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Homogenized Elastic-Viscoplastic Behavior of Plain-Woven GFRP Composites Subjected to In-Plane Tensile Load

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Summary

In this study, the in-plane anisotropic elastic-viscoplasic behavior of plain-woven glass fiber/epoxy composites is analyzed by using a homogenization theory. To this end, assuming the in-phase and out-of-phase stacks of plain fabrics, the homogenization theory of nonlinear time-dependent composites is applied to the analysis. Moreover, it is pointed out that the plain-woven composites with in-phase and out-of-phase stacks of plain fabrics have point-symmetric internal structure. The point-symmetry of internal structure is then utilized to reduce the domain of analysis, leading to significant computational efficiency. As a result of analysis, it is shown that the present analysis is successful in reproducing the nonlinear behavior of plain-woven GFRP composites subjected to off-axial load as well as the linear behavior of the composites under axial load. It is also shown that the way of stacking of plain fabrics has influence on the elastic-viscoplastic behavior of plain-woven GFRP composites.

Introduction

Plain-woven composites are now important engineering materials because of their high specific strength, high specific stiffness, and so on. The plain-woven composites generally exhibit marked nonlinear behavior due to the inelasticity of matrix materials especially when the composites are subjected to off-axial load, resulting in the considerable in-plane anisotropy. In practical use of plain-woven composites, the composites can be subjected to not only axial load but also off-axial load. It is therefore of significance to analyze the off-axial nonlinear behavior of plain-woven composites.

Complex internal structures of plain-woven composites, however, cause the difficulty with analyzing the deformation behavior of the composites. The homogenization theory based on unit cell problems [e.g. 1] is one of the most promising theories for the analysis of plain-woven composites because the FEM based analysis generally employed in the theory enables us to take into account the complex microscopic structures of composites. The theory therefore has already been applied to analyzing the nonlinear behavior of plain-woven composites with the microscopic damages in fiber bundles under axial load [2-4]. But the off-axial nonlinear behavior given rise to by the inelastic deformation of matrix materials has not been considered.

The present authors [5,6] constructed the homogenization theory of nonlinear time-dependent composites. The theory was then applied to analyzing the elastic-viscoplastic

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behavior of long fiber-reinforced laminates, and succeeded in predicting experimental results accurately [7]. This successful application was owing to the features of the theory: The theory analyzes not only the macroscopic time-dependent behavior of composites but also the distributions of microscopic stress and strain rates in composites. This unique advantage of the theory can make it possible to analyze the off-axial nonlinear behavior of plain-woven composites.

The present authors [8,9] further showed the following: If the internal structure of a composite has point-symmetry, the perturbed velocity field in the composite also has point-symmetry. Using the point-symmetry of perturbed velocity field as a boundary condition of unit cell problems, we are able to reduce the domain of analysis, leading to considerable computational efficiency. In general, the internal structures of plain-woven composites assumed in the homogenization analysis have the point-symmetry as well as the *Y*-periodicity, which will be discussed in the section after next. One can therefore introduce the point-symmetric boundary condition into the homogenization analysis of plain-woven composites so that the domain of analysis can be reduced. But, so far, whole unit cells have been taken as the domain of analysis for the use of the *Y*-periodic boundary condition of perturbed velocity [2-4].

In this study, we analyze the in-plane elastic-viscoplastic deformation of plain-woven GFRP composites using the homogenization theory of nonlinear time-dependent composites. It is thus shown that the analysis reproduces the in-plane anisotropic elastic-viscoplastic behavior of plain-woven GFRP composites. Moreover, the point-symmetric internal structure of plain-woven composites is utilized to reduce the domain of analysis, resulting in considerable computational efficiency.

Homogenization Theory

In this section, we briefly review the homogenization theory of nonlinear time-dependent composites. Let us consider that a composite with periodic internal structure is subjected to macroscopically uniform load and deforms infinitesimally. We then take the unit cell *Y* of periodic composite and the Cartesian coordinates y_i $(i = 1, 2, 3)$ for *Y*. Each constituent in *Y* is assumed to exhibit elastic-viscoplastic behavior characterized as

$$\dot{\sigma}_{ij} = c_{ijkl}(\dot{\varepsilon}_{kl} - \beta_{kl}), \qquad (1)$$

where $\dot{\sigma}_{ij}$ and $\dot{\varepsilon}_{kl}$ indicate microscopic stress and strain rates, respectively, c_{ijkl} denotes elastic stiffness, and β_{kl} represents a viscoplastic function and vanishes if elastic.

