Shape Optimization of Laminated Composite Shell

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Summary

Shape design optimization of shell structure is implemented on a basis of integrated framework of geometric modeling and finite element analysis which is constructed on the geometrically exact shell surface representation. This shell theory enables more accurate and robust analysis for complicated shell structures, and it fits for the nature of B-splines function which is popular in CAD field. Shape of laminated composite shells is optimized through genetic algorithm and sequential linear programming. Sequential adaptation of global and local optimization makes the process more efficient.

Introduction

Shell is a thin curved structural component, used to sustain loads with light weight structure, or to make large containing space. To fulfill this laminated composite material is widely used for shell structure, because composite material has better performance in specific stiffness and strength than conventional metallic materials. However, composite shell structure has complexities in numerical analysis, so various and consecutive efforts to understand the mechanical behaviors of shell have been made in the fields of computational mechanics and experimental tests.

For shape optimization of shell structures, three kind of research area need to be addressed, and connected with each other systematically. They are finite element analysis part, computer aided geometric modeling part, and shape optimization part. The currently used FEA(Finite Element Analysis) program imports geometric information from CAD software by introducing neutral file formats. But this formats sometimes make troubles while connecting CAD with FEA program is attempted, because the shell surface reconstructed by interpolating function of FEA program is not precise as much as geometric modeling of CAD, besides, iterative data transfer and recalculation are followed by error accumulation and a lot of efforts.[1,2] Thus it is essential to make an integrated design tool that uses geometric information of CAD directly, and simultaneously modifies the model according to analysis results.

The present study introduces an integrated design framework using B-spline modeling and geometrically exact shell element to deal successfully with those problems. Optimization module on a basis of this framework is consisted of genetic algorithm for global optimization and gradient-based method for local optimization. In numerical examples, shape optimization of laminated composite shell structure is presented.

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Review of General Shell Theory and Shell Element

In the Naghdi's shell model[3], the displacement vector based on general curvilinear coordinate and tensor is assumed as

$$\vec{V} = (u_\alpha + \theta^3 \psi_\alpha)\vec{a}^\alpha + w\vec{a}_3 \tag{1}$$

, where u_α and w are translational displacement, ψ_α is rotational displacement. From the given displacement field in Eq.(1), we obtain the membrane, bending, and shear measures in terms of mid-surface displacement, surface metric, and curvature tensors.

The partial mixed variational functional for the proposed first order shear deformable shell model is given as,

$$\begin{aligned}
&\Pi^P\left(\varepsilon_{\alpha\beta}, \gamma_\alpha, u_\alpha, w, \psi_\alpha \right) \\
&= \int_\Omega B^{\alpha\beta\gamma\mu} h \varepsilon_{\alpha\beta} \frac{1}{2}\left(u_\gamma \|_\mu + u_\mu \|_\gamma - 2b_{\mu\gamma} w\right)\sqrt{a}\, d\theta^1 d\theta^2 - \frac{1}{2}\int_\Omega B^{\alpha\beta\gamma\mu} h \varepsilon_{\alpha\beta}\varepsilon_{\gamma\mu}\sqrt{a}\, d\theta^1 d\theta^2 \\
&+ \int_\Omega Gha^{\alpha\beta}\gamma_\alpha \left(w_{,\beta} + \psi_\beta + b^\lambda{}_\beta u_\lambda\right)\sqrt{a}\, d\theta^1 d\theta^2 - \frac{1}{2}\int_\Omega Gha^{\alpha\beta}\gamma_\alpha\gamma_\beta\sqrt{a}\, d\theta^1 d\theta^2 \\
&+ \frac{1}{2}\int_\Omega B^{\alpha\beta\gamma\mu}\frac{h^3}{12}\{\frac{1}{2}[\psi_\alpha \|_\beta + \psi_\beta \|_\alpha + \frac{1}{2}\{b_\alpha^\lambda (u_\beta \|_\lambda - u_\lambda \|_\beta) + b_\beta^\lambda (u_\alpha \|_\lambda - u_\lambda \|_\alpha)\}]\} \\
&\quad \times \{\frac{1}{2}[\psi_\gamma \|_\mu + \psi_\mu \|_\gamma + \frac{1}{2}\{b_\gamma^\lambda (u_\mu \|_\lambda - u_\lambda \|_\mu) + b_\mu^\lambda (u_\gamma \|_\lambda - u_\lambda \|_\gamma)\}]\}\sqrt{a}\, d\theta^1 d\theta^2 \\
&- \int_\Omega \left(p^\alpha u_\alpha + pw\right)\sqrt{a}\, d\theta^1 d\theta^2
\end{aligned} \tag{2}$$

