Multi–Disciplinary Optimization for the Conceptual Design of Innovative Aircraft Configurations

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Summary

The paper presents an overview of recent work by the authors and their collaborators on multi–disciplinary optimization for conceptual design, based on the integrated modeling of structures, aerodynamics, and aeroelasticity. The motivation for the work is the design of innovative aircraft configurations, and is therefore first–principles based, since in this case the designer cannot rely upon past experience. The algorithms used and the philosophy behind the choices are discussed.

Introduction

The aim of this paper is to present an overview of the work of the authors and their collaborators in the field of MDO/CD (*Multi–Disciplinary Optimization for Conceptual Design*), for innovative aircraft configurations. In this paper, we emphasize the philosophy behind the choices being used in developing the methodology.

This paper is rather projected toward the future, towards what needs to be done and what are the criteria to be used – the work performed in the past is briefly reviewed in this paragraph. The main motivation and source of inspiration for our work has been a specific innovative aircraft configuration, which has, as a distinguishing feature, a low induced drag. This was proposed by Frediani [1] and by him denoted as *Prandtl–Plane*, in honor of the Prandtl [2] work on unswept box wings.[3] The proposed configuration is a counter–swept box–wing, *i.e.*, a biplane with a backward–swept low front wing and forward–swept high back wing (which acts as a horizontal stabilizer as well); these are connected to each other by vertical streamlined connections. The emphasis has been on wing design, with the fuselage assumed as given. Numerical and experimental studies performed in the past few years by the authors and their collaborators (Refs. [5], [6], [7], [8], and [9]) have confirmed that the induced drag of this configuration is considerably lower than the induced drag of an equivalent monoplane. This fact allows one to reduce the wing–span of this configuration without major drag penalties, thereby facilitating the capability of respecting the maximum spanwise dimensions (critical for the NLA – New Large Airplanes – with classical wing configuration), as required by existing airport regulations. Moreover, the low induced drag could allow one to reduce community noise (lower take–off power required, Ref. [10]) and possibly chemical pollution. The methodology has been successfully applied also to the optimal design of a Blended–Wing–Body configuration (see Ref. [11]).

Within this context (that is, in the process of assessing the pros and cons of the Prandtl–Plane), the authors have developed (and are still in the process of developing) a computer code called MAGIC (Multidisciplinary Aircraft desiGn of Innovative Configurations). MAGIC is an evolution of the code FLOPS developed by Mc Cullers (see, *e.g.*, Ref. [12]), which is essentially based on elementary and/or

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[3] This is closely related to the joined–wing concept. Comprehensive overviews on joined–wing configurations are presented in Refs. [3] and [4].

empirical algorithms. Such an approach is not possible for innovative configurations, for which the designer cannot rely upon past experience. Thus, the first and foremost criterion used in developing MAGIC is that the algorithm be based on first principles, whenever possible. This is the approach used in particular in developing the MAGIC modules for structure, aerodynamics, and aeroelasticity (see, *e.g.*, Refs. [13], [14], [15], [16], and [17]), which are all first–principles based and are described in this paper.

Another aspect of the philosophy used stems from the fact that we are interested in the conceptual design phase (MDO/CD). Hence, it is highly desirable to use algorithms that produce accurate predictions with a relatively small computational effort. Accordingly, in this work we address the advantages of modeling (modal approach, boundary elements, reduced order models) over simulation (finite elements, computational fluid dynamics) in the context of MDO/CD. In addition, the code MAGIC is geared specifically towards for civil aviation; hence, advantage is taken of this aspect whenever possible. In summary, the physical models chosen must be able to capture the essence of the phenomenon within the specific application of interest, with the corresponding numerical algorithms being very efficient and at the same time adequately accurate (and apt to be refined as much as necessary).