Then, in accordance with our previous papers [5,6], we can derive the evolution equation of microscopic stress σ_{ij} and the relation between macroscopic stress rate $\dot{\Sigma}_{ij}$ and strain rate $\dot{E}_{kl}$ as follows:

$$\dot{\sigma}_{ij} = c_{ijpq}(\delta_{pk}\delta_{ql} + \chi^{kl}_{p,q})\dot{E}_{kl} - c_{ijkl}(\beta_{kl} - \varphi_{k,l}), \qquad (2)$$

$$\dot{\Sigma}_{ij} = \left\langle c_{ijpq}(\delta_{pk}\delta_{ql} + \chi^{kl}_{p,q})\right\rangle \dot{E}_{kl} - \left\langle c_{ijkl}(\beta_{kl} - \varphi_{k,l})\right\rangle, \qquad (3)$$

where $(\ )_{,i}$ stands for the differentiation with respect to y_i, δ_{ij} signifies Kronecker's delta, and $\langle \# \rangle = |Y|^{-1} \int_Y \# \, dY$. Here, $|Y|$ denotes the volume of Y. Moreover, χ_i^{kl} and φ_i in Eqs. (2) and (3) indicate the characteristic functions determined by solving the following boundary value problems

$$\int_Y c_{ijpq} \chi_{p,q}^{kl} v_{i,j} dY = -\int_Y c_{ijkl} v_{i,j} dY , \tag{4}$$

$$\int_Y c_{ijpq} \varphi_{p,q} v_{i,j} dY = \int_Y c_{ijkl} \beta_{kl} v_{i,j} dY , \tag{5}$$

where v_i indicates any field of perturbed velocity satisfying the Y-periodicity. In general, the above boundary value problems (4) and (5) can be solved using FEM by imposing the Y-periodicity of χ_i^{kl} and φ_i.

Basic Cell and Boundary Condition

In the analysis based on the homogenization theory, the way of stacking of plain fabrics in plain-woven composites is generally assumed as shown in Fig. 1 [2-4]. In Fig. 1(a), all the plain fabrics are stacked with no offset. On the other hand, in Fig. 1(b), the plain fabrics are stacked with the phase shift π in the y_1- and y_3-directions. Thus, from now forth, the stacking patterns of plain fabrics illustrated in Fig. 1(a) and (b) will be referred to as "in-phase stacking" and "out-of-phase stacking", respectively [2].

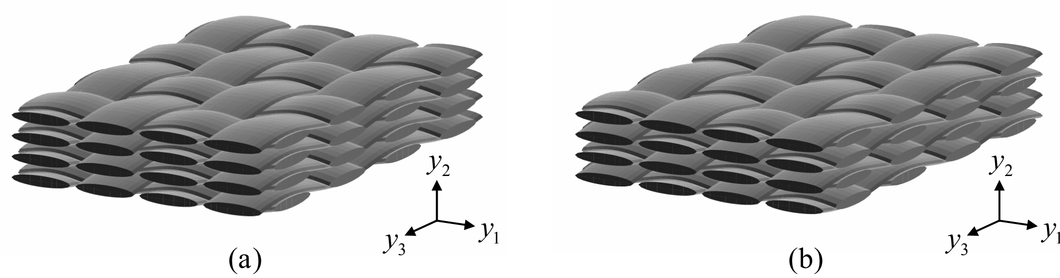


Fig. 1. Two kinds of stacking patterns of plain fabrics in plain-woven composites; (a) in-phase stacking, (b) out-of-phase stacking.

