Study of a lid-driven cavity in an axisymmetric geometry

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Abstract. We numerically study flow in a toroidal lid-driven cavity. It is shown that the radial geometry gives rise to interesting flow patterns. By comparing such a cavity to its planar counterpart, we highlight the effects of a radial geometry on the process of internal flow separation. A qualitative analogy is suggested between an axisymmetric cavity and a tall planar cavity. We also present results for internal flow-separation when the motion of the lid is periodic.

Key words: internal flow-separation, lid-driven cavity, axisymmetry, incompressible flow.

1. Introduction

At high Reynolds numbers, accelerating and decelerating flows are not mirror images of each other. The left-right asymmetry caused by the presence of a steady laminar wake downstream of a bluff-body at even moderately low Reynolds numbers is all too well-known. With this notion of asymmetry in mind, we study here the axisymmetric counterpart of the planar lid-driven cavity (Fig. 1). Here onwards, we refer to an axisymmetric lid-driven cavity as an ALDC and a planar lid-driven cavity as a PLDC. Our literature survey indicates that [1, 2, 3] seem to be the only published studies for flows in an ALDC.

As the Reynolds number is increased, 2D flow in a PLDC of finite length tends to becomes 3D due to transverse pressure gradients produced by boundary layer effects at the end-walls. Additionally at higher Reynolds numbers, the flow also suffers from centrifugal instabilities which make the flow 3D. On the other hand, flow in the ALDC does not have end walls but it still remains prone to centrifugal instabilities at higher Reynolds numbers. The studies [1, 2, 3] have looked at the ALDC mainly as an attempt to remove the end-wall effects which is necessarily present in any experimental set-up of the PLDC. However, we have studied the ALDC from a slightly different view-point. Our motivation was a recent finding [4] of a flow situation where axisymmetry was responsible for flow-phenomena not possible in planar geometries. Further, in an axisymmetric closed geometry containing an incompressible fluid, conservation of mass necessitates that fluid as it spreads radially outwards must decelerate on an average. The lid-driven cavity having sharp corners is a good test problem for studying internal separated flows (as opposed to external separated flows like that occurring in the flow over an airfoil at high angles of attack). The aim of this study is to examine if the internal flow-separation occurring inside the PLDC can be modified significantly by a radial geometry as that of an ALDC and to understand the physics of these geometry-induced separated flows.

The paper is organised as follows. We start with a brief description of the numerical schemes used for solving the steady and unsteady Navier-Stokes equations for the ALDC geometry. The steady-state solutions for the square ALDC are compared to those available for the PLDC. We then make a limited comparison between the ALDC and the PLDC with large aspect ratio to point out certain qualitative similarities. The unsteady solutions are presented for the special case where the ALDC lid moves with a sinusoidal velocity. We make a brief comparison between this and the known results for a PLDC, especially with respect to variation of Reynolds number and curvature. In the discussion section, a previously known model for the PLDC is used to suggest a physical explanation for the kind of flow separation observed in the ALDC. We finally conclude with a summary of the main results and scope for further work.

2. Numerical calculations

The 2D, incompressible, axisymmetric continuity and Navier-Stokes equations in cylindrical coordinates, written in streamfunction-vorticity formulation are:

$$\omega_\theta = \frac{1}{r}\frac{\partial^2\psi}{\partial z^2} - \frac{1}{r^2}\frac{\partial\psi}{\partial r} + \frac{1}{r}\frac{\partial^2\psi}{\partial r^2} \tag{1}$$

$$\frac{\partial\omega_\theta}{\partial t} + \left(\frac{\partial\psi}{\partial z}\right)\left(\frac{\partial\omega_\theta}{\partial r}\right) - \left(\frac{\partial\psi}{\partial r}\right)\left(\frac{\partial\omega_\theta}{\partial z}\right) = \frac{r}{\mathrm{Re}}\left[\frac{\partial^2\omega_\theta}{\partial z^2} + \frac{\partial^2\omega_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\omega_\theta}{\partial r} - \frac{\omega_\theta}{r^2}\right] \tag{2}$$

Referred to a cylindrical coordinate system the symbols represent :

ω_θ - Azimuthal component of vorticity
ψ - Streamfunction
r - Radial coordinate of the cylinder
z - Axial coordinate along the axis of the cylinder
Re - Reynolds number based on the lid velocity and the length of lid