, where the infinitesimal area element is $\sqrt{a}\, d\theta^1 d\theta^2 = dA$, and $a = \det(a_{\alpha\beta})$. Using the partial mixed functional and assumed strain parameters, finite element discretization is performed and the discretized functional is given as the following symbolic form.

$$\Pi^p\left(\vec{d},\vec{\alpha},\vec{\beta}\right) = \frac{1}{2}\vec{d}^T\mathbf{K_b}\vec{d} - \frac{1}{2}\vec{\alpha}^T\mathbf{H}_m\vec{\alpha} + \vec{\alpha}^T\mathbf{G}_m\vec{d} - \frac{1}{2}\vec{\beta}^T\mathbf{H}_\gamma\vec{\beta} + \vec{\beta}^T\mathbf{G}_\gamma\vec{d} - \vec{d}^T \cdot \vec{f} \tag{3}$$

By taking first variation with respect to $\vec{d}$, $\vec{\alpha}$, and $\vec{\beta}$, the following conventional element stiffness equation is obtained for a given partial mixed assumed strain functional.

$$\mathbf{K_b}\vec{d} + \left(\mathbf{G}_m{}^T\mathbf{H}_m{}^{-1}\mathbf{G}_m + \mathbf{G}_\gamma{}^T\mathbf{H}_\gamma{}^{-1}\mathbf{G}_\gamma\right)\vec{d} = \vec{f} \tag{4}$$

$$\mathbf{K^e} = \mathbf{K_b} + \mathbf{G}_m{}^T\mathbf{H}_m{}^{-1}\mathbf{G}_m + \mathbf{G}_\gamma{}^T\mathbf{H}_\gamma{}^{-1}\mathbf{G}_\gamma \tag{5}$$

B-spline Surface Representation

A B-spline curve[4,5] is defined by the following equation,

$$C(t)=\sum_{i=0}^{n} N_{i,p}(t)P_i \tag{6}$$

, where $N_{i,p}(t)$ are B-spline basis functions, P_i are vectors composed of coordinates of the control points, p is the order of a B-spline curve, and $[t_{0,}t_{1,}.....,t_{n+p}]$ is the knot vector which specifies the distribution of the parameter t along the curve. B-spline basis functions $N_{i,p}(t)$ are defined by

$$N_{i,1}(t)=\begin{cases}1 & \text{if } t_i \le t < t_{i+1} \\ 0 & \text{otherwise}\end{cases}, \quad N_{i,p}(t)=\frac{t-t_i}{t_{t+p+1}-t_i}N_{i,p-1}(t)+\frac{t_{i+p}-t}{t_{t+p}-t_{i+1}}N_{i+1,p-1}(t) \tag{7}$$

The interval $[t_i, t_{i+1}]$ is called the i th knot span.

A $(p\times q)$ th order B-spline surface is defined as follows,

$$S(u,v)=\sum_{i=0}^{m}\sum_{j=0}^{n} V_{i,j}M_{j,q}(v)N_{i,p}(u) \quad (u_{p-1}\le u \le u_{m+1},\ v_{q-1}\le v \le v_{n+1}) \tag{8}$$

, where $(m+1)\times(n+1)$ control points $V_{i,j}$ build up the control net in 3-D space, and $N_{i,p}(u)$, $M_{j,q}(v)$ are the B-spline basis functions of order p and q in the u and v directions, respectively.

The geometric values are calculated directly from B-spline surface expression like Eq.(8) which is constructed on convected coordinate system, so remeshing or file transfer are not necessary even when structure is modified. Fig.1 shows a scene of program execution which is made on the basis of integrated framework B-spline modeling and shell finite element analysis.

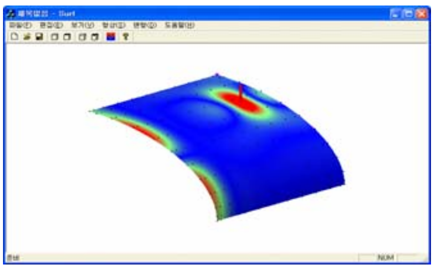

Fig.1 Integrated Program of Modeling and Analysis developed on B-spline Representation

Shape Design Optimization

We formulate the objective function as strain energy. Strain energy as an objective function[6] is expressed as

$$\min_{s} \sum_{i=1}^{L} \sum_{j=1}^{M} d_i K^{ij} d_j \tag{9}$$

, where s is the set of design variables, L , M are the number of nodes, d is the nodal displacement vector, and K is the stiffness matrix.

