Dynamic behaviour of 2.5D multi-layered systems with elastic and acoustic materials

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Summary

A number of frequency domain solutions for the dynamic behavior of layered media are already known for the pure 2D and 3D cases. For the case of 2.5D systems, the authors have defined analytical solutions for specific configurations. This paper extends this earlier work to allow the analysis of generic layered media built as a sequence of solid and fluid layers, excited by 2.5D loads.

The proposed model expresses the solutions for sinusoidal line loads as a summation of the effects of plane waves with different inclinations, taking into account the appropriate boundary conditions. This procedure allows the presence of the multiple layers and the interaction between them to be properly modeled. 3D responses can then be computed using a similar procedure, requiring the solution of a sequence of 2D problems for 2.5D loads with different wavenumbers. The model thus defined is used to compute the wavefield generated inside a fluid-filled waveguide with an elastic ground. It is conveniently integrated into a Boundary Element code in order to allow the presence of submerged inclusions to be taken into account.

Introduction

The development of analytical solutions for wave propagation in solid and fluid media has interested researchers for many years. Although they can only be defined for simple geometries, these solutions are useful as benchmark solutions, as a practical approach for simple engineering problems or as Green´s functions that may be used in conjunction with numerical methods. The case of stratified media, made of different fluid and solid layers, is particularly interesting, since these simple configurations can be found in research fields such as acoustics or oceanography. A class of solutions employs the wavenumber integration technique, allowing the wavefield to be written as a continuous integral of the effects of waves with different inclinations. The technique was first introduced by Lamb [1], and since then many researchers have developed it and applied it to different domains (Pekeris [2]; Bouchon [3]). More recently, this methodology has been generalized to allow the analysis of generic multi-layered domains, and is usually known as the Direct Global Matrix (DGM) approach (Jensen et al [4]). However, a significant limitation of the DGM is that it only allows the use of 2D or axisymmetric loads, and so it fails to model any 3D non-axissymetric configuration.

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This happens, for example, when the propagation domain is excited by a load with a harmonic variation along its axis. That particular case is usually designated as a 2.5D problem, and is of particular interest since, in conjunction with adequate numerical methods, it allows the study of systems with 2D geometry subjected to 3D loads.

In this paper, previous work by the authors (Tadeu et al [5]) is extended to give the analytical solutions for wave propagation in multi-layered systems formed as a sequence of solid and fluid layers, and subjected to the incidence of 2.5D pressure loads in a fluid layer. To establish these solutions, the wavenumber integral is conveniently transformed in a discrete summation, assuming the presence of an infinite number of virtual sources, equally spaced along one direction. A damping factor is used to avoid the contamination of the response by the virtual sources. A similar procedure is used to define the 3D solution as a discrete summation of 2.5D solutions. An applied example is presented, corresponding to a specific case in the field of oceanography. In this example, the wave propagation is studied in a fluid channel, with an elastic ground over rigid bedrock, subjected to a point load, and in the presence of a submerged rigid circular inclusion. A direct BEM formulation is used in conjunction with the analytical solutions to model the rigid inclusion. Results in the time domain are presented.

Mathematical formulation

Consider a layered system of infinite extent built from a series of fluid and solid layers with different thicknesses and properties, excited by a spatially sinusoidal harmonic pressure load located in a fluid layer at (x_0, y_0). The incident field generated by this source can be defined by

$$\mathrm{p}^{full} = -\frac{\mathrm{i}}{2}\mathrm{H}_0^{(2)}\left[k_{\alpha f}\sqrt{(x-x_0)^2+(y-y_0)^2}\right] \text{ with } \operatorname{Im} k_{\alpha f} \leq 0 \qquad (1)$$

with $\mathrm{H}_0^{(2)}(...)$ being second kind Hankel functions of order 0, $k_{\alpha f} = \sqrt{k_f^{\,2} - k_z^{\,2}}$, $k_f = \omega/\alpha_f$, α_f is the acoustic wave velocity of the host fluid medium, ω the excitation frequency, $\mathrm{i} = \sqrt{-1}$ and k_z is the wavenumber in the z direction. In the layer containing the source, the wavefield can be expressed as the sum of the incident field with the surface terms needed to satisfy the required boundary conditions at the medium boundaries. For this purpose, it is useful to express p^{full} as a continuous integral of plane waves with different inclinations. This integral can be discretized into a summation of discrete terms assuming the existence of an infinite number of virtual sources placed along the x direction at equal intervals, L_x, which are large enough to prevent the virtual loads from contaminating the response. Following that methodology, equation (1) can be written as