The last criterion we follow is that strong emphasis be given to the integration of the various disciplines. This implies not only that special care be given to the interfaces, but also that the concurrency of certain types of analysis be exploited whenever possible. For instance, the fact that the natural modes of vibration must be evaluated for the dynamic aeroelastic analysis implies that a modal analysis may be used for the stress analysis as well. Similar considerations hold for steady and unsteady aerodynamics algorithms. What we are saying here is that the final objective is to develop a code that is not a collection of the codes used for the individual disciplines. It is necessary to start from scratch. Indeed, the methods that are the most convenient for the individual disciplines are not necessarily the most convenient in the global context. Therefore, our work is based upon a critical analysis of the methodologies that are best suited for the stated goal.

On the basis of all these considerations, our choices have been towards the following methodologies: (i) a linear elastic finite–element method for the wing structure, (ii) quasi–potential flows (*i.e.*, flows that are potential everywhere except for a zero–thickness wake surface emanating from the trailing edge) for the aerodynamic analysis, with an integral boundary–layer analysis for the viscous effects (this is a good example of taking advantage of the civil–aviation applications), and (iii) modal analysis and reduced order model (ROM) for aeroelasticity. All three of them assure the high efficiency required, with an accuracy that is quite adequate within civil aviation applications. We will address these issues in some details in the remainder of the paper.

Modeling vs. simulation in structural dynamics

We begin with structural dynamics, which presents a clear exemplification of what we mean by "modeling" and "simulation," and for which the advantages of modeling over simulation are apparent (aerodynamics and aeroelasticity are examined in the following sections). In the following, we discuss the linear formulation, which is standard in conceptual aircraft design. Combining the linearized momentum equation with the constitutive equations for linear elastic material and the linear strain tensor yields $\rho\ddot{\mathbf{u}}+\mathbf{L}\mathbf{u}=\mathbf{f}$, where $\mathbf{L}$ denotes a tensor operator with Cartesian components $L_{ik}(\ldots)=-[c_{ijkl}(\ldots)_{/l}]_{/j}$.[4]

[4]We prefer to derive the structural dynamics equations from the differential approach to emphasize the relationship between finite–element (simulation) and modal (modeling) methods, as well as the commonality between solids and fluids. The same

Using the Bubnov–Galerkin method, Ref. [18], one seeks an approximate solution of the type $\mathbf{u}(\mathbf{x},t) = \sum_{n=1}^{N} u_n(t)\Psi_n(\mathbf{x})$, where $\{\Psi_n\}$ is a set of linearly independent functions which satisfy suitable boundary conditions (for the issues related to the distinction between essential and natural boundary conditions, see, *e.g.*, Ref. [18]). The Galerkin equations are obtained by projecting the resulting expression in direction Ψ_n, to obtain a system of linear second–order differential equations, in the unknown $\mathsf{u} = \{u_n\}$, given by $\mathsf{M}\ddot{\mathsf{u}} + \mathsf{K}\mathsf{u} = \mathsf{f}$, where the elements of $\mathsf{M} = [M_{kn}]$, $\mathsf{K} = [K_{kn}]$, and $\mathsf{f} = \{f_k\}$ are given by (neglecting volume forces)

$$M_{kn} = \int_{\mathcal{V}_S} \rho\, \Psi_k \cdot \Psi_n d\mathcal{V} \qquad K_{kn} = \int_{\mathcal{V}_S} \Psi_k \cdot \mathbf{L}\Psi_n d\mathcal{V} \qquad f_k = \oint_{\mathcal{S}} \mathbf{t}\cdot\Psi_k d\mathcal{S} \tag{1}$$

