Advances in Computational & Experimental Engineering & Science

Entropic Lattice Boltzmann Models

Bruce M. Boghosian

Department of Mathematics, Tufts University, Medford, MA 02155, U.S.A.

May 4, 2004

Abstract

In the early 1990's, it was noticed that the Chapman-Enskog analysis that is used to derive the Navier-Stokes equations from the Boltzmann equation is very insensitive to gross truncations of the velocity space. That is, even Boltzmann equations on certain discrete velocity spaces may yield continuum Navier-Stokes behavior in the limit of small Knudsen number. The resulting lattice Boltzmann equation has become an important new tool in computational fluid dynamics and materials science [2, 3].

In the drive to lower the viscosity of lattice Boltzmann models, and to extend them to complex fluids, including immiscible and amphiphilic flow, it has been noted that their utility is limited by the onset of numerical instabilities. Such instabilities are precluded in continuum kinetic theory by the celebrated H-Theorem of Boltzmann. For this reason, it has been proposed that the retention of an H-Theorem in lattice Boltzmann models will eliminate the instabilities, and thereby increase the range of applicability of the method [4, 5, 6]. This has given rise to much recent interest in *entropic lattice Boltzmann models*, whose dynamics possess a Lyapunov function, and are therefore nonlinearly and unconditionally stable [7, 8, 1, 9].

It has been shown [1] that the requirement of Galilean invariance uniquely determines the form of the H function used in a wide class of entropic lattice Boltzmann models for the incompressible Navier-Stokes equations in D dimensions. The form obtained was that of the Burg entropy for $D = 2$, and the Tsallis entropy with nonextensivity parameter $q = 1 - 2/D$ for $D \neq 2$. This result was originally derived for particles of a single mass and speed on a Bravais lattice, and later

Proceedings of the 2004 International Conference on Computational & Experimental Engineering & Science 26-29 July, 2004, Madeira, Portugal

Advances in Computational & Experimental Engineering & Science

generalized to allow for multiple masses and speeds [9]. In the latter instance, the required H function for these models must be determined by solving a certain nonlinear functional differential equation. Remarkably, the solutions to this equation have the form of the Tsallis entropy, where q is determined by the solution to a certain transcendental equation, involving the dimension and symmetry properties of the lattice, as well as the masses and speeds of the particles.

Other equations of motion yield other unique forms for the H function. In particular, the application of the entropic lattice Boltzmann methodology to the Burgers' equation requires an H function that involves the exponential integral function [10]. The resulting model has been shown to retain stability, though at the expense of accuracy, in the limit of small viscosity and concomitantly small shock width.

Finally, the presence of a Lyapunov function in entropic lattice Boltzmann models has led to the proposal that such models constitute a new class of large-eddy simulations of turbulence [11]. This remains an active and exciting area of research on these novel and interesting models of hydrodynamics.

References

[1] B.M. Boghosian, P.J. Love, P.V. Coveney, I.V. Karlin, S. Succi, J. Yepez, *Phys. Rev. E Rapid Communications* **68** (2): Art. No. 025103 Part 2 (2003).

[2] R. Benzi, S. Succi, M. Vergassola, *Phys. Reports*, **222** (1992).

[3] S. Succi, "The Lattice Boltzmann Equation - For Fluid Dynamics and Beyond," Oxford University Press (2001).

[4] I.V. Karlin, A.N. Gorban, S. Succi, V. Boffi, *Phys. Rev. Lett.* **81** (1998).

[5] I.V. Karlin, A. Ferrante, H.C. Öttinger, *Europhys. Lett.* **47** (1999).

[6] B.M. Boghosian, J. Yepez, P.V. Coveney, A. Wagner, *Proc. R. Soc. (London) A.* **457** (2001).

[7] S. Ansumali, I.V. Karlin, *Phys. Rev. E* **62** (2000) 7999.

[8] S. Ansumali, I.V. Karlin, *Phys. Rev. E* **65** (2002).

Advances in Computational & Experimental Engineering & Science

[9] B.M. Boghosian, P.J. Love, J. Yepez, P.V. Coveney, "Galilean-invariant multi-speed entropic lattice Boltzmann models," *Physica D*, in press (2004).

[10] B.M. Boghosian, P.J. Love, J. Yepez, " Entropic Lattice Boltzmann Model for Burgers' Equation," *Philosophical Transactions of the Royal Society of London Series A*, in press (2004).

[11] S. Succi, I.V. Karlin, H. Chen, *Rev. Mod. Phys.* (2002).