$$\mathrm{p}^{full} = -\frac{\mathrm{i}}{L_x}\sum_{n=-N}^{n=+N}\left[\frac{E_f}{\nu_n^f}\right]E_d\,, \tag{2}$$

where $E_d = e^{-\mathrm{i}k_n(x-x_0)}$, $k_n = \frac{2\pi}{L_x}n$, $E_f = e^{-\mathrm{i}\nu_n^f|y-y_0|}$, and $\nu_n^f = \sqrt{k_f^{\,2} - k_z^{\,2} - k_n^{\,2}}$ with $\mathrm{Im}\left(\nu_n^f\right) \le 0$.

In a similar manner, the wavefield in any fluid layer can be expressed by taking into account the surface terms generated at its top and bottom boundaries, also written as a superposition of plane waves. For this purpose, two pressure potentials should be taken into account, which can be written as

$$\phi^{f_i,top} = -\frac{\mathrm{i}}{L_x}\sum_{n=-N}^{n=+N}\left[\frac{-(\alpha^{f_i})^2}{\omega^2\lambda^{f_i}}\frac{E_{f0}^{f_i}}{\nu_{nf}^{f_i}}A_n^{1,f_i}\right]E_d\,;\;\phi^{f_i,bottom} = -\frac{\mathrm{i}}{L_x}\sum_{n=-N}^{n=+N}\left[\frac{-(\alpha^{f_i})^2}{\omega^2\lambda^{f_i}}\frac{E_{f1}^{f_i}}{\nu_{nf}^{f_i}}A_n^{2,f_i}\right]E_d \tag{3}$$

where $E_{f0}^{f_i} = \mathrm{e}^{-\mathrm{i}\nu_{nf}^{f_i}\left|y-y^{i,top}\right|}$, $E_{f1}^{f_i} = \mathrm{e}^{-\mathrm{i}\nu_{nf}^{f_i}\left|y-y^{i,bottom}\right|}$, $y^{i,top}$ and $y^{i,bottom}$ are the y coordinates of the upper and lower boundary of the fluid layer i, defined in the global system x, y, z, $\nu_{nf}^{f_i} = \sqrt{k_{f_i}^{\,2} - k_z^{\,2} - k_n^{\,2}}$, with $\mathrm{Im}\left(\nu_{nf}^{f_i}\right) \le 0$ and $k_{f_i} = \omega/\alpha^{f_i}$; A_n^{1,f_i} and A_n^{2,f_i} are the unknown amplitude factors.

For the case of a solid layer, each boundary requires considering one dilatational and two shear potentials, again written as a superposition of plane waves. The required potentials are:

$$\phi^{y,s_i,top} = E_A^{s_i}\sum_{n=-N}^{n=+N}\left[E_{b0}^{s_i}A_n^{y,1,s_i}\right]E_d \qquad \phi^{y,s_i,bottom} = -E_A^{s_i}\sum_{n=-N}^{n=+N}\left[E_{b1}^{s_i}A_n^{y,4,s_i}\right]E_d$$

$$\psi_z^{y,s_i,top} = E_A^{s_i}\sum_{n=-N}^{n=+N}\left[\frac{k_nE_{c0}^{s_i}}{\gamma_n^{s_i}}A_n^{y,2,s_i}\right]E_d \qquad \psi_z^{y,s_i,bottom} = E_A^{s_i}\sum_{n=-N}^{n=+N}\left[\frac{k_nE_{c1}^{s_i}}{\gamma_n^{s_i}}A_n^{y,5,s_i}\right]E_d$$

$$\psi_x^{y,s_i,top} = E_A^{s_i}k_z\sum_{n=-N}^{n=+N}\left[\frac{-E_{c0}^{s_i}}{\gamma_n^{s_i}}A_n^{y,3,s_i}\right]E_d \qquad \psi_x^{y,s_i,bottom} = E_A^{s_i}k_z\sum_{n=-N}^{n=+N}\left[\frac{-E_{c1}^{s_i}}{\gamma_n^{s_i}}A_n^{y,6,s_i}\right]E_d \tag{4}$$