Next, we discuss the choice for the functions Ψ_n. In the *finite–element method*, u_n typically denotes nodal values of the displacement components, whereas $\Psi_n(\mathbf{x})$ are suitable interpolation functions. We refer to this approach as *simulation*. Generally speaking, in order to have a good approximation of the solution, the number N of the unknowns u_n is very high (for a wing treated as a beam, $N = 10 \div 30$; for the complete aircraft configuration, $N = 10^4 \div 10^6$). Thus, the numerical solution of the above equations is highly computer intensive. As mentioned above, as an alternative, one may use a modal approach (spectral method), *i.e.*, set $\mathbf{u}(\mathbf{x},t) = \sum_{m=1}^{M} q_m(t)\Phi_m(\mathbf{x})$, where $\{\Phi_n\}$ are the normalized natural modes of vibration of the structure (eigenfunction of the operator $\mathbf{L}$), which satisfy the equation $\mathbf{L}\Phi = \rho\lambda\Phi$, with homogeneous boundary conditions. In this case, $\mathsf{M} = \mathsf{I}$ and $\mathsf{K} = \Omega^2$, where $\Omega^2 = [\omega_n^2\delta_{kn}]$ (with $\omega_n^2 = \lambda_n$) and the structural dynamics equations reduce to $\ddot{\mathsf{q}} + \Omega^2\mathsf{q} = \mathsf{e}$, where $\mathsf{e} = \{e_k\}$, with $e_k = \oint_{\mathcal{S}} \mathbf{t}\cdot\Phi_k d\mathcal{S}$.

It is a rather common belief that the advantage of the modal approach ($\ddot{\mathsf{q}} + \Omega^2\mathsf{q} = \mathsf{e}$) over the finite element approach ($\mathsf{M}\ddot{\mathsf{u}} + \mathsf{K}\mathsf{u} = \mathsf{f}$) is the fact that in the first the equations are uncoupled. Whereas this fact is true in structural dynamics, this property does not apply in aeroelasticity and aircraft dynamics, because in this case coupling appears through the aerodynamic forces, which are functions of the unknown $\mathbf{u}$. Nonetheless, the modal approach is more advantageous in aeroelasticity and aircraft dynamics as well – the reason is that, for smooth functions, the convergence of an expansion in terms of orthogonal functions, such as the natural modes of vibration, has a very high rate of convergence.[5] This implies that even approximate natural modes are adequate, because the relevant aspect is the orthogonality of the base functions, not the decoupling of the equations (in the code MAGIC, the approximate finite–element modes of vibration are obtained by using the finite–element method described above, as applied to the solution of the eigenvalue problem – it is easy to show that these approximate modes satisfy the same orthogonality conditions as the exact ones).

Finally, it is easy to show that using the Galerkin method with the base functions given by the approximate eigenfunctions is fully equivalent to diagonalizing the finite–element equations and truncating the system to the first M modes. From this observation, we gather that the approximate–mode equations may (in the limit as the number of modes M tends to the number of finite element unknowns N) contain as much information as the finite–element equations. However, the modal model may be reduced considerably in number, typically without much loss in accuracy. Indeed, in aeroelastic analysis, $M << N$ (the modes corresponding to the lowest frequencies are used; for a complete configuration, $M = 10 \div 30$, a considerable reduction with respect to the finite–element approach, where for the full

results would be obtained using the Lagrange equations of motion.

[5]For a discussion of the convergence properties of *spectral methods* (of which the expansion in terms of the natural mode of vibration is a particular case), the reader is referred to Gottlieb and Orszag [19].

configuration $N = 10^4 \div 10^6$). Consequently, it is apparent that, within a MDO/CD context, structural–dynamics modeling (*i.e.*, approximate–mode approach) is preferable to structural–dynamics simulation (*i.e.*, finite elements).[6]

In all of the papers cited above a simple equivalent–beam model has been used for the wing structure. Recently, a more sophisticated model (beams plus in–plane loaded plates) has been added to MAGIC (see Ref. [20] for details). A full three–dimensional finite–element method (which is based upon the Hermite interpolation, and has been developed specifically for optimization but is not yet included in MAGIC), is discussed in Ref. [21].

These models with different levels of sophistication may be combined by implementing an optimization procedure proposed by Alexandrov and Lewis [22]; this procedure allows one to use a sophisticated model used to "calibrate" a simple one, through an affine transformation which is kept constant during several iterations of the optimization procedure. This yields the accuracy of the sophisticated model with an efficiency only slightly lower than that obtained with the simple model.