The steady state version of equations (1) and (2) are solved numerically subject to no-slip and no-penetration boundary conditions at the walls. The initial condition is that of no flow inside the ALDC. At time t > 0 the top lid is impulsively moved radially outwards with a constant velocity U. At t > 0 the vorticity at the wall is calculated using the Thom's boundary condition. The equations are solved by an iterative finite-difference scheme using successive over-relaxation [6]. The method uses central differencing and is second order accurate in space. Convergence is said to have been achieved when the cumulative difference between two successive iterations was less than 10^{-6}. The unsteady equations (1) and (2) are solved using a finite-difference scheme having second order accuracy in space and the time derivative is approximated using a fourth order explicit Runge-Kutta method. As shown in [7], this results in a scheme which is stable under the CFL condition given by the convective terms. A 128 x 128 grid has been used for all the results presented and grid-independence of the results has been verified using 64 x 64 and 256 x 256 grids. The correctness of the numerical schemes has been verified by reproducing the previously published results of [8] and [9] for a PLDC and these are presented in Fig 2 and Fig. 3. The ALDC code was validated by recovering the results for a PLDC when the inner radius was very large. All streamline plots presented at the end of the paper are in the r-z plane in the cylindrical coordinate system defined above.

2.1. STEADY STATE RESULTS

Before presenting the results for a square ALDC, it is useful to briefly summarise the well-known results for a square PLDC as the Reynolds number is increased. At Re = 1000, the flow inside a square PLDC consists of a primary vortex and two secondary vortices at the corners [8]. At Re = 5000, a third vortex starts developing at the upstream corner near the moving lid [8]. As the Reynolds number is increased to 10000, the downstream corner displays two secondary vortices [8]. It is possible to go higher in Reynolds number and still obtain steady 2D solutions in a PLDC. The important thing to be noted here is that for a square PLDC upto Reynolds 10000 there is only one primary vortex. Multiple primary vortices start appearing only when the aspect ratio of the PLDC exceeds one [5].

The non-dimensional numbers for flow in a square ALDC shown in fig 1 are: Reynolds number ($Re = UL/\upsilon$) and curvature $\delta = 2*L / \{R1 + R2\}$ defined in the same way as [1, 2]. It was shown in [3] that the 2D axisymmetric flow in an ALDC is unstable in the linear sense, at higher Reynolds numbers to disturbances of the centrifugal type, for all values of curvature δ. The critical Reynolds number was shown to reduce significantly with increasing δ. Unless otherwise specified, we only present results in the linearly stable regime. Fig. 2 is a contour plot of the steady-state streamfunction ψ with non-dimensional parameters δ = 0.66, Re = 100 and a radially outward lid velocity. Comparing fig 4 to fig. 2, it is seen that at δ = 0.66 and Re = 100, the radial geometry hardly produces any noticeable difference in flow patterns and a PLDC and an ALDC looks almost the same. However, in fig. 5, for the same value of δ = 0.66 but Re = 500, the ALDC flow pattern now looks very different from a PLDC. The flow depicted in figs. 2 & 5 are in the subcritical range. It is seen from the figures that now there are 2 primary vortices in the flow (the corner eddies are still present although this might not be visible in the figure). As remarked in the previous paragraph, this is reminiscent of a PLDC with aspect ratio greater than 1 where too multiple primary vortices have been reported [5]. Since there are two non-dimensional parameters for an ALDC viz. Re and δ, it is interesting to see what the flow would look like in the double limit $Re \to 0$ and $\delta \to \infty$. In our study we find that as $Re \to 0$, the curvature δ no matter how large, seems to have a negligible effect on the flow. We may conclude that the most significant effect of axisymmetry is through the convective term. As we increase Reynolds number, axisymmetry starts producing more pronounced effects on the flow. As an illustration, fig. 6 shows the streamfunction plot for $\delta = 1.81$ and $Re = 1000$. As seen from the figure, as we increase Reynolds no. the orientation of the two primary vortices start changing and they align themselves roughly along the diagonal of the cavity. This is in contrast to a PLDC with an aspect ratio greater than unity, where the multiple recirculation regions are always oriented parallel to the bottom. It should be however remembered that at $\delta = 1.81$, the critical Reynolds no. is less than 530 [3] and hence fig. 6 represents a flow which is linearly unstable. Nevertheless it gives us an idea as to how at high Reynolds numbers, a radial geometry can produce substantial changes in internal flow separation. Interestingly the effects of axisymmetry at high Reynolds numbers are pronounced only when the lid is driven radially outwards. As can be seen from fig. 7 if the ALDC lid is driven radially inwards, the flow inside looks very similar to a PLDC [3]. That it should be so is not immediately obvious, because a radially inward accelerating flow at the lid has to be compensated by a corresponding outward decelerating flow at the bottom of the ALDC. We might thus expect flow-separation and development of multiple recirculation regions at the bottom where there is an outward flow, but the numerical results show none. The steady-state results presented in this section will be discussed more later in the paper.