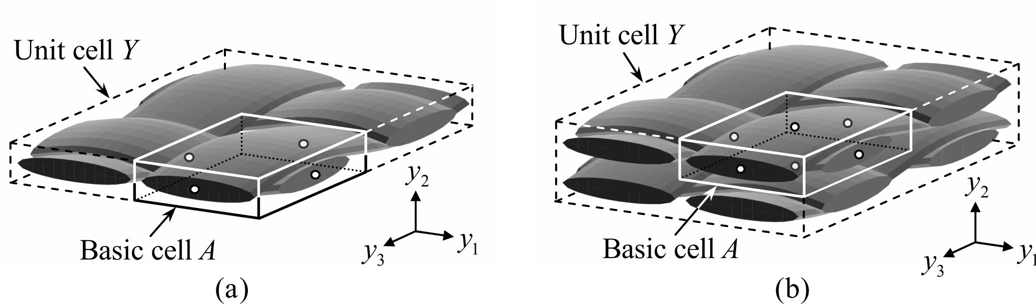


Fig. 2. Unit cell Y (dashed lines) and basic cell A (solid lines) of plain-woven composites; (a) in-phase stacking, (b) out-of-phase stacking.

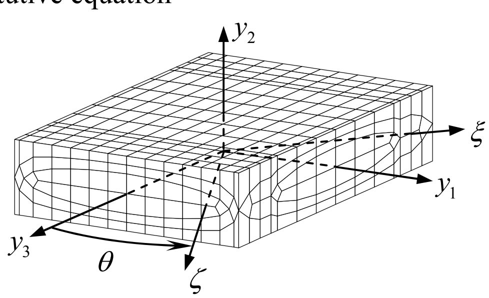
For the plain-woven composites with the above-mentioned stacking patterns of plain fabrics, unit cells Y can be taken as shown in Fig. 2(a) and (b), respectively. As stated in the Introduction, so far, the unit cells in Fig. 2 have been taken as the domain of analysis [2-4]. But, in the present study, we pay attention to a part of unit cells, which is indicated by the solid lines in Fig. 2(a) and (b), and will be referred to as a basic cell A hereafter. It is noted that both the composites have the same basic cell.

Then, a close look at Fig. 2 tells us the following: The composite with in-phase stacking has the point-symmetric internal structure with respect to the centers of lateral facets of A, which are denoted by the small open circles in Fig. 2(a), and has the Y-periodicity in the stacking direction. On the other hand, the composite with out-of-phase stacking has the point-symmetric internal structure with respect to the centers of top and bottom facets as well as lateral facets of A, which are also denoted by the small open circles in Fig. 2(b). Thus, when the plain-woven composites are subjected to macroscopically uniform load, the perturbed velocity in the composites distributes point-symmetrically with respect to the open circles indicated in Fig. 2. Employing this point-symmetry of perturbed velocity field as a boundary condition of the boundary value problems mentioned previous section, the basic cell A instead of the unit cell Y can be taken as the domain of analysis [8,9], leading considerable computational efficiency.

Analysis of In-Plane Elastic-Viscoplastic Behavior of Plain-Woven GFRP Composites

In the present analysis, the plain-woven glass fiber/epoxy composites with the in-phase and the out-of-phase stacking patterns of plain fabrics shown in Fig. 1 are considered. Then, the basic cell illustrated in Fig. 3 is taken as the domain of analysis for both the composites, and discretized into 8-node isoparametric elements (1624 elements, 1995 nodes).

Fiber bundles are regarded as elastic materials. The elastic constants of fiber bundles are calculated by using the homogenization theory on the assumption that the fiber bundles are unidirectional glass fiber/epoxy composites which have transversely hexagonal fiber array and 65% fiber volume fraction. On the other hand, the epoxy matrix is regarded as an elastic-viscoplastic material which obeys the following constitutive equation



(a)

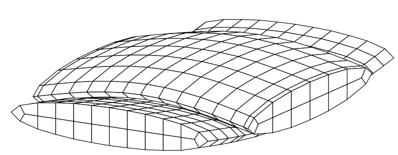

(b)

Fig. 3. Basic cell of plain-woven GFRP composites and finite element mesh (1624 elements, 1995 nodes); (a) full view of basic cell, (b) fiber bundles in basic cell.