Modeling vs. simulation in aerodynamics

Next, consider aerodynamics. Again, we begin with simulation methodologies, as useful background for discussing the modeling methodology proposed here. In our definition, aerodynamics simulation is based upon solution of the equations of conservation of mass (continuity), momentum (Euler or Navier–Stokes), and energy, by a methodology broadly known as CFD (Computational Fluid Dynamics. The CFD method most commonly used is the finite–volume technique, which consists of writing a discretized form of the conservation principles for a small volume. This may be considered as a special approach to obtain finite–difference expressions, and also as a very crude finite–element formulation for the above equations (with weight functions equal to one within the element, and to zero otherwise – partition of unity). Again, the number of degrees of freedom for a complete aircraft configuration is very high (*e.g.*, $10^5 \div 10^7$, the lower numbers being obtained in the inviscid case, or when a inviscid–viscous coupling is used, *e.g.*, with a finite–volume Euler external–flow analysis coupled with a boundary–layer or thin–Navier–Stokes viscous–flow analysis). Thus, these techniques are highly computer intensive; while fundamental in a simulation environment, they are not suitable in a MDO/CD context, in which it is desirable to utilize simpler methods, able to yield accurate solutions with computational efforts reduced as much as possible.

Indeed, in the case of interest here – civil aviation – we are dealing primarily with high–Reynolds–number attached flows, and traditional numerical methods in aerodynamics (where a boundary element code is coupled with an integral boundary-layer analysis) are tools more convenient than CFD.[7] Specifi-

[6] A few comments on convergence rate are in order. The convergence rate of the approximate–mode expansion is initially similar to that of spectral methods (*i.e.*, much higher than that for finite–elements) since, for $M << N$, the Mth approximate mode is virtually identical to the exact one. On the other hand, as M increases to its maximum value, N, the convergence rate becomes gradually poorer, since for $M = N$, the modal expansion is fully equivalent to the finite–element one, as the two span exactly the same space.

[7] Here, at the risk of oversimplifying the situation, we think of a fluid dynamicist as someone starting with very low Reynolds number, in the limit $Re = 0$, and working his way up; indeed, much of the work in CFD started with low Reynolds number flows. On the contrary, an aerodynamicist starts from attached flows with very high Reynolds number, in the limit $Re = \infty$, and works his way down. Indeed, classical aerodynamic formulations are based upon Prandtl's work on viscous/inviscid interaction, with thin attached boundary layers, which imply very high Reynolds numbers. For the attached high–Reynolds–number flows of interest here, the aerodynamicist's approach is at least as accurate as that of the computational fluid dynamicist [23].

cally, the method we propose for MDO/CD is a boundary–element analysis for compressible (subsonic) quasi–potential flows (*i.e.*, flows that are potential everywhere except for the wake surface, which is the locus of the points emanating from the trailing–edge, Ref. [17]), coupled with an integral boundary–layer analysis; the potential–viscous coupling is based upon the Lighthill [28] transpiration–velocity approach (see last section). The reason for this choice is that boundary elements for quasi–potential subsonic flows requires the same order of magnitude of computational effort as the methods typically used in industry for conceptual design (*e.g.*, vortex–method analysis for incompressible potential flows), while at the same time being obviously more sophisticated than those methods in terms of physical/geometrical representation and considerably more accurate.

