PIV measurement of a subharmonic "mode C" three-dimensional instability behind inclined square cylinders

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Abstract

The flow over a cylinder with a square cross-section oriented at angles of incidence with respect to the direction of flow is investigated experimentally. Reynolds numbers above the onset of three-dimensional flow are considered, and Particle Image Velocimetry (PIV) is used to measure three-dimensional flow features in order to determine the instability modes present in the flow. The findings of these experiments are compared against predictions from recently published linear stability analysis computations.

Introduction

Cylinders with square cross-section present an interesting alternative to the canonical circular cylinder for the investigation of low-Reynolds-number bluff-body wake flows. Following the development of unsteady flow, the wakes behind square and circular cylinders are two-dimensional and time-periodic. The circular cylinder wake naturally possesses a half-period-flip symmetry, whereby a shift of half a period in time and reflection about the wake centreline recovers the original wake (Barkley & Henderson 1996). This symmetry restricts available three-dimensional instability modes to either synchronous or quasi-periodic modes (Blackburn, Marques & Lopez 2005). This same symmetry is also found behind a square cylinder when oriented at either 0° (a square) or 45° (a diamond) to the oncoming flow. In these instances, it is known that these flows are unstable to three-dimensional instability modes consistent with (synchronous) Modes A and B, as well as a quasi-periodic mode (Blackburn & Lopez 2003). Behind a square cylinder at zero degrees, linear stability analysis (Sheard, Fitzgerald & Ryan 2009) predicted these modes to become unstable at critical Reynolds numbers $Re = 164$, 197 and 215, with spanwise wavelengths $\lambda/d = 5.2$, 1.1 and 2.6, respectively.

Recently (Sheard, Fitzgerald & Ryan 2009) it has been shown that the effect of increasing the incidence angle of a square cylinder from zero degrees is to cause a corresponding increase in the critical Reynolds numbers of modes A and B, while it was also shown that at 7.5°, the wake is no longer unstable to a quasi-periodic mode, but instead is unstable to a subharmonic instability mode (one with a period twice that of the base flow). The critical Reynolds number of the subharmonic mode decreases with increasing incidence angle; finally becoming the first-occurring instability mode for incidence angles greater than approximately 12°. Beyond 26°, the first-occurring instability reverts to mode A, which persists through to 45°.

When comparing observations of wake flows in the laboratory beyond the onset of a first-occurring three-dimensional instability mode, disparities emerge between observed behaviour and the behaviour predicted using a stability analysis, owing to the difference between the physical three-dimensional wake and the two-dimensional wake used as a basis for the stability analysis. For example, considering the circular cylinder, stability analysis predicts mode A to emerge at $Re = 188$, and mode B to become unstable at $Re = 259$ (Barkley & Henderson 1996). However, experiments reveal that while the first transition is accurately predicted (notwithstanding hysteresis at the onset of the instability), evidence of wake structures consistent with the predicted mode B instability are observed with increasing prevalence beyond approximately $Re = 230$.

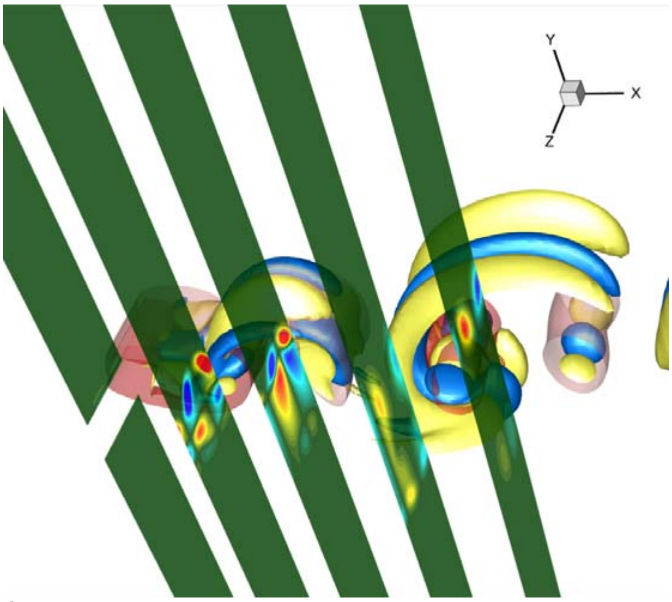

Figure 1. A plot of streamwise vorticity in the wake behind a square cylinder inclined at 22.5 degrees at Re = 130. Translucent red isosurfaces plot spanwise vorticity, while yellow and blue isosurfaces plot streamwise vorticity, as per figure 13 in Sheard, Fitzgerald & Ryan (2009), in which this wake was identified as having developed from a mode C instability. Adjacent to these isosurfaces, 5 y-z planes equi-spaced at x-locations between 0d and 9d from the cylinder centreline are also shown. On these planes, streamwise vorticity is plotted. Flow is left to right, and the width in the spanwise direction of each of the planes is 2.09d, corresponding to the predicted peak wavelength of the mode C instability from linear stability analysis.