Shape optimization has various local optima not only by the ways of variable selection, but also by the paths of searching. Thus it is required to adopt global optimization strategy. Genetic algorithm is a suitable choice for this purpose. Genetic algorithm is a way of searching based on the biological evolution theory, and genetics. The best solution is searched by iterative selection, crossover, mutation of population, which has a form of chromosome. In this study, the z -coordinate values of selected design variables in Fig.2 are discretized into the 5 level in z -direction, and the combinations of coordinate values are made into chromosomes.[7]

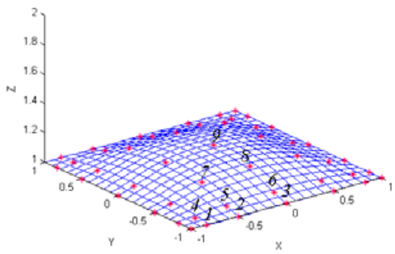

Fig.2 Control Points Selected as Design Variables

Once the global optimization is executed, local optimization is carried out for minute adjustment. Sequential linear programming(SLP) as a local optimization method is given as

$$\text{Min } f(x_o) + \sum_{i=1}^{n} (x_i - x_{oi}) \left(\frac{\partial f}{\partial x_i}\right)_{x_o}, \quad \text{s.t. } g_j(x_o) + \sum (x_i - x_{oi}) \left(\frac{\partial g_j}{\partial x_i}\right)_{x_o} \geq 0 \tag{10}$$

by substituting objective function, and constraint functions with linearly approximated equations at initial variables. And $j = 1, ..., n_g$, $a_{li} \leq x_i - x_{oi} \leq a_{ui}$. The replaced objective function by SLP[8] requires the derivatives of strain energy to be calculated with respect to design variables. By differentiating strain energy in Eq.(9), and using semi-analytics, sensitivity is formulated as

$$\frac{\partial f}{\partial x} = -\frac{1}{2}\vec{d}^{T}\underset{\sim}{K}'\vec{d}\ , \qquad \underset{\sim}{K}' = \frac{\underset{\sim}{K}(x+\Delta x) - \underset{\sim}{K}(x)}{\Delta x} \tag{11}$$

Numerical Examples

The shape of laminated composite shell is optimized with selected design variables considering geometry and layup symmetry and effects on the whole shape[9-11]. In this example, layup angle of $[0^{o}/90^{o}]_{s}$ and $[45^{o}/-45^{o}]_{s}$ is considered. The z-coordinate values of design variables are transformed to chromosomes for genetic algorithm. The simple genetic algorithm[12] used in this study does not contain elitism. The population size is 30, and probability of mutation is 3%. The material properties and geometric conditions of laminated composite shell are as followed. $E_1 = 2.0\times10^{6}$, $E_2 = 0.1\times10^{6}$, $\nu = 0.3$, $L = 2$, $h = 0.01$. Pin-jointed boundary conditions are applied at four corners. Dead load is given to the structures. Figs.3 show the optimized shape of fixed layup $[0^{o}/90^{o}]_{s}$, $[45^{o}/-45^{o}]_{s}$,and isotropic material, respectively. The strain energy is reduced to about $1/10^{4}$ of initial value through shape and layup optimization.

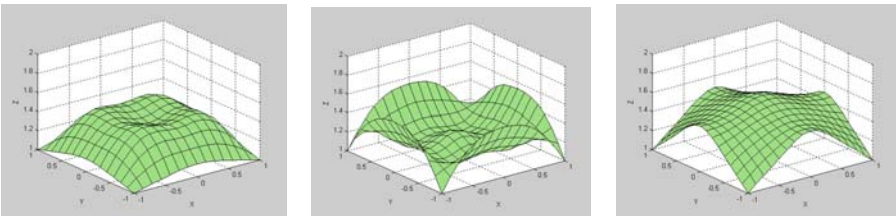

Fig.3 Optimized Shape of $[0^{o}/90^{o}]_{s}$, $[45^{o}/-45^{o}]_{s}$ Laminated Composite, and Isotropic Shell

Conclusion

In this study, shape optimization of composite shell structures was established which is based on integrated framework of geometrically exact shell element and B-spline modeling. Geometrically exact shell element has a natural connection with surface construction method of CAD that more accurate analysis is enabled. And the element is suitable for optimization because optimization requires huge computational cost for iterative design modification and analysis process.

Optimization of laminated composite shell was accomplished by genetic algorithm and semi-analytic method. Shape optimization provides so many local optima according to its selection method of design variables, searching paths, and constraints that global optimization for the whole domain of design is necessary. In this study, genetic algorithm for roughly discretized design domain and gradient-based local search were adopted for effective use of computational resources.

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