In this section, we present the formulation for the limited case of incompressible quasi–potential flows, since the extension of the formulation to compressible flows is treated extensively in Refs. [14] and [17], to which the reader is referred for details. An inviscid, incompressible, initially–irrotational flow remains, at all times, quasi–potential. In this case, the velocity field, $\mathbf{v}$, may be expressed as $\mathbf{v} = \nabla\varphi$ (where φ is the velocity potential). Combining with the continuity equation for incompressible flows, $\nabla \cdot \mathbf{v} = 0$, yields $\nabla^2\varphi = 0$. The boundary conditions for this equation are as follows. The surface of the body, $\mathcal{S}_B$, is assumed to be impermeable; this yields $(\mathbf{v} - \mathbf{v}_B) \cdot \mathbf{n} = 0$, *i.e.*, $\partial\varphi/\partial n = \chi := \mathbf{v}_B \cdot \mathbf{n}$, where $\partial/\partial n = \mathbf{n} \cdot \nabla$, whereas $\mathbf{v}_B$ is the velocity of a point $\mathbf{x} \in \mathcal{S}_B$, and $\mathbf{n}$ is the outward unit normal to $\mathcal{S}_B$. At infinity, in a frame of reference fixed with the unperturbed air, we have $\varphi = 0$. The boundary condition on the wake surface, $\mathcal{S}_W$, are obtained from the principles of balance of mass and momentum across a surface of discontinuity and are given by: *(i)* the wake surface is impermeable, and *(ii)* the pressure, p, is continuous across it. These imply that, for $\mathbf{x}$ on $\mathcal{S}_W$, *(i)* $\Delta(\partial\varphi/\partial n) = 0$, where Δ denotes discontinuity across $\mathcal{S}_W$, and *(ii)* $\Delta\varphi$ = constant in time following a wake point $\mathbf{x}_W$ (whose velocity is the average of the fluid velocity on the two sides of the wake), *i.e.*, $\Delta\varphi(\mathbf{x}_W, t) = \Delta\varphi(\mathbf{x}_{TE}, t - \tau)$, where τ is the time required to the material point to move from the trailing edge point $\mathbf{x}_{TE}$ to the wake point $\mathbf{x}_W$. Hence, $\Delta\varphi$ on the wake equals the value it had when $\mathbf{x}_W$ left the trailing edge. Finally, the trailing–edge condition states that, at the trailing edge, $\Delta\varphi$ on the wake equals $\varphi_2 - \varphi_1$ on the body, where the subscripts 1 and 2 denote the two sides of the wing surface (for a detailed analysis of this issue, see Morino and Bernardini [16]). Once the above problem has been solved, the pressure is obtained from the Bernoulli theorem.

In the methodology used in the code MAGIC, the above problem for the velocity potential is solved by a boundary–element formulation. The boundary integral representation for this problem, using the above wake boundary conditions, is given by (see Refs. [14] and [17])

$$\varphi(\mathbf{x}, t) = \oint_{\mathcal{S}_B} \left(G\chi - \varphi \frac{\partial G}{\partial n} \right) d\mathcal{S}(\mathbf{y}) - \int_{\mathcal{S}_W} \Delta\varphi_{TE}(t - \tau) \frac{\partial G}{\partial n} d\mathcal{S}(\mathbf{y}), \tag{2}$$

with $G = -1/4\pi\|\mathbf{y} - \mathbf{x}\|$ and χ prescribed from the above impermeability boundary condition. Note that, in the absence of the wake, Eq. 2, in the limit as $\mathbf{x}$ tends to $\mathcal{S}_B$, yields a boundary integral equation for φ on $\mathcal{S}_B$, with χ on $\mathcal{S}_B$ known from the boundary condition. Once φ on the body is known, φ (and hence $\mathbf{v}$ and, by using Bernoulli's theorem, p) may be evaluated everywhere in the field. The situation is similar in the presence of the wake, since, by applying the wake and trailing–edge conditions, $\Delta\varphi$ on the wake may be expressed in terms of φ over the body at preceding time steps. It should be noted that the geometry of the wake is not known *a priori*. However, in the case of airplanes one may assume, without much loss of accuracy, the wake to be parallel to the undisturbed flow (small–disturbance assumption), which we take to be the direction of the x-axis (for a free wake analysis, see Ref. [14]); consistently, we have that τ is given by $\tau = (x_W - x_{TE})/U_\infty$. Note that now the integral operator is linear. Then, taking the Laplace transform of Eq. 2, one obtains

$$\tilde{\varphi}(\mathbf{x}) = \oint_{\mathcal{S}_B} \left(G\tilde{\chi} - \tilde{\varphi}\frac{\partial G}{\partial n} \right) d\mathcal{S}(\mathbf{y}) - \int_{\mathcal{S}_W} \Delta\tilde{\varphi}_{TE}\, e^{-s\tau} \frac{\partial G}{\partial n} d\mathcal{S}(\mathbf{y}), \tag{3}$$

