CYLINDER WAKE DESTABILISATION DUE TO ELASTIC MOUNTING

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INTRODUCTION

The transition from a steady flow to periodic vortex shedding in the wake of a circular cylinder is one of the most well-studied in fluid mechanics. It is has been shown that this transition occurs around $Re = 47$ (Dušek *et al.*, 1994), when the flow undergoes a supercritical Hopf bifurcation (Provansal *et al.*, 1987). Essentially, this means that this transition shows no hysteresis.

The cylinder wake flow has also been studied as a slowly-varying parallel flow, using the theory proposed by Bers (1983), and developed and applied by, for example, Huerre & Monkewitz (1985); Triantafyllou *et al.* (1986); Monkewitz & Nguyen (1987); Monkewitz (1988); Le Dizès *et al.* (1996); Huerre & Rossi (1998); Pier (2002); Chomaz (2005); Khor *et al.* (2008). These theories allow regions of the wake to be characterised as either globally or convectively unstable. Globally unstable regions allow any introduced perturbation to grow at the location where it is introduced. For a flow to become globally unstable (such as occurs when vortex shedding commences) a region of absolute instability is a necessary, but not sufficient, condition (Chomaz, 1991).

Monkewitz (1988) showed that a region of absolute instability exists in the wake of a cylinder for Reynolds numbers as low as $Re = 25$. This suggests that if some amplification mechanism is introduced, it may be possible to see the transition to vortex shedding occur at lower Reynolds numbers than usual.

A variety of such mechanisms has been recently tested. For example, a perturbation introduced as a controlled periodic oscillation across the stream has been used numerically (Le Gal *et al.*, 2001), and experimentally (Buffoni, 2003). Both of these studies saw periodic shedding occur for $Re \leqslant 25$, depending on the amplitude of the introduced forcing.

Mittal & Singh (2005) studied the onset of vortex shedding from an elastically-mounted cylinder, free to oscillate both in the cross-stream and streamwise directions. Following a function for the natural structural frequency as a function of Re, this study showed that vortex shedding could occur at values as low as $Re = 20$.

The study undertaken here aims to further extend these works, and attempts to unify some of the previous parallel flow study results with the experimental and numerical simulation results. The system studied is that of an elastically-mounted cylinder, constrained to oscillate in the cross-stream direction only. The oscillating cylinder system is assumed to have no mechanical damping; this results in a system with three independent parameters. These parameters are the mass ratio, $m^* = 4m/\rho\pi D^2$, Reynolds number, $Re = UD/\nu$, and the nondimensional spring constant, $k^* = D^2k/U^2m$, where m is the cylinder mass, ρ is the fluid density, D is the cylinder diameter, U is the freestream velocity, ν is the kinematic viscosity, and k is the spring constant. The nondimensional spring constant can also be expressed as the nondimensional natural frequency of the cylinder system *in vacuo*, $f_N = sqrt(k^*)$.

The simulations for this study were conducted for a mass ratio $m^* = 1$. Simulations were conducted across a range of natural frequencies for a given value of Re to determine the cylinder response. For all cases where vortex shedding was observed, the vortex shedding was purely periodic. The response was therefore characterised by the amplitude of oscillation, A^*, and the frequency of response, f, as a function of f_N. This was repeated for a range of Re below $Re = 47$, and the value of f_N at which the maximum value of A^* occurred was recorded. The frequency of response at this f_N was also recorded. In this way, the maximum response amplitude, A^*_{max}, and the response frequency at maximum amplitude (which could be interpreted loosely as the most unstable frequency), f_{max}, could be expressed as functions of Re. The natural frequency required to elicit the maximum

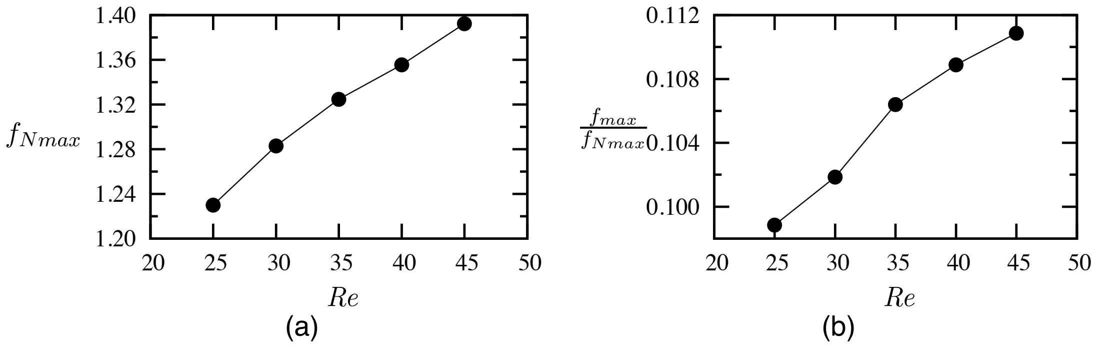

Figure 1. (a) The natural frequency, at the maximum amplitude of response, f_{Nmax}, as a function of Re. (b) The ratio of the primary frequency of response, f, and the natural frequency, f_N. No clear trend can be identified, but (b) highlights the discrepancy between f and f_N.

response, f_{Nmax}, was also expressed as a function of Re.

It was found that very high values of f_N were required to elicit any response from the cylinder, and that the frequency selected by the system was far lower than f_N. This effect could not be corrected by simply including the inviscid added mass in the natural frequency definition. This result suggests that the added mass effect is much larger at low Re. The frequency of response did not follow the extrapolated Strouhal curve, but was closer to the global frequency determined from the parallel flow theory applied to the steady base flow. This result is interpreted as implying that the premature vortex shedding is caused by a feedback between the area of absolutely unstable flow, and the cylinder.

