On Computational Approaches to Coupling Poroelasticity and Damage Mechanics

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Summary

This paper examines the mechanics of a fluid-saturated poroelastic medium the porous skeleton of which can experience micromechanical damage, which can alter the deformability and fluid transmissivity of the porous skeleton. In contrast to damage that evolves with time, the concept of *stationary damage* is an approximate procedure whereby alterations in the porous skeleton due to damage that occur at the commencement of the poroelastic processes are held stationary. The paper applies these procedures to examine the problem of the indentation of a poroelastic damage susceptible halfspace.

Introduction

The mechanics of fluid saturated porous media has applications to a variety of engineering applications ranging from geomechanics to biomechanics. The initial studies in this area commence with the work of Terzaghi, Fillunger, Rendulic and others (de Boer [1]). The classical theory of poroelasticity proposed by Biot [2] deals with the mechanics of a porous elastic medium that is saturated with a compressible pore fluid. The porous skeleton is assumed to exhibit isotropic Hookean elasticity and the mechanics fluid transport through the porous structure is described by Darcy's law. The resulting linear theory has been extensively applied via analytical methods, boundary element and finite element schemes to develop solutions to boundary value problems of engineering interest. The assumption of classical linear elasticity and Darcy flow are limitations to the application of Biot's model for the study of poroelastic geomaterials where stress-induced processes can lead to alterations in both the deformability and fluid transport characteristics of the porous skeleton. The mechanical processes that contribute to such alterations are varied and will depend on the type of geomaterials and the levels of stress sustained by the porous skeleton. In this paper, we consider geomaterials in which the porous skeleton of which can exhibit micromechanical damage. The damage in materials as proposed by Kachanov [3] is phenomenological and can be interpreted through the evolution of micro-mechanical defects such as voids, cracks and cavities. In the context of its applications to porous geomaterials, we implicitly assume that the voids that are generated during damage are at a scale larger than the naturally occurring pores in the geomaterials. Although damage evolution is an anisotropic process, in this paper, we consider an isotropic form of damage evolution, where the micro-mechanical manifestations can be identified with the nucleation and growth of voids in the porous skeleton of a geomaterials. The manifestations of damage phenomena in terms of the

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poroelastic responses are largely in the alterations in the deformability and the fluid transmissivity characteristics of the porous medium. A review of the contributions in terms of experimental, analytical and computational modelling of damage in poroelastic media is given by Selvadurai [4]. An approach advocated in this paper is a departure from the conventional modelling of the poroelastic damage in that damage that occurs at the start of the transient processes is assumed to remain unaltered throughout the transient poroelastic processes. This simplifies the study of the damaged poroelastic medium to that of the modelling of an inhomogeneous poroelastic medium, the spatial distribution of the properties of which is determined through a combination of analytical results and experimental observations. The analysis of the stationary damage problem in poroelasticity is therefore equivalent to the study of a classical problem in poroelasticity of an inhomogeneous medium. The methodology is used to examine a poroelastic contact problem, related to the indentation of a fluid-saturated halfspace with a smooth cylindrical indentor with a flat base. The numerical results derived through a finite element analysis of the contact problem demonstrate the influenced of both the stationary damage and evolving damage modelling on the time-dependent displacement of the indentor.

Poroelasticity and Damage

The partial differential equations governing the displacement and pore pressure fields in a poroelastic medium are given by

$$\mu\,\nabla^2\mathbf{u}+\frac{\mu}{(1-2\nu)}\nabla(\nabla.\mathbf{u})+\alpha\,\nabla p=\mathbf{0}\quad;\quad\kappa\,\beta\,\nabla^2 p-\frac{\partial p}{\partial t}+\alpha\beta\,\frac{\partial}{\partial t}(\nabla.\mathbf{u})=0 \tag{1}$$

where **u** is the displacement vector, p is a scalar pressure and ∇^2 is Laplace's operator. Considering thermodynamic requirements for a positive-definite strain energy potential it can be shown that the material parameters should satisfy the following constraints: $\mu>0$; $0\le\widetilde{B}\le 1$; $-1<\nu<\nu_u\le 0.5$; $\kappa>0$. For isotropic damage and considering a 'strain equivalence hypothesis', the constitutive equation for the damaged skeleton of the poroelastic medium can be written as

$$\boldsymbol{\sigma}=2(1-D)\mu\,\boldsymbol{\varepsilon}+\frac{2(1-D)\mu\nu}{(1-2\nu)}(\nabla.\mathbf{u})\,\mathbf{I}+\alpha\;p\,\mathbf{I} \tag{2}$$

which implies that Poisson's ratio remains constant. We adopt the damage evolution, criterion proposed in the literature [5], which takes the form

$$\frac{\partial D}{\partial\xi_d}=\eta\,\frac{\gamma\xi_d}{(1+\xi_d)}\left(1-\frac{D}{D_c}\right) \tag{3}$$

where ξ_d (the equivalent shear strain) is related to the second invariant of the deviator strain dyadic and η and γ are positive material constants. Also, $\xi_d = \mathrm{tr}\,\mathbf{e}^2$ and $\mathbf{e} = \boldsymbol{\varepsilon} - (1/3)\mathrm{tr}\boldsymbol{\varepsilon}\,\mathbf{I}$. Integrating (3) between appropriate limits, the evolution of D with ξ_d can be prescribed as follows:

$$D = D_c - \left(D_c - D_0\right)\left(1 + \psi\xi_d\right)^{\eta/\psi D_c} \exp\left(-\eta\xi_d / D_c\right) \tag{4}$$