2.2. UNSTEADY RESULTS

In this section we present unsteady results for the ALDC obtained from the solution of equations (1) and (2). The boundary conditions remain the same but instead of an impulsively started plate with constant radially outward velocity U, we have a radial sinusoidal velocity $U\sin\Omega t$ at the lid. The non-dimensional parameters governing the system are Reynolds number ($Re = UL/\upsilon$), curvature $\delta = 2*L / \{R1 + R2\}$) as before and additionally a Strouhal number ($St = \Omega L/U$) arising out of the new time-scale $1/\Omega$. Fig. 3 shows a contour plot of the streamfunction at $t = 2\pi + 2St/8$, inside a PLDC where the non-dimensional parameters are $Re = 400$ and $St = 1$. The 2π additive is in order to look at the periodic flow after the initial transients. This result agrees well with the results presented in [9] and thus validates the unsteady numerical scheme [7] that we have used. We mention here that to the best of our knowledge, a study of the flow in an ALDC with an oscillating lid has not been reported in literature before.

As in the previous section, before discussing the unsteady results for an ALDC, we briefly summarise the known results for a PLDC with a sinusoidally oscillating lid. Such results exist for example in [9, 14]. It was shown in [9] that for very high St, the presence of the side-walls have almost no effect on the bulk of the flow and the effect of the motion of the lid was restricted to a thin-boundary layer of constant thickness attached to the main wall, similar to Stokes second problem. On the other hand, for very low St, the flow looked like that inside a steady PLDC as discussed in the last section. For St ~ O(1), the flow displayed sensitivity to changes in Reynolds number. A complete parametric study of this unsteady problem is beyond the scope of this paper and here we restrict ourselves to St ~ O(1) and examine only the effects of varying Re and the curvature δ. Fig 9 presents a streamfunction plot at Re = 800, St = 1, δ = 0.66 at t = 2π. Notice that a separated region is present at the top right hand corner. The bias of the primary vortex towards the left corner is due to the effect of the inward motion of the lid in the second half of the previous cycle. Compare this to fig. 8 where the parameters are Re = 400, St = 1, δ = 0.66 at t = 2π. The effect of Reynolds no. on the flow separation region is clearly visible. However, these differences are minimal at other phases (fig 10). It should also be noted that fig 9, looks qualitatively the same as a PLDC with an oscillating lid. We have some results which indicate that if δ is pushed very high, then the flow inside an ALDC may start looking very different from a PLDC. However more careful cross-checks are required to validate the accuracy of these results and hence these are not presented here.

2.3. Discussion of results

In this section we make an attempt to qualitatively interpret the results presented so far. To start with, we try to address the question of how are multiple primary eddies formed inside an ALDC at steady state, when the lid moves radially outwards with constant velocity. As demonstrated in [11], in cavities with finite aspect ratio, as the Reynolds no. is increased, the steady flow consists of a inviscid core of uniform vorticity with very thin viscous layers close to the walls. When the thicknesses of these viscous layers are small compared to the dimensions of the cavity, it is possible to interpret the formation of multiple recirculation regions inside the cavity as arising out of boundary layer separation. For example, as discussed by Schlichting [10], the corners are stagnation points where most of the kinetic energy of the outer inviscid flow gets converted into pressure. This imposes a very strong adverse pressure gradient on the boundary layer, which thus separates and causes the development of recirculation regions near the corners. Such a physical interpretation in terms of boundary layer separation was also suggested in [13]. However this explanation becomes unsatisfactory in the Stokes limit where we can no longer speak of boundary layers and viscosity starts becoming important everywhere in the cavity. It is well known that flow in a PLDC with a high aspect ratio exhibits multiple eddies even in the Stokes limit. Shankar [12] suggested a mechanism of formation of these multiple eddies. As the aspect ratio of the PLDC increases, the corner eddies grow in size and they merge. This is a continuous process in which corner eddies merge to form primary eddies while new corner eddies are created. We are inclined to think that a similar mechanism could be at work even in an ALDC except that here the role of aspect ratio is played by curvature δ. However, we presently do not understand why the separating streamline orients itself roughly along the diagonal as the Reynolds number is increased. We have also made only a very brief study of the case of the oscillating lid. In order to interpret the unsteady results physically, more parametric studies are required.

