play an important role in determining the heat transfer characteristics of the flow.

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