METHODOLOGY

The simulations were conducted using a highly accurate spectral-element method to solve the incompressible Navier-Stokes equations. The spatial discretisation over each element was achieved using seventh-order Lagrange polynomials as shape functions, associated with Gauss-Legendre-Lobatto quadrature points. A three-way fractional step method was used for the time stepping, applying a semi-implicit predictor-corrector method for the advection, solving a Poisson equation for the pressure, and using a Crank-Nicholson scheme for the diffusion. To account for the body motion, the equations were solved in an accelerating frame of reference attached to the cylinder. The body motion was solved for in a coupled iterative process during the advection step, hence the requirement to use a predictor-corrector method. The code has been extensively used and validated for previous similar simulations, for examples see Thompson *et al.* (1996); Leontini *et al.* (2006, 2007).

RESULTS

Natural frequency and synchronisation

As stated in the introduction, very high values of f_N were required to elicit an oscillatory response from the cylinder. The value of f_N at the maximum response increased with increasing Re. This is clearly demonstrated in figure 1a. Comparison with the frequency of response, plotted in figure 2a, shows that the eventual synchronised frequency was well below the natural frequency, indicating that a large added-mass-type effect is present at such low values of Re. To highlight this fact, the ratio of f and f_N as a function of Re is plotted in figure 1b.

This added-mass effect could not be accounted for by using the inviscid added mass. This is not surprising, considering the significant role of viscosity at these low values of Re. Since there is no monotonic trend in the plots of figure 1, the "true" added mass is not simply a function of Re. It seems likely that it is a function of Re, A^*, and f. However, no definitive relationship is offered at this stage.

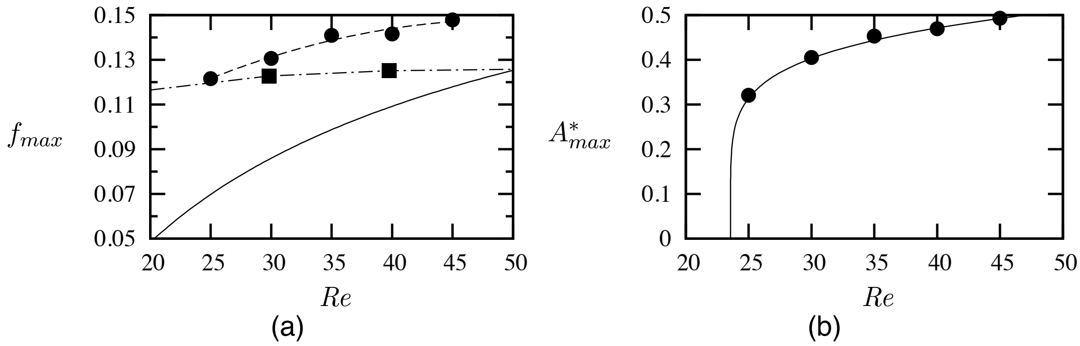

Figure 2. (a) Frequency of response at maximum response, f_{max} (•), parallel flow theory frequency (■, and extrapolated Strouhal frequency (solid line) as functions of Re. It is clear that near the onset of shedding, that the parallel flow theory frequency predicts the saturated frequency much better than the extrapolated Strouhal curve. (b) The maximum amplitude of response, A^*_{max}, as a function of Re. This plot indicates that oscillation is possible for $Re \geqslant 23$.

Frequency selection

Figure 2a compares the final saturated frequency at maximum amplitude, f_{max}, the global frequency predicted from parallel flow theory (using a saddle-point criterion) applied to the steady base flow from Pier (2002), and the Strouhal frequency extrapolated using the three-term fit given by Williamson & Brown (1998), as functions of Re. It is clear from the figure that the extrapolated Strouhal curve does not represent the saturated oscillation frequency. However, the saddle point frequency extracted from the steady base flow is much closer, but the comparison becomes progressively worse with increasing Re.

This can be explained as follows. At low values of Re, the amplitude of oscillation is small (see figure 2), and therefore the nonlinear terms do not have as great an impact on the eventual saturated frequency. For wake flows behind stationary cylinders, it has been shown by Hammond & Redekopp (1997), Pier (2002), and Leontini *et al.* (2010) that applying the parallel flow theory to the mean flow, rather than the steady flow, results in a very accurate prediction of the saturated frequency. At values of Re only slightly above those required for vortex shedding, the difference between the steady base flow and the mean flow is very small, and therefore the frequency compares well at Reynolds numbers where the maximum amplitude of oscillation is close to zero.

CONCLUSIONS

It has been confirmed that vortex shedding from an elastically-mounted cylinder occurs at much lower values of Re than from a stationary cylinder. This vortex shedding leads to a periodic oscillation of the cylinder, the amplitude of which is a function of the natural frequency of the mechanical system. The frequency of this oscillation is significantly lower than the natural frequency, suggesting that the fluid contributes an added mass to the combined fluid-structure system that is significantly larger than the inviscid added mass.

The frequency of response does not follow an extrapolation of the Strouhal curve for a stationary cylinder. However, at least near the onset of instability, the frequency of response is close to the global frequency predicted from a slowly-varying parallel flow analysis. This suggests that the onset of vortex shedding at low Re is caused by a feedback between a region of absolutely unstable flow in the near wake, and the cylinder.

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