$$\dot{\varepsilon}_{ij} = \frac{1+\nu_m}{E_m}\dot{\sigma}_{ij} - \frac{\nu_m}{E_m}\dot{\sigma}_{kk}\delta_{ij} + \beta_{ij}\,,\ \ \beta_{ij} = \frac{3}{2}\dot{\varepsilon}_0^p\left[\frac{\sigma_e}{g(\bar{\varepsilon}^p)}\right]^n \frac{s_{ij}}{\sigma_e}\,, \tag{6}$$

where E_m , ν_m and n are material constants, $g(\bar{\varepsilon}^p)$ denotes a hardening function depending on accumulated viscoplastic strain $\bar{\varepsilon}^p = \int[(2/3)\beta_{ij}\beta_{ij}]^{1/2}dt$, $\dot{\varepsilon}_0^p$ indicates a reference strain rate, s_{ij} stands for the deviatoric part of σ_{ij} , and $\sigma_e = [(3/2)s_{ij}s_{ij}]^{1/2}$. The material constants and function employed in the present analysis are listed in Table 1.

The composites are subjected to in-plane uniaxial elongation at a constant strain rate. The loading condition is as follows:

$$\dot{E}_{\zeta\zeta} = 10^{-5}\,\mathrm{s}^{-1},\ \dot{\Sigma}_{\xi\xi} = \dot{\Sigma}_{\xi\zeta} = 0,\ \dot{\Sigma}_{22} = \dot{\Sigma}_{2\xi} = \dot{\Sigma}_{2\zeta} = 0\,, \tag{7}$$

where the subscripts ξ and ζ denote the axes which make an angle θ with the y_1- and y_3-axes, respectively [Fig. 3(a)]. Four loading directions, i.e., $\theta = 0°, 15°, 30°, 45°$, are considered in the present analysis.

The macroscopic stress-strain relations of plain-woven GFRP composites are shown in Fig. 4. As seen from the figure, in the case of $\theta = 0°$, the composites deform almost linearly. By contrast, the composites subjected to the off-axial load, i.e., in the case of $\theta = 15°, 30°, 45°$, exhibit considerable nonlinearity due to the viscoplasticity of epoxy matrix. Flow stress decreases according as θ increases, and becomes the lowest at $\theta = 45°$. We can therefore say that the plain-woven GFRP composites have marked in-plane elastic-viscoplastic anisotropy, which is observed in experimental results. Next, let us compare the results of in-phase stacking with those of out-of-phase stacking. As shown in Fig. 4, both the results have the difference about 4~10%, meaning that the offset stacking of plain fabrics can affect the macroscopic behavior of plain-woven GFRP composites.

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Table. 1. Material constants.

Glass fiber	$E_f = 8.0\times10^4$ $\nu_f = 0.3$
Epoxy	$E_m = 5.0\times10^3$ $\nu_m = 0.35$ $\dot{\varepsilon}_0^p = 10^{-5}$ $n = 31$ $g(\bar{\varepsilon}^p) = 10.0(\bar{\varepsilon}^p)^{0.10} + 20$

MPa (stress), mm/mm (strain), s (time).

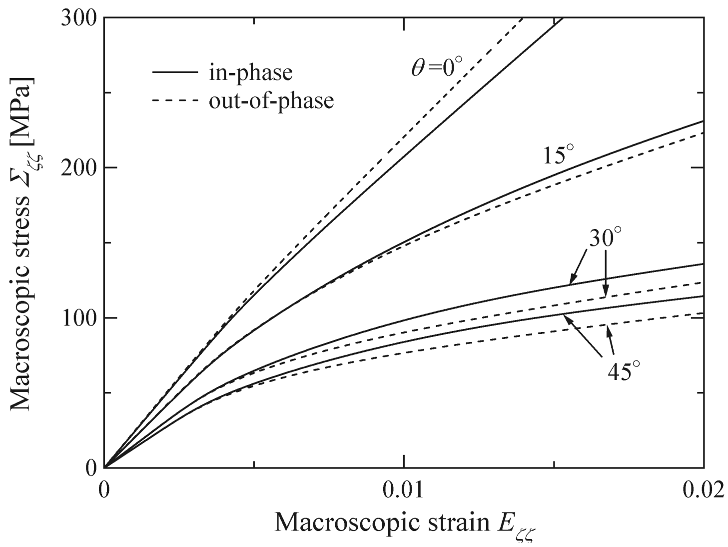

Fig. 4. Macroscopic stress-strain relations of plain-woven GFRP composites at $\dot{E}_{\zeta\zeta} = 10^{-5}\,\mathrm{s}^{-1}$.

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