For the wake behind a square cylinder, the only study known to the authors which visualized the three-dimensional wake structure at non-zero angles of incidence was performed using dye visualization by Tong, Luo & Khoo (2008). However, the reported experiments were perfomed at an angle of 10°, below the angle of 12° at which the subharmonic mode C is predicted to become the first-occurring mode, and hence structures corresponding to mode A were observed. Simulated dye visualization computed by Sheard, Fitzgerald & Ryan (2009) at 15° demonstrated that three-dimensional wake structures resembling the spanwise wavelength of mode C may be observed at Reynolds numbers slightly above the predicted onset of three-dimensional flow. However, this is yet to be observed in a physical experiment, motivating the experiments proposed in this study.

The elusive nature of the mode C instability is not restricted to cylinders with a square cross-section. Three other bluff-body flows in which the mode C instability have been predicted by computations each share a geometric feature which breaks the half-period-flip symmetry of the wake, a slender ring aligned normal to the flow (Sheard, Thompson & Hourigan 2003), staggered circular cylinders in tandem (Carmo, Sherwin, Bearman & Willden 2008), and a circular cylinder wake tripped by a wire located near to the cylinder (Zhang, Noack, König & Eckelmann 1995). Visualizing this mode is difficult as it is rarely the first-occurring instability mode. Rings with ratios of mean diameter to cross-section diameter in the range of approximately 4 to 7 are an exception, with mode C predicted to be the first-occurring instability (Sheard, Thompson & Hourigan 2003). Dye visualization experiments conducted using a towed buoyant tethered ring in a water tank (Sheard *et al.* 2005) reported what is believed to be the only known evidence of three-dimensional structures consistent with mode C. Velocimetry measurement confirming the presence of mode C in a second geometric configuration would mark a significant milestone towards our understanding of three-dimensional instabilities to time-periodic wake flows.

The aim of the present study is to employ particle image velocimetry (PIV) measurements of the three-dimensional wakes behind a square cylinder to ascertain which three-dimensional instability modes emerge in the physical wake, and how these compare with predictions of the linear stability analysis.

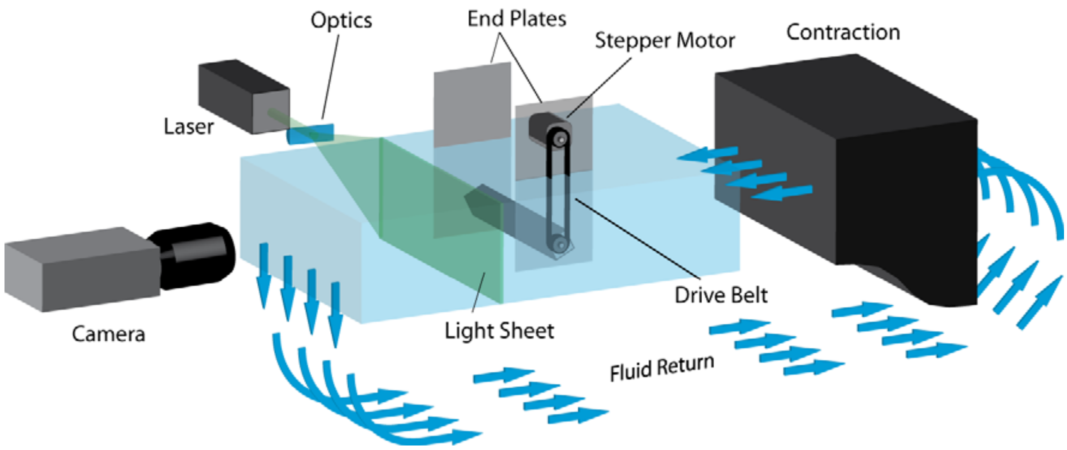

Figure 2. Schematic diagram of configuration used for PIV experiments showing water tunnel, cylinder – and adjustment mechanism, laser, optics and PIV camera.

Methodology

Experiments are conducted in the FLAIR closed-loop free-surface water channel in the Department of Mechanical and Aerospace Engineering, Monash University. The working section measures 600mm wide by 800mm high and 4m in length. A square cylinder of length 400mm and cross-section side length 19mm, fabricated from anodised aluminium, is affixed to a rig permitting variation of the cylinder incidence angle, and is placed in the working section such that the cylinder axis is perpendicular to the oncoming flow.

To allow for the the low Reynolds numbers required for this study a low-flow-rate pumping and particle seeding system has been designed and built. The Reynolds number is controlled by adjusting the channel flow rate, and with the cylinder model employed in this study, a range of Reynolds numbers (based on the side length (d) of the cylinder cross-section) of 50 to 200 are possible. It is noted that an alternative definition for the Reynolds number in studies of the flow past a square cylinder employ the projected frontal height (h) instead.

PIV measurements are performed by seeding the flow with 100micro-meter diameter polystyrene particles, and planes normal to the bulk flow are illuminated using a Nd:Yag pulsed laser (Continuum, USA). Image pairs are captured using a PCO4000 camera (PCO, Germany). PIV analysis is performed using the same approach as detailed in Fouras, LoJacono and Hourigan (2008).

The experimental protocol used in this investigation involves fixing the cylinder at a desired incidence angle, then running the channel to a Reynolds number slightly below the predicted Reynolds numbers for the onset of three-dimensional flow. The Reynolds number is then elevated incrementally, and PIV measurements of the near wake are taken. A large sample size is captured to permit processing methods such as phase averaging to be employed.

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