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Growth of Planetesimals in a Gaseous Ring

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Introduction

The aggregation of a large number of planetesimals into a single body is a problem that has proved to be a stumbling block for many theories of planetary formation. This difficulty has mainly arisen because of the tendency of an orbiting stream of bodies to disperse over the equatorial plane as a result of the combined effects of collisions and gravitational interactions (Trulsen 1972; Brahic 1975). Once such a chaotic distribution of orbiting bodies has developed, it is extremely difficult for these bodies to subsequently accumulate into a single entity. This problem does not arise, however, when aggregation takes place within a differentially rotating gaseous ring as proposed by Prentice (1978). It has been shown previously (Hourigan 1977) that the gas drag overcomes the disruptive nature of collisional interactions, resulting in the formation of a thin concentrated torus of orbiting bodies.

The process leading to the formation of a population of large bodies through the gravitational fragmentation of the dense torus of grains that forms at the mean circular orbit of a gas ring has been investigated previously (Hourigan and Prentice 1979). As a result of this process, it can be shown that bodies of radii in the kilometre or tens-of-kilometre range may form at the orbit of the Earth. In the discussion below, the further growth of these planetesimals as a result of the accretion of infalling grains, in addition to collisions between the planetesimals themselves, is considered.

The Model

Consider the mass distribution function $n(m,t)$ of the planetesimal population at time t. The N_0 planetesimals of total mass M_0, which form as a result of the fragmentation of the grain torus, are assumed to have a mass distribution function which is peaked around the mean value $\overline{m} = M_0/N_0$. This mean value $\overline{m}$ represents the mass of the fragments produced by the mode of maximum instability during the fragmentation of the grain torus. In order to reflect this initial mass distribution, $n(m,t)$ is taken to have the initial form $n(n,0) = ame^{-bm}$, where $a = 4N_0/\overline{m}^2$ and $b = 2/\overline{m}$.

To investigate the evolution of the mass distribution function, the integral form of the coagulation equation is used (ref. Hourigan 1981):

$$\frac{\partial}{\partial t} n(m,t) = \tfrac{1}{2}\int_0^m A(m', m-m')\, n(m',t)\, n(m-m',t)\, dm' - n(m,t)\int_0^\infty A(m,m')\, n(m',t)\, dm' - \frac{B}{M_T}\frac{\partial}{\partial m}(n(m,t)m), \quad (1)$$

where $A(m,m')$, the coagulation coefficient, is the collision and aggregation probability for bodies of mass m and m', B is the uniform mass segregation rate of the grains, and $M_T(t)$ is the total mass of the planetesimal population at time t. The first two terms on the right of equation (1) represent the growth of planetesimals as a result of collisions with other planetesimals. The last term represents the growth as a result of the accretion of incoming grains. In order to give a qualitatively accurate description of the accumulation process across the whole spectrum of planetesimal sizes, the coagulation coefficient is chosen to have the form given by $A(m,m') = (m + m')A_1$, where A_1 is a constant (Safronov 1962).

Taking the Laplace Transform of equation (1) and solving the resultant partial differential equation gives the result

$$F(p,t)\left[p + (3N_0 - N)/M_T - \frac{1}{M_T}\left(\frac{N_0}{N} - 1\right)F(p,t)\right]^2 = 4N_0^2N/M_T^2, \quad (2)$$

where $N(t)$ is the number of planetesimals at time t and $F(p,t)$ is the Laplace Transform of $n(m,t)$. For small times t, the cubic equation (2) can be solved for $F(p,t)$ and the inverse Laplace Transform taken to given

$$M_T\, n(m,t) = \begin{cases} 4\dfrac{(1-\tau)}{M_T}N_0^3 m \exp\left(-\dfrac{N_0}{M_T}(2+\tau)m\right); \\ \tau << 1, m\alpha << 1, \\ \dfrac{3^{-1/6}}{\pi}\Gamma\left(\dfrac{4}{3}\right)M_T\dfrac{(1-\tau)}{\tau}\alpha^{2/3} \times \\ \times \exp\left(-\dfrac{N_0}{M_T}(2-3(2\tau)^{1/3}+\tau)m\right)m^{-1/3} - \\ -8(1-\tau)\dfrac{N_0^3}{M_T}m\exp\left(-\dfrac{N_0}{M_T}(2+\tau)m\right); \\ \tau << 1, m\alpha >> 1, \end{cases} \quad (3)$$

where $\tau = 1 - N/N_0$ and $\alpha = \dfrac{3N_0}{M_T}(2\tau)^{1/3}$.