In these equations, $E_A^{s_i} = \frac{1}{2\rho^{s_i}\omega^2L_x}$, $E_{c0}^{s_i} = \mathrm{e}^{-\mathrm{i}\gamma_n^{s_i}\left|y-y^{i,top}\right|}$, $E_{c1}^{s_i} = \mathrm{e}^{-\mathrm{i}\gamma_n^{s_i}\left|y-y^{i,bottom}\right|}$, $E_{b0}^{s_i} = \mathrm{e}^{-\mathrm{i}\nu_n^{s_i}\left|y-y^{i,top}\right|}$, $E_{b1}^{s_i} = \mathrm{e}^{-\mathrm{i}\nu_n^{s_i}\left|y-y^{i,bottom}\right|}$, $\nu_n^{s_i} = \sqrt{\left(k_p^{s_i}\right)^2 - k_z^{\,2} - k_n^{\,2}}$, with $\mathrm{Im}\left(\nu_n^{s_i}\right) \le 0$, $\gamma_n^{s_i} = \sqrt{\left(k_s^{s_i}\right)^2 - k_z^{\,2} - k_n^{\,2}}$, with $\mathrm{Im}\left(\gamma_n^{s_i}\right) \le 0$, $k_p^{s_i} = \omega/\alpha^{s_i}$ and $k_s^{s_i} = \omega/\beta^{s_i}$, and α^{s_i} and β^{s_i} being the P and S wave velocities in the solid layer. For a generic multilayered system, the derivation of the above potentials and the imposition of adequate boundary conditions allow the wavefield at any point of the domain to be computed.

The solutions thus defined can be integrated into a boundary element code as Green´s functions, allowing the analysis of multilayered systems containing inclusions without needing to discretize the interfaces between layers. Assuming the presence of a rigid 2.5D inclusion inside a fluid layer, the relevant boundary integral equation can be written

$$C\,\mathrm{p}(\underline{x}_P) + \sum_{n=1}^{N} \mathrm{p}^n \int_{S_n} \mathrm{g}^*(\underline{x}_P, \underline{x}_n)\, dS_n = \mathrm{p}_{inc}(\underline{x}_0, \underline{x}_P) \tag{5}$$

where $\mathrm{p}(\underline{x}_p)$ is the pressure at $\underline{x}_p$, p^n is the pressure at the nodal point n, $\mathrm{g}^*(\underline{x}_P, \underline{x}_n)$ is the Green's function for the pressure at $\underline{x}_n$ due to a unit load at $\underline{x}_p$, C is a constant that assumes the value 0.5 for a smooth boundary and $\mathrm{p}_{inc}(\underline{x}_0, \underline{x}_P)$ represents the pressure at $\underline{x}_p$, generated by a 2.5D pressure source placed at $\underline{x}_0$ in the absence of any inclusion. One should note that the Green's function $\mathrm{g}^*(\underline{x}_P, \underline{x}_n)$ is defined by summing the incident wave field, generated directly by the virtual source, with the wavefield generated at the boundaries of the different layers.

When the excitation source is a point pressure load, the full 3D response can be computed as a discrete summation of the effects of 2.5D loads with different wavenumbers along z (k_z). For this purpose, a second set of virtual loads, equally spaced L_z along the z axis has to be assumed. The 3D pressure field can then be written as

$$\mathrm{p}^{full,3D}(\omega) = \frac{2\pi}{L_z} \sum_{n=-N}^{n=+N} \mathrm{p}^{full}(\omega, k_z)\, \mathrm{e}^{-\mathrm{i}k_z z} \tag{6}$$

After computing the 3D pressure field in the frequency domain, the pressure in the spatial-temporal domain can be calculated by applying a numerical fast inverse Fourier transform in ω. For this, the pressure point source is assumed to generate a Ricker pulse. Using this technique, it is possible to analyze a total time window of $T = 2\pi/\Delta\omega$ ($\Delta\omega$ is the frequency increment). To avoid interference from aliasing phenomena, complex frequencies of the form $\omega_c = \omega - i\eta$ (with $\eta = 0.7\Delta\omega$) are used. In the time domain, the response must then be rescaled by applying an exponential window $\mathrm{e}^{\eta t}$.

Applied example

The model defined in the preceding sections is now applied to simulate the wave propagation inside a fluid waveguide filled with water ($\rho_f = 1000\,\mathrm{Kg/m^3}$, $\alpha_f = 1500\,\mathrm{m/s}$), 20.0m deep, excited by a point pressure load placed 0.5m from its bottom. The bottom of the waveguide is assumed to be a sedimentary ground 10.0m thick, lying over rigid bedrock. It is considered that the axis of a rigid circular inclusion inside the fluid waveguide is parallel to the global z axis, and has a radius of 1.0m. This inclusion is modeled with a number of constant boundary elements defined so that each element is at least 12 times smaller than the wavelength of the incident pressure waves. In no case is the number of elements less than 20. The pressure responses are registered over a horizontal line of receivers, placed along the *z* direction (Figure 1). The response is computed for a set of 128 frequencies, ranging from 10.0Hz to 1280.0Hz, with an increment of 10.0Hz. Time responses are then calculated, assuming that the excitation source generates a Ricker pulse with a characteristic frequency of 400.0Hz.