where ˜ denotes Laplace transformed functions. Equation 3 may be discretized by dividing the surfaces $\mathcal{S}_B$ and $\mathcal{S}_W$ into small elements, $\mathcal{S}_j$ ($j = 1\cdots, N_B$), and $\mathcal{S}_n$ ($n = 1, \cdots, N_W$) respectively, and assuming $\tilde{\varphi}$, $\tilde{\chi}$, and $\Delta\tilde{\varphi}$ to be constant within each element (zeroth–order boundary–element formulation, see Ref. [17]; for a third–order formulation, see Ref. [16]). This yields the matrix E_{IE} (used in the next section), which relates the vector of the values of the velocity potential (evaluated at the element centers), to the vector of the normal–wash, $\chi = \partial\varphi/\partial n$ (also evaluated at the element centers).

The above formulation is used for steady as well as unsteady aerodynamics. For the evaluation of the steady–state potential–aerodynamics loads, we use the formulation of Ref. [24] – an exact extension of the Trefftz formulation, Ref. [25]; this is completed by the steady boundary–layer analysis of the viscosity effects, discussed in the last section. The unsteady aerodynamics formulation is used for flutter and gust response (when the viscosity effects are typically negligible). Its coupling with the structural dynamics formulation is discussed in the next section.

Finally, note that for compressible unsteady flows, the mesh required for convergence is much finer than that for unsteady incompressible flows. Thus, this is another item to which apply the Alexandrov and Lewis [22] procedure discussed at the end of the preceding section. The same holds true for the transonic steady–state analysis, which requires the use of volume elements to take into account the non–linear terms (see, *e.g.*, Ref. [26]). On the other hand, for the unsteady case (needed for the flutter analysis), it may be shown that is still possible to use a linear formulation (*i.e.*, linearizing the volume terms, Ref. [27]). Nonetheless, this is still considerably more computer intensive than the subsonic case, and hence the Alexandrov and Lewis procedure is a good candidate here as well.

Aeroelastic modeling

In this section, we show how the structural and the aerodynamic modeling may be coupled to obtain the formulation for aeroelasticity. This is accomplished by noting that the vector of the generalized aerodynamic forces is given by $\tilde{\mathsf{e}} = q_D \mathsf{E}(\breve{s})\tilde{\mathsf{q}}$, where $q_D = \frac{1}{2}\rho_\infty U_\infty^2$ is the dynamic pressure, whereas $\breve{s} = s\ell/U_\infty$ is the dimensionless Laplace parameter (also known as complex reduced frequency; note that the reduced frequency is given by $k = Imag(\breve{s})$). The matrix E is given (in the Laplace domain) by $\mathsf{E}(\breve{s}) = \mathsf{E}_{GF}\mathsf{E}_{BT}(\breve{s})\mathsf{E}_{IE}(\breve{s})\mathsf{E}_{BC}(\breve{s})$ where: (i) the matrix E_{BC} (obtained from the boundary condition, $\chi = (U_\infty\mathbf{i} + \sum_n \dot{u}_n\Phi_n)\cdot\mathbf{n}$) relates the vector $\tilde{\mathsf{f}}_\chi$ of the dimensionless normalwash at the element centers, to the generalized coordinates vector o$\tilde{\mathsf{q}}$, as $\tilde{\mathsf{f}}_\chi = \mathsf{E}_{BC}\tilde{\mathsf{q}}$; ($ii$) E_{IE} (obtained from the integral equation, see paragraph that follows Eq. 3) relates the vector $\tilde{\mathsf{f}}_\varphi$ of the dimensionless velocity potential at the element centers, to $\tilde{\mathsf{f}}_\chi$, as $\tilde{\mathsf{f}}_\varphi = \mathsf{E}_{IE}\tilde{\mathsf{f}}_\chi$; ($iii$) the matrix E_{BT} (obtained from the linearized Bernoulli theorem, $\mathsf{c}_p = -2(\dot{\varphi} + U_\infty\partial\varphi/\partial x)/U_\infty^2$) relates the vector $\tilde{\mathsf{c}}_p$ of the pressure coefficient at the element centers, to $\tilde{\mathsf{f}}_\varphi$, as $\tilde{\mathsf{c}}_p = \mathsf{E}_{BT}\tilde{\mathsf{f}}_\varphi$; ($iv$) the matrix E_{GF} (obtained from the definition of e_k) relates the vector $\tilde{\mathsf{e}}$ of the generalized aerodynamic forces, to $\tilde{\mathsf{c}}_p$, as $\tilde{\mathsf{e}} = q_D\mathsf{E}_{GF}\tilde{\mathsf{c}}_p$.