It can be shown from equation (3) that as time proceeds, an increased proportion of the mass becomes concentrated in the large bodies of the planetesimal population. In particular, the largest body grows the most rapidly of all. The asymptotic solution of the transformed coagulation equation (2) for small m and for large times ($t \to \infty$ or $\tau \to 1$) is given by

$$n(m,\tau) \overset{\tau \to 1}{\sim} 4\frac{N_0^2N}{M_T^2}m.$$

Thus, after a sufficiently long time, the number of bodies N left is small and most of the mass is concentrated in the largest body. The statistical analysis afforded by the coagulated equation cannot be used to adequately model the very late stages of planetary formation when there are only a few bodies remaining. More individual attention must then be paid to the evolution of each planetesimal. In particular, the relative accretion rates of the largest body of mass m_1 and radius r_1 and the second largest body of mass m_2 and radius r_2 are given by (ref. Hourigan 1981)

$$\frac{\dot{m}_1/m_1}{\dot{m}^2/m_2} \approx \frac{1+h}{1+h\,r^2/r_1} > 1,$$

where $h = 3/2\ \varrho v_{rel}\theta M_T/\sigma r_1 B$. Here, ϱ is the spatial density of planetesimals, v_{rel} is the characteristic relative velocity between bodies, θ is a constant $\sim$ 10, σ is the intrinsic density of the planetesimals. This equation again demonstrates that the largest body grows more-than-proportionately faster than the second largest body, thereby increasing its mass relative to the latter. That is, a runaway effect occurs in the accretion process, eventually leading to the emergence of a single dominant body amongst the population of planetesimals.

Conclusions

It has been found in the above analysis that the aggregation of planetesimals in a gaseous ring leads naturally to the development of a dominant body amongst the planetesimal population. The presence of the gas in the form of a differentially rotating ring serves to constrain the orbits of the planetesimals and grains to within a thin toroidal region through the action of gas drag. This situation allows for the efficient aggregation of bodies and, as a result of the low resultant relative velocities, the minimization of collisional fragmentation effects.

At present, a large-scale numerical code incorporating the effects of collisions and gravitational encounters is being developed in order to provide a more quantitatively correct description of the aggregation process outlined above.

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The Solar Neutrino Problem Revisited

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1. Introduction

The discrepancy between the observed and predicted flux of neutrinos from the Sun is well known. Past attempts at reconciling this difference have been unconvincing (Kuchowicz 1976, Rood 1978), and hence investigations in this area continue.

The original reports (Reines *et al.* 1980 and Lubimov *et al.* 1980) of a non-zero lower limit for the neutrino rest-mass received mixed reactions (De Rújula and Glashow 1980). At this stage it is impossible to either confirm or deny the early claims. However, the possibility of oscillations between the three neutrino states (or 'flavours'; electron, muon, tauon) has been suggested as early as 1968 (Pontecorvo 1968) as a means for reducing the predicted neutrino capture rates. Recently (Bahcall *et al.* 1980) it has been shown that if the broad energy spectrum of the solar neutrinos is included in the calculations (an effect previously neglected) then neutrino oscillations alone are not sufficient to reduce the predicted capture rate to the (best-estimate) observed rate (Davis 1978). A decrease by a factor of 2.6-4.0 is needed to explain the observations, whilst oscillations can only give a factor of 2.

Prentice (1976, referred to as P76) suggested that if the Sun initially possessed a metal-rich, supersonically turbulent central core of mass fraction $q_c \approx 0.02$, the predicted neutrino flux would be lowered considerably. This core would have formed during the very early stages of the Sun's contraction from interstellar densities, when grains rich in CNO ices rapidly sank to the centre of the protosolar cloud. It was later shown by Krautschneider (1977) that gravitational collapse from a typical Bok globule could indeed lead to this metal-rich core. This increased opacity in the core will make the region less stable against convection, and for a high enough Z_{core} perhaps the convective motions will become supersonic. This phenomenon has been discussed elsewhere (Prentice 1973, referred to as P73) in some detail. It is the aim of this study to verify and extend the preliminary results given in P76 using a modified version of the Mt Stromlo Stellar Structure Program.

2. The Effect of Turbulence

The supersonic convective turbulence gives rise to a radial stress (see P73), not an isotropic pressure, which takes the form

$$P_t = \beta\varrho GM(r)/r, \tag{1}$$

where all symbols have their usual meaning, and β is the 'turbulence parameter'. This stress increases outwardly with radius r, and is zero at the centre.

Preliminary studies including the effects of turbulent stress were made in P76. These were done using simple polytropic models. An $n = 1.5$ polytrope was used to model the core, and this was matched to an $n = 3$ polytrope for the surrounding envelope, presumed to be radiatively stable. Turbulent stress was confined to the high Z core. The effects of overshooting beyond the core are also ignored in the present paper, for the sake of comparative studies.

To isolate the effects of turbulence from those due to the high central Z, we firstly consider models of homogeneous abundances, and various β. For comparative purposes we choose values as used in P76:

$$X_c = X_e = 0.75;\ Z_c = Z_e = 0.02,\ q_c = 0.02,\ \beta = 0,\ 2,\ 5,\ 10$$

(Subscripts 'c' and 'e' refer to core and envelope, respectively.) The core is defined as all points having $q = M(r)/M \leq q_c$. Note that $\beta \equiv 0$ for $q > q_c$. In these models the entire core is assumed convective and supersonically turbulent, except for the non-turbulent case $\beta = 0$.

In the convective core the abundances of all elements are mixed to homogeneity. In the event of a convective region developing outside the core, convective mixing is treated in the usual manner. In no case, however, is there allowed to be mixing of higher Z material (from the core) with the lower Z