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On the Optimization of a Radial Induction Heating Problem

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Summary

Efficiency of induction heating processes can be improved considerably when designed in accordance with a respective optimal control problem solution. In order to achieve a required temperature field evolution in a heated workpiece efficiently, considering both, minimization of the input energy and time needed for the accomplishment of the heating process, while respecting all technological limitations, the process parameters must be correspondingly tuned. Since a highly demanding numerical approach is needed, in general, for the optimal solution of the considered physically coupled problem, any trustful initial estimation, though based on a physically simplified model, is beneficial and advantageous. As a first attempt to shorten the computational time the investigation of a corresponding radial problem is performed. Conditions, that characterize two specific classes of induction heating processes, surface and volume heating, respectively, are addressed.

Introduction

Induction heating is a convenient method used in industrial processes for surface and through volume heating of metal workpieces [1]. It relies on the fact that by electromagnetic induction, which results from an independent alternating electric source applied to an inductor, heat is generated in an electrically conducting workpiece, thus giving rise to a corresponding temperature distribution in it. Evidently, the problem is physically coupled, with thermal problem being dependent directly on the induced electric power, while electromagnetic problem being dependent indirectly through the established temperature dependence of the respective material physical properties. Due to the latter characteristic, particularly in regard to passing of the temperature beyond the Curie temperature and consecutive abrupt change in magnetic permeability, the problem could become also highly non-linear. To find a solution to the considered problem, considering actual boundary conditions as imposed by the geometry of practical inductor-worpiece assembly, is definitely a hard task even in the case, when a direct problem is considered. The only feasible and really efficient way is a numerical approach, in particular a combination of two powerful methods, FEM and BEM, respectively. This was clearly demonstrated for the assumed axisymmetrical cases, which are, fortunately, dominating in industrial applications [2].

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When trying to find optimal solutions to the above addressed problems, considering a chosen set of process parameters that are subject to optimization, the usual strategy is to perform corresponding sensitivity analyses, and based on those analyses to iteratively approach to the optimal solution in a suggested direction. Concerning numerical evaluation of respective parameter sensitivities, which is mostly done by the finite difference approach, difficulties often arise. Since, in addition, such computations are also very time consuming, it is rather important to be able to predict good initial guess for the optimal design. Practical technological experience is undoubtedly the basis, which we can rely on. Unfortunately, this support can not assist us in more complex cases. In consequence, further estimations, relying on simplified but still physically objective mathematical models, could be sought for the optimal solution.

As an approximation to a real axisymmetrical case we consider in the sequel a radial case, which is most elementary. Taking characteristic heat source distributions as evolved by the eddy currents into account, analytic expressions can be obtained for the respective temperature field evolutions. This definitely reduces the effort needed for the optimal design solution.

Solution of the Radial Thermal Problem

We consider an infinite cylinder of external radius R_e, which is exposed along its axis to radial thermal loading. The cylinder domain will be denoted by Ω, including the interval $R_i \le r \le R_e$, where $R_i \ne 0$ stays for the case of a hollow cylinder. Accordingly, the respective domain boundary, including both external and eventual internal radius, will be denoted by Γ. In order to cover both, high and low frequency induction with respective surface and volume developed heat generation, the respective heat agents are considered in the governing heat conduction equation for the temperature field evolution $T(r,t)$

$$\frac{1}{\kappa}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{\lambda} q_V(r,t) \quad ; \quad r \in \Omega \tag{1}$$

where r is the radial coordinate, t is time, κ and λ are respectively thermal diffusivity and thermal conductivity, and q_V is the volume heat source. A physically consistent solution to the above equation is obtained by taking associated initial and boundary conditions into account

$$T(r,0) = F(r) \quad ; \quad r \in \Omega \tag{2}$$

$$\lambda \frac{\partial T}{\partial n} + \alpha T = f(r,t) \quad ; \quad r \in \Gamma \tag{3}$$

In Eq. (3), which expresses convective type of boundary conditions with α being heat transfer coefficient, the first term is actually covering the surface heat source q_S, when specified.