Combining with the structural dynamics equation discussed above ($\ddot{\mathsf{q}} + \Omega^2\mathsf{q} = \mathsf{e}$), one obtains, in the

Laplace domain,

$$s^2\tilde{\mathsf{q}} + \Omega^2\tilde{\mathsf{q}} = q_D \mathsf{E}(\breve{s})\tilde{\mathsf{q}} \tag{4}$$

The solution of this equation involves some traditional methods, such as the *V–g* and the *p–k* methods, briefly illustrated in Ref. [9]. These methods are cumbersome and not apt for use in MDO/CD. In recent years a new trend has emerged that consists of a matrix rational approximation of the function $\mathsf{E} = \mathsf{E}(k)$, such that the resulting equations form a system of first–order ordinary differential equations, whose stability analysis requires simply the use of a root locus of the eigenvalues of a matrix by varying U_∞ (finite–state aeroelasticity, or reduced–order model). Probably, the earliest example of this approach is the work of Jones [29] who gives a rational approximation for the Theodorsen function and the corresponding time–domain approximation for the Wagner function. In the matrix approach, the concept was introduced by Roger [30]. A widely used approach is that by Karpel [31]. The specific reduced–order model of interest here is based on the model presented in Ref. [15]. This consists of expressing the aerodynamic matrix $\mathsf{E}(\breve{s})$ as[8]

$$\mathsf{E}(\breve{s}) \simeq \hat{\mathsf{E}}(\breve{s}) = \mathsf{E}_2\breve{s}^2 + \mathsf{E}_1\breve{s} + \mathsf{E}_0 + (\breve{s}\mathsf{I} + \mathsf{F})^{-1}\mathsf{G} \tag{5}$$

where E_k, G, and F are fully populated square matrices, which are independent of $\breve{s}$. These matrices are evaluated by a least square procedure on a set of numerical data for the matrix of the aerodynamic forces $\mathsf{E}(\breve{s}$. The aeroelastic system resulting from Eqs. 4 and 5 is equivalent to

$$s^2\tilde{\mathsf{q}} + \Omega^2\tilde{\mathsf{q}} = \frac{1}{2}\rho_\infty U_\infty^2 \left(\breve{s}^2\mathsf{E}_2\tilde{\mathsf{q}} + \breve{s}\mathsf{E}_1\tilde{\mathsf{q}} + \mathsf{E}_0\tilde{\mathsf{q}} + \tilde{\mathsf{r}}\right) \qquad (\breve{s}\mathsf{I} + \mathsf{F})\tilde{\mathsf{r}} = \mathsf{G}\tilde{\mathsf{q}}, \tag{6}$$

which may be easily transformed into the time domain to yield a system of linear homogeneous first–order differential equations of the type $\dot{\mathsf{x}} = \mathsf{A}(U_\infty)\mathsf{x}$, where $\mathsf{x}^T = [\mathsf{q}^T, \dot{\mathsf{q}}^T, \mathsf{r}^T]$. This approach allows one to perform the flutter analysis through a root locus of the eigenvalues of the matrix $\mathsf{A}(U_\infty)$, thereby avoiding the above mentioned traditional methods, which unnecessarily complicate the optimization procedure.