3. Conclusions

We have studied the effect of radial geometry on the steady and unsteady flow inside an ALDC. In the steady state, the effects of axisymmetry are pronounced only at high Reynolds numbers. The ALDC with multiple separated regions looks qualitatively like a PLDC whose aspect ratio is greater than unity. The process of creation of multiple eddies in an ALDC may happen in a way analogous to that suggested for a PLDC with high aspect ratio. The flow in an ALDC with the lid moving inwards looks the same as a PLDC.

The high Reynolds nos. flows for the ALDC need to be checked with uniform vorticity models like that of Batchelor [15]. Similarly the high frequency and low frequency regimes need to be studied. It is expected that both Reynolds number and curvature would have a strong influence on the low frequency regimes where the flow is expected to closely resemble that of a steady ALDC.

Figures

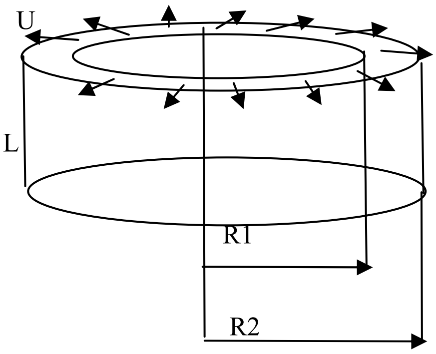

Figure1. ALDC, Lid moving outwards

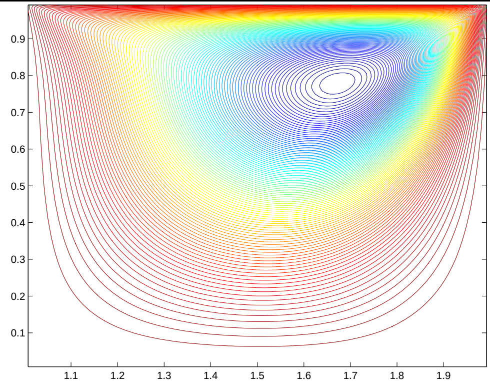

Figure2. ALDC, Re – 100 - δ – 0.66

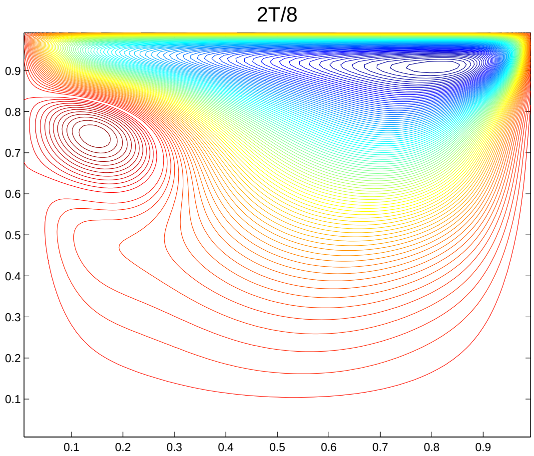

Figure 3. PLDC – St – 1 – Re – 400 – t = 2T/8

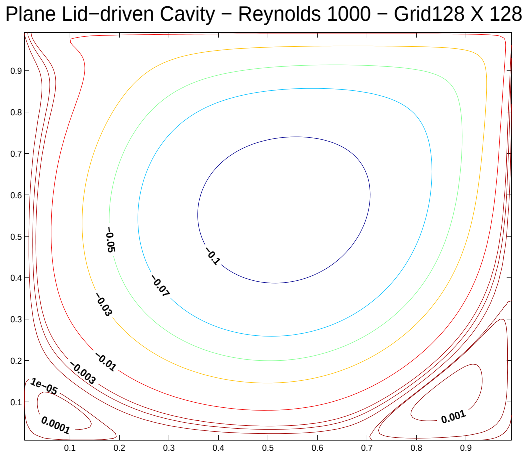

Figure 4. PLDC – Re - 1000 – left to right

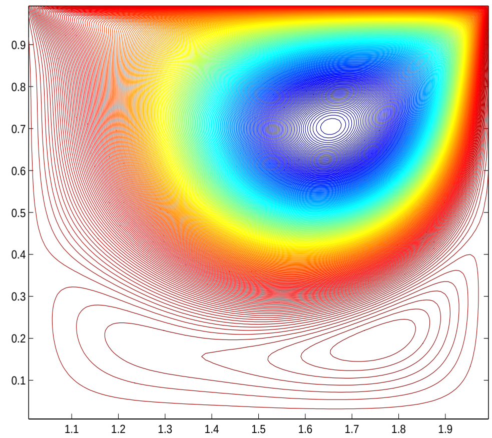

Figure 5. ALDC – Re - 500 – δ - 0.66

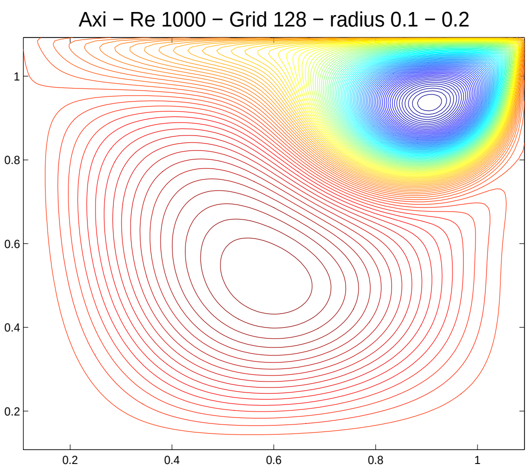

Figure 6. ALDC –Re – 1000 –δ – 1.81

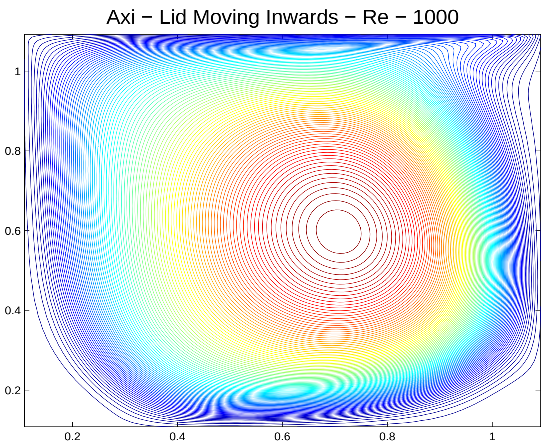

Figure 7. ALDC –Re – 1000 –δ – 1.81

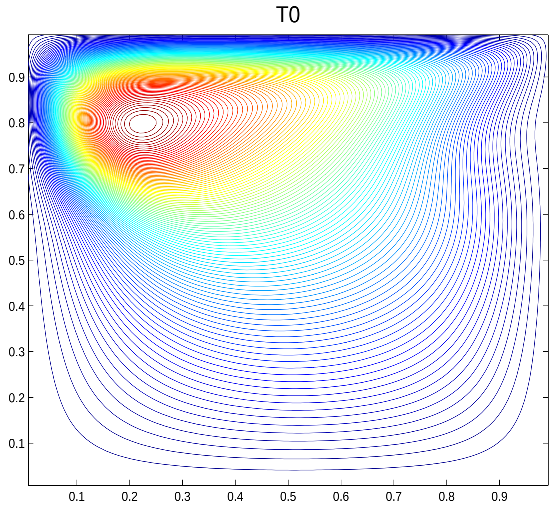

Figure 8. ALDC – Re - 400 – St – 1 - δ - 0.66

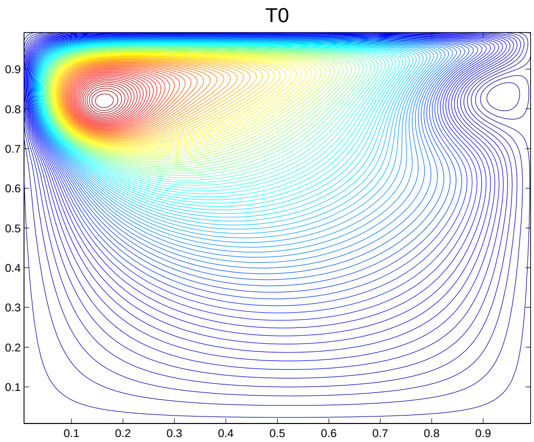

Figure 9. ALDC – Re - 800 – St – 1 - δ - 0.66

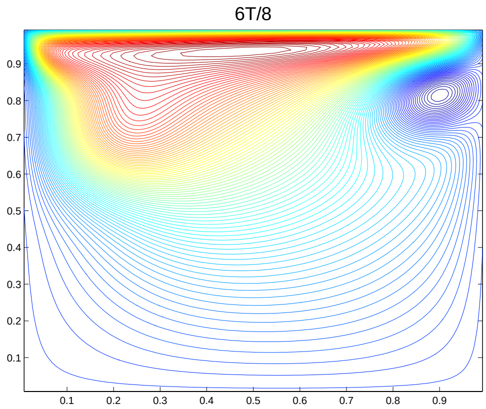

Figure 10. ALDC – Re - 800 – St – 1 - δ - 0.66 – t = 6T/8

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