When assuming material properties as temperature independent the solution $T(r,t)$, that fulfils the above problem equations, can be efficiently derived in terms of the corresponding Green's functions $G(r,t|r',\tau)$ [3]. For the considered problem the following expression can be derived for the temperature field evolution

$$
\begin{aligned}
T(r,t) = & \int_{\Omega} G(r,t\,|\,r',0)F(r')2\pi r'\,dr' + \int_{\tau=0}^{t}\int_{\Omega}\frac{\kappa}{\lambda}G(r,t\,|\,r',\tau)q_V(r',\tau)2\pi r'\,dr'\,d\tau + \\
& + \kappa\int_{\tau=0}^{t}\left[\frac{f(r',t)}{\lambda}G(r,t\,|\,r',\tau)2\pi r'\right]_{\Gamma} d\tau
\end{aligned}
\tag{4}
$$

where individual summands include consecutively the effects of initial conditions, volume heat source and surface direct or indirect heat source.

Because, due to space limitation, only a specific boundary heating case will be considered in the sequel, we write down the Green's function which is covering the convective radiative boundary condition in case of solid cylinder (R_e=R)

$$
G(r,t\,|\,r',\tau) = \frac{1}{\pi R^2}\sum_{m=1}^{\infty} e^{-\frac{\beta_m^2\kappa(t-\tau)}{R^2}} \frac{\beta_m^2 J_0\left(\beta_m \frac{r}{R}\right)J_0\left(\beta_m \frac{r'}{R}\right)}{J_0^2(\beta_m)\left(Bi^2+\beta_m^2\right)}
\tag{5}
$$

where Bi is the Biot number and β_m are the roots of the equation

$$
-\beta_m J_1(\beta_m) + Bi J_0(\beta_m) = 0 \qquad ; \qquad Bi = \frac{\alpha R}{\lambda}
\tag{6}
$$

Considering Eqs. (4) and (5), the following analytical expression is obtained for the temperature field evolution in case of prescribed ambiental temperature which is assumed sectionally constant in time, i.e. in the i-th time interval $t\in[t_{i-1},t_i] \Rightarrow T^{\infty}(t)= T_i^{\infty}$,

$$
\begin{aligned}
T(r,t) = & 2T_0\sum_{m=1}^{\infty} C_m A_m(r) e^{D_m t} + \\
& + 2Bi\sum_{i=1}^{n-1} T_i^{\infty}\sum_{m=1}^{\infty} B_m A_m(r)\left(e^{D_m(t-t_i)} - e^{D_m(t-t_{i-1})}\right) + 2Bi T_n^{\infty}\sum_{m=1}^{\infty} B_m A_m\left(1-e^{D_m(t-t_{n-1})}\right)
\end{aligned}
\tag{7}
$$

Here, functions $A_m(r)$ and coefficients B_m, C_m and D_m are defined in terms of Bessel functions and roots β_m in the following way

$$
A_m(r) = \frac{J_0\left(\beta_m \frac{r}{R}\right)}{J_1^2(\beta_m)+J_0^2(\beta_m)}\;;\; B_m = \frac{J_0(\beta_m)}{\beta_m^2}\;;\; C_m = \frac{J_1(\beta_m)}{\beta_m}\;;\; D_m = -\frac{\beta_m^2\kappa}{R^2}
\tag{8}
$$

The Optimal Problem Definition

With respect to the above considered thermal problem and adopted assumptions we formulate the following optimization problem, which consists in finding the shortest time τ at which an arbitrary axisymmetric layer $r\in[r_{in}, r_{out}]$ of the solid cylinder is heated into a pre-defined temperature range $T(r)\in[T_{low}, T_{high}]$. The heat source being acting from the outside of the cylinder, the heat is conducted from the surface $r=R$ inwardly during the heating period, causing lagging of the temperature rise of inner layers.