Viscous flow modeling

In this section, we consider the modeling for the viscous–flow correction. The analysis is limited to steady attached high–Reynolds–number flows, where the vortical region (*i.e.*, boundary layer and wake) has a small thickness (as mentioned above, viscosity effects are usually not included in conceptual design for unsteady aerodynamics, which is only needed for the linear analysis of flutter and gust response). Outside boundary layer and wake, the flow is irrotational and is solved by using a potential–flow model obtained by introducing, in the boundary integral formulation described above, a viscous–flow correction based on Lighthill's equivalent sources approach [28]. This consists of modifying the impermeability boundary conditions $\partial\varphi/\partial n = \mathbf{v}_B$, into $\partial\varphi/\partial n = \mathbf{v}_B \cdot \mathbf{n} + \chi_V$, where the transpiration velocity χ_V is given by

$$\chi_V = \frac{\partial}{\partial s_1}\int_0^\delta (u_e - u)\, d\eta + \frac{\partial}{\partial s_2}\int_0^\delta (v_e - v)\, d\eta \tag{7}$$

[8]The leading term being of $O(\breve{s}^2)$ is motivated by the fact that we want $\hat{\mathsf{E}}(\breve{s})$ to have the same order as $\mathsf{E}(\breve{s})$ (*i.e.*, $O(\breve{s}^2)$), which stems from $\mathsf{E}_{BC}(\breve{s}) = O(\breve{s})$, $\mathsf{E}_{IE}(\breve{s}) = O(1)$, $\mathsf{E}_{BT}(\breve{s}) = O(\breve{s})$, whereas $\mathsf{E}_{GF}(\breve{s})$ is independent of $\breve{s}$).

where s_1 and s_2 are local orthogonal arclengths over the wing surface, δ is the thickness, and u_e and v_e the velocities at the external edge of the boundary layer respectively in s_1 and s_2 directions; a similar correction is used on the wake surface, with $\Delta(\partial\varphi/\partial n) = (\chi_v)_2 + (\chi_v)_1$ (see Ref. [32] for an in–depth analysis of this point). An integral formulation is used for the boundary layer (for attached flows, this approach yields results as accurate as those obtained by differential methods, with considerably reduced computational effort).

We have considered three models with different levels of sophistication: (1) a very simple model based upon the classical Blasius theory used as strip theory, (2) a two–dimensional integral boundary–layer formulation used as strip theory (see below), and (3) a three–dimensional integral boundary layer formulation (see also below). Note that, in general, three–dimensional effects within the boundary layer may be neglected with a minor loss of accuracy for applications to wings with large aspect ratio and reduced sweep angle; this is even more applicable in the case of the Prandtl wing, where no tip effect exists. In all the models, the viscosity correction to the potential flow is evaluated through the Lighthill [28] transpiration velocity, as mentioned above (through both $\mathcal{S}_B$ and $\mathcal{S}_W$).

The first model, limited to laminar flows, with an empirical correction for turbulent flows, was used initially, just to have an order of magnitude of the correction. In the second one, the laminar portion is computed by the Thwaites [33] method, the turbulent portion by the Green [35] 'lag-entrainment' method; the transition from laminar to turbulent flow is detected by the Michel [34] method. Matching of the boundary–layer solution with the viscous–flow–corrected potential–flow solution is obtained through classical direct iteration. The viscous drag is evaluated with the Squire and Young [36] approach. Other contributions to the drag (such as wave drag) are currently evaluated by empirical corrections from Ref. [12]. Finally, the three–dimensional integral boundary–layer algorithm uses two equations for momentum (extension of von Kármán equation to three–dimensional flows) coupled by two auxiliary equations: the first is the kinetic energy equation and the second is the transport equation for the maximum shear stress coefficient ('lag' equation; see Ref. [6] for details).

The two–dimensional integral boundary–layer formulation, used as 'strip–theory' in three–dimensional applications, has been validated by comparison with experimental results available in literature, in the case of: *(i)* isolated wing, *(ii)* biplane, and *(iii)* box–wing configuration (see Ref. [6], which presents in particular the polar at $Re = 5.1 \cdot 10^5$ of a box–wing configuration; the results are in good agreement with the experimental and numerical results by Gall and Smith [37].

On the basis of these results, we believe the strip–theory approach to be a better candidate for MDO/CD in that yields comparable results with less computational effort; however, the verdict is still open and the issue should be addressed within the context of the methodology by Alexandrov and Lewis [22] discussed above.

Reference

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