The normalizing damage measure in (4) is the critical damage D_c, which is associated with the damage corresponding to a residual value of the strength of the geomaterial under uni-axial compression; the normalizing value could equally well be taken as the damage measure at the attainment of the peak load. We shall assume that the alterations in the hydraulic conductivity characteristics also follow an isotropic form. This is an approximation with reference to the mechanical response of brittle geomaterials that tend to develop micro-cracking along the dominant direction of stressing, leading to differing permeabilities in orthogonal directions. Mahyari and Selvadurai [6] have proposed the following relationship for the evolution of hydraulic conductivity as a linear function of the parameter ξ_d :i.e.

$$k^d = (1 + \Omega\xi_d)k \tag{5}$$

where k^d is the hydraulic conductivity applicable to damaged material, k is the hydraulic conductivity of the undamaged material and Ω is a material constant. In computational procedures that *do not* invoke the *stationary damage* concept, damage evolution criteria are used to define the time- and position-dependent alteration of elastic stiffness and hydraulic conductivity properties. The conventional computational approaches to the modelling of poroelastic behaviour can be conveniently adopted for the modelling both the concept of *stationary damage* and *evolving damage* [7].

Indentation of a Poroelastic Halfspace

We consider the axisymmetric smooth indentation of the surface of an isotropic poroelastic halfspace by a cylindrical indentor with a flat base. In the stationary damage modelling, the state of stress in the elastic halfspace is evaluated analytically using the results given by Sneddon [8]. This stress state is then used, with $\nu = 1/2$, to determine the spatial variation of ξ_d within the indented halfspace region and through (2), (4) and (5), the spatial distribution of elastic stress-strain parameters and hydraulic conductivity characteristics within the poroelastic halfspace region at time $t = 0$. In the stationary damage modelling, these values are assumed to be fixed in time and the analysis of the consolidation process can be carried out using any standard computational approach for poroelastic problems. In modelling evolving damage, the elasticity and hydraulic conductivities are allowed to change with time as the stresses are transferred from the pore fluids to the soil skeleton during consolidation, resulting in additional damage as defined by the damage evolution function (4). This process involves an iterative time-stepping approach that accounts for continued changes in both μ and k_d with time. The details of the procedures and the computational algorithms used in the computations are described in the literature [6, 9]. The damage susceptible poroelastic material chosen for the numerical simulations is a porous sandstone, the poroelasticity parameters are as follows: $k = 1\times10^{-6}\ m/s$; $\gamma_w = 1\times10^4 N/m^3$; $E = 8.3GPa$; $\nu = 0.195$; and the corresponding coefficient of consolidation, $c = (2\mu\, k/\gamma_w) = 0.6946\ m^2/s$. The indentor is subjected an axial load of $P = 5.02\times10^8 N$ and the radius of the rigid circular indentor is taken as $R = 2m$.

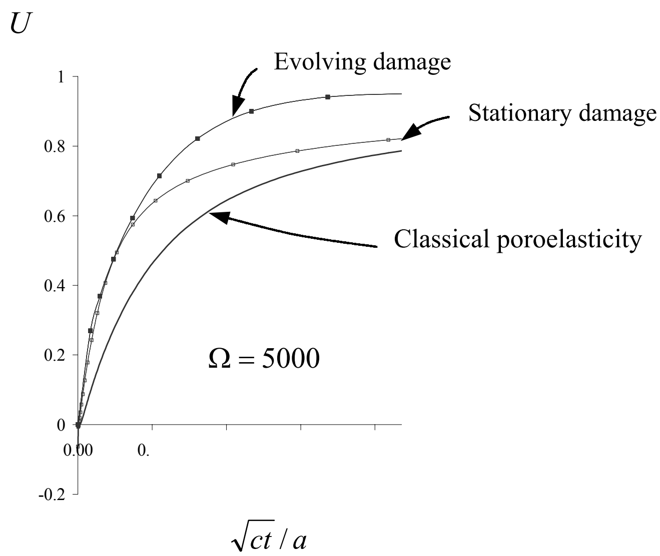

Figure 1. Consolidation displacement of a rigid cylindrical indentor

The level of stress induced by the external loading is sufficient to prevent the development of plastic failure of the sandstone consistent with the typical strength properties for the sandstone and an assumed theory of plastic failure for the material. The hydraulic conductivity evolution is based on the linear relationship (5) and the parameter Ω is taken as 5×10^3. The dominant changes in the poroelastic response will occur when Ω increases. The results of the computational modelling can be best illustrated through the definition of a "*Degree of Consolidation*", defined by $U = [\Delta(t)-\Delta_0)/(\Delta_\infty-\Delta_0)]$, where $\Delta(t)$ corresponds to the rigid displacement of the indentor at any arbitrary time *t*, Δ_0 and Δ_∞ correspond to the rigid indentor displacement at $t = 0$ and $t\to\infty$, respectively. Fig. 1 illustrates the results for U for the rigid indentor for of poroelastic responses corresponding to classical poroelasticity, poroelasticity with stationary damage and poroelasticity with evolving damage.

Concluding Remarks

The concept of stationary damage in poroelastic materials is a convenient approximation for examining the time-dependent mechanical behaviour of fluid saturated media that experience damage in the pore structure. It is shown that the consolidation response of the stationary damage model gives a bound for the complete poroelastic damage modelling where the elasticity and fluid transmissivity characteristics of the porous medium evolves during the development of damage. The incorporation of micromechanical damage that maintains the poroelastic character of the fluid-saturated medium generally has an appreciable effect on the duration of the consolidation process.

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