The minimization will be focused to such cases, where due to high heat flux rate from the heater the prescribed temperature range $[T_{low}, T_{high}]$ can not be obtained by a single heating. In this particular case the surface temperature is approaching material temperature limit T_{max}/T_{high}, even before the temperature $T(r_{in})$ reaches the lower prescribed temperature T_{low}. In consequence, the heat supply has to be turned off, causing cooling of the surface. Meanwhile, the cylinder core is still heating due to conduction of the heat stored in the outer layers. The described heating-cooling sequences are continued considering the above material constraint, till prescribed temperature values are obtained.

The minimization procedure is conducted in two steps. In the first step a rough approximation of the shortest process time τ (heating + cooling) is found, succeeded by a refined minimum search in the second step. In this step not the whole domain of interest $0\le r_{in}\le r\le r_{out}\le R$ is checked against the allowable temperature range $T(r)\in[T_{low}, T_{high}]$. Instead, only temperatures $T(r_{in})$, $T(r_{out})$ at the inner and outer radius are examined. Times, when those temperatures fall within the required temperature range, are denoted as τ_{in} and τ_{out}. In general heating-cooling sequences those two times are not equal, thus the greater is the time, when temperature requirements in the investigated layer are satisfied, $\tau=max[\tau_{in}, \tau_{out}]$.

This is true only, if in the continuation of the heat transfer, when $t>\tau$, the temperatures $T(r_{in})$ and $T(r_{out})$ become equal, and if this equal temperature T_{sect} is still inside the prescribed temperature range

$$T(r_{in},t)=T(r_{out},t)=T_{sect} \quad , \quad t>\tau \quad \wedge \quad T_{sect}\in\left[T_{low},T_{high}\right] \tag{9}$$

In the minimization procedure all relevant parameters, which govern the heat transfer in the cylinder, except heating and cooling times t_i, are kept constant. The subscript i stays for the number of a treatment period, regardless the nature of treatment, i.e. heating or cooling. Cumulative number of heating plus cooling periods is n. The cost function $W(\mathbf{x})$, which is to be minimized by a proper selection of parameters vector $\mathbf{x}$, is written as

$$W(\mathbf{x})=W\left(x_1,...,x_p\right)=\tau\left(t_1,...,t_{n-1}\right) \tag{10}$$

where minimization parameters x_i are actually times t_i, corresponding to the termination of the i-th heating or cooling period. The end of the last cooling period is in fact time τ, which is to be minimized in the ongoing optimization. Thus, it does not represent an optimization parameter, but the cost function itself.

The aforementioned minimization problem constraints, i.e.: prohibited overheating of the cylinder surface at the end of each heating period, increasing values of subsequent heating and cooling period times, and holding temperature T_{sect} inside the prescribed temperature range, are written in a normalized form as

$$g_j(\mathbf{x}) = \frac{T_i\left(r_{out}, t_{2j-1}, t_0, \ldots, t_{2j-2}, T_{\infty 1}, \ldots, T_{\infty(2j-1)}\right)}{T_{max}} - 1 \le 0, \quad j = 1, \ldots, j_0, \quad j_0 = \frac{n}{2}$$

$$g_j(\mathbf{x}) = \frac{t_{j-j_0}}{t_{j+1-j_0}} - 1 \le 0, \quad j = j_0 + 1, \ldots, j_1, \quad j_1 = j_0 + n - 2 \tag{11}$$

$$g_{j_1+1}(\mathbf{x}) = 1 - \frac{T_{sect}}{T_{low}} \le 0 \quad , \quad g_{j_1+2}(\mathbf{x}) = \frac{T_{sect}}{T_{high}} - 1 \le 0$$

Subject to the above constrains the minimization problem is finally solved by using a quadratic programming routine [4].