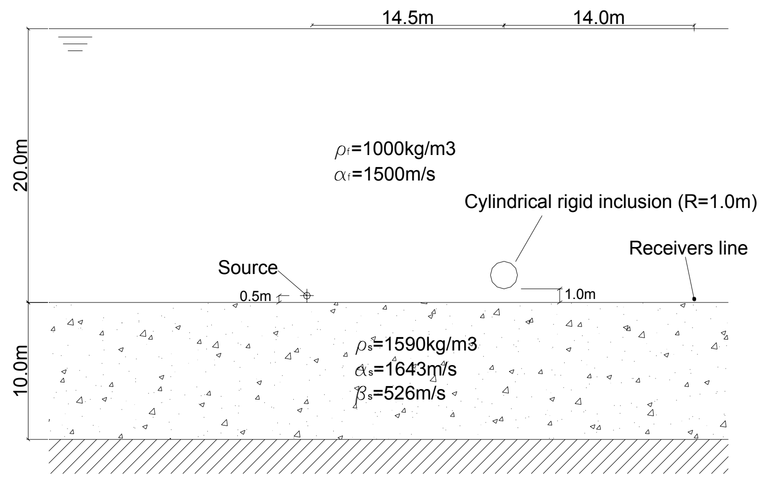
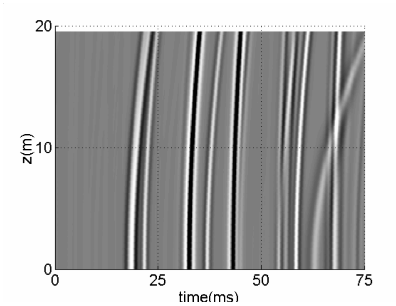
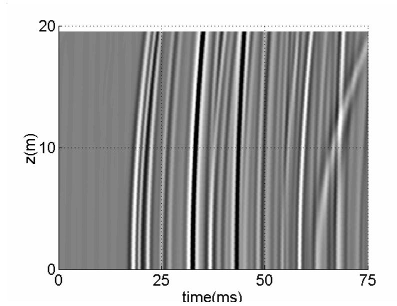


Figure 1: Simulation model.

Figure 2 illustrates the response computed at the first line of receivers. The amplitude of the response is represented by a grayscale, ranging from white to black as the amplitude decreases. In Figure 2a, the first signal registered corresponds to the incident pulse combined with a first reflection occurring at the top of the sediment layer, and it arrives at successively later times at receivers placed further away in *z*. Also visible are pulses resulting from multiple reflections between the free surface and the solid-fluid interface. Each time a wavefront strikes the solid-fluid interface, part of the energy is transmitted into the sediment layer, traveling as P, S and surface waves, while the remaining part is reflected back into the fluid. Within the sediment layer, multiple reflections occur between the rigid bottom and the interface with the fluid, causing multiple mode conversions between P and S waves. When these pulses hit the top interface, part of their energy is transmitted into the fluid again, and its arrival is visible in the time plots. At later times, it is possible to identify the arrival of the Scholte surface wave, traveling along the solid-fluid interface at a velocity below the S wave velocity in the sediment layer. When a rigid inclusion is introduced into the waveguide (Figure 2b), additional pulses become visible, due to reflection and diffraction effects on the surface of the inclusion. The wavefield thus generated is now more complex, since the multiple waves described will interact with the inclusion, generating new sets of pulses.





a) b)

Figure 2: Time response registered when R=1.0m: a) waveguide without obstacles; b) waveguide with rigid inclusion.

Conclusions

Frequency domain analytical solutions for wave propagation in generic layered media excited by 2.5D loads have been described. These solutions are obtained as the superposition of the effects of plane waves with different inclinations, and they allow the 3D response in the presence of 2.5D configurations to be computed as a discrete summation of 2.5D responses. These solutions have been used in conjunction with a BEM formulation as Green´s functions, making it possible to take into account the interfaces between the various layers without requiring their discretization. An example has been presented to demonstrate the applicability of these solutions, together with the BEM, to model a fluid-filled waveguide containing a rigid inclusion with constant geometry along one direction, excited by a point pressure load.

References

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