Since in the first step of the minimization procedure only temperatures at two radii $T(r_{in})$ and $T(r_{out})$ are taken into consideration, it is possible, that some layers within r_{in} and r_{out} at time τ may fall out of the prescribed temperature range, defined by T_{low} and T_{high}. This happens actually in the case, when cylinder's surface is gradually cooled while $T(r_{out})=T_{high}$. At this instant, the maximum temperature can be found at a certain depth from the surface, say at radius r_x. To find this radius, the spatial derivative of the temperature field $\angle T/\angle r$ is equalized to zero. With the third checking radius introduced into minimization procedure a new loop of iterations, representing in fact the second step of the considered optimization, is started.

Numerical Example

Infinite solid cylinder of radius R=117mm is initially at uniform temperature of 20°C. The desired final temperature is confined between T_{low} = 205°C and T_{high} = 220°C for the whole cylinder, $r\in[r_{in}=0, r_{out}=R]$, while maximum material temperature is T_{max}=285°C. The heating protocol provides two heating and two cooling periods, given in the form of alternating surrounding air temperature: $T_{\infty 1}$=2000°C, $T_{\infty 2}$=20°C, $T_{\infty 3}$=2000°C, $T_{\infty 4}$=20°C. Since the first heating period t_1 can be calculated directly by imposing the maximum temperature to be obtained on the surface, while time t_4 is actually the sought time τ, there are only two remaining parameters which act as optimization design variables, namely $x_1=t_2$ and $x_2=t_3$. The respective material data considered in the numerical investigation are respectively, density ρ=7800kg/m^3, specific heat c=460J/kgK, conductivity λ=5.95W/mK and surface convection film coefficient α=10W/m^{2}K.

Starting from initial values of x_1=2586s and x_2=2869s, and using computer program NLPQL [4] optimal solution x_1^{opt}=2489s and x_2^{opt}=2743s is found in 5 iterations. The respective temperature time evolution at $r_{in}=0$, $r_{out}=R$ is displayed in Fig. 1, where in the part of the diagram under the abscissa, the history of the imposed heating regime is also plotted. Graphic representation of the time evolution of the temperature field distribution, represented in Fig. 2, clearly exposes the need for a second minimization step. White and black regions of the diagram represent temperatures, which are (in terms of radius and time) higher than T_{high} and lower than T_{low}. A minimum time τ is found by pushing a vertical line crossing the gray shadowed region of the diagram to the most left position.

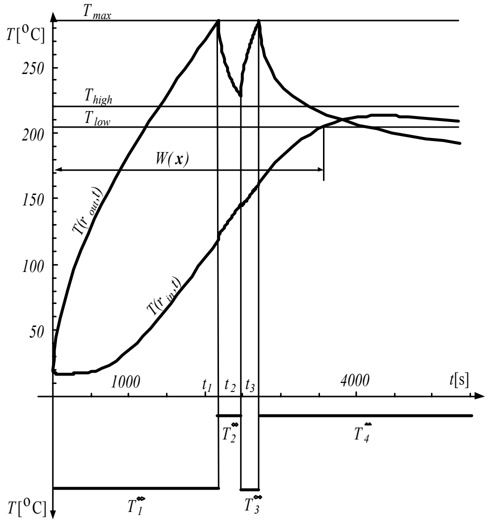

Figure 1: Surface and axis temperature evolution by optimized heating regime

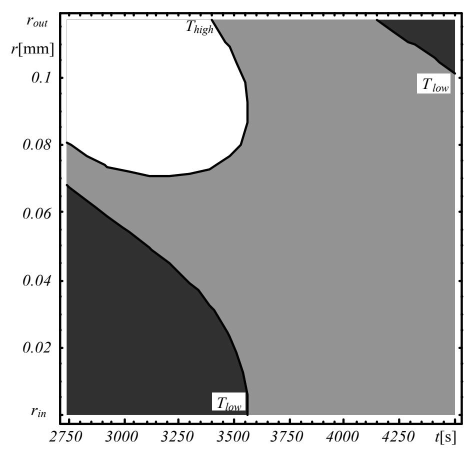

Figure 2: Time evolution of the temperature field distribution

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