Elastoplastic Finite Element Analysis of the Trapdoor Problem

T. Ozaki[1], T. Okayasu[2], K. Hashiguchi[2], D. Yajima[2]

Summary

Deformation behavior of the ground at the trapdoor descent is analyzed by the finite element method program in which the subloading surface model falling within the framework of unconventional plasticity is incorporated. The stress acting on the trapdoor predicted by this program agrees well with the experimental results and its accuracy is higher than classical Terzaghi's solutions for all cases having overburden depth. Progressive failure behavior due to lowering of the trapdoor, in particular, the formation of arch action in the overburden ratio $H/B=3$ (deep ground) is described realistically. Furthermore when the adjacent trapdoor is displaced downward the large arch action is newly formed the surrounding the arch formed by the first trapdoor descent.

Introduction

Response of embedded geotechnical structures such as tunnels, culverts, pipes, etc. is governed by the deformation behavior of soils around the structures. Especially, when the tunnel is constructed in an urban area, influences on existing embedded structures and foundations and settlement of the surface due to stress redistribution at the tunnel excavation are fully estimated in advance. This problem is often called the *trapdoor problem* and numerous studies have been performed from both experimental and theoretical view points up to the present [1-3].

Recently, the finite element method (FEM) has been frequently used for the analysis of trapdoor problem because it is possible to evaluate not only the steady stress state but also the soil deformation behavior around the embedded structure. The accuracy of the prediction, however, depends on the efficiency of the constitutive model introduced into the FEM program. The *subloading surface model* [4, 5] falling within the framework of the unconventional plasticity, which does not put the premise that the interior of the yield surface is an elastic domain, is capable of realistically describing the smooth elastic-plastic transition and thus the softening behavior observed in the over-consolidated soils. The validity of this model has been proved for various kinds of soils such as sands and clays (cf. e.g. [5]).

In this paper, the FEM program incorporating the subloading surface model is adopted for the analysis of the trapdoor problem. First, the FEM results are compared

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[1] Kyushu Electric Engineering Consultants Inc., Fukuoka 810-0005, JAPAN

[2] Department of Bioproduction Environmental Sciences, Graduate School of Kyushu University, Fukuoka 812-8581, JAPAN

with the experimental results and the classical Terzaghi's solutions for various cases of overburden depth. Second, the progressive failure phenomenon due to the lowering of the trapdoor is discussed. Finally, the ground deformation behavior for the state that two adjacent trapdoors are displaced downward is also simulated.

Governing Equations and Analytical Condition

The rate description of virtual work based on the updated Lagrangian formulation is adopted to analyze the trapdoor problem mentioned above as follows

$$\int_v \{\overset{\circ}{\boldsymbol{\sigma}} + (\mathrm{tr}\,\mathbf{D})\boldsymbol{\sigma} - \boldsymbol{\sigma}\mathbf{D} + \mathbf{W}\boldsymbol{\sigma}\} : \delta\mathbf{L}\,\mathrm{d}v = \int_s \dot{\boldsymbol{\pi}}\cdot\delta\mathbf{v}\,\mathrm{d}s\ , \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress. $\mathbf{D}$ and $\mathbf{W}$ are the stretching (the symmetric part of the velocity gradient $\mathbf{L} \equiv \partial\mathbf{v}/\partial\mathbf{x}$; $\mathbf{v}$: velocity) and the continuum spin (the anti-symmetric part of the velocity gradient $\mathbf{L}$), respectively. $\dot{\boldsymbol{\pi}}$ is the nominal traction rate in the current configuration. v and s denote the volume and the area, respectively, of the body in the current configuration, respectively. $\mathrm{tr}(\ )$ and $\delta(\ )$ stand for the trace and the virtual increment. The proper corotational Cauchy stress rate $\overset{\circ}{\boldsymbol{\sigma}}$ is related into $\mathbf{D}$ as follows

$$\overset{\circ}{\boldsymbol{\sigma}} = \mathbf{C}^{ep}\mathbf{D} \tag{2}$$

where $\mathbf{C}^{ep}$ is the fourth-order elastoplastic stiffness tensor based on the subloading surface model [5].

Fig. 1 illustrates the finite element mesh used in the analysis under the plane strain condition, while the four-noded quadrilateral isoparametric elements are adopted. The bottom of the ground is rough, i.e., both the horizontal and vertical displacements are fixed. Both the sides are the smooth boundary allowing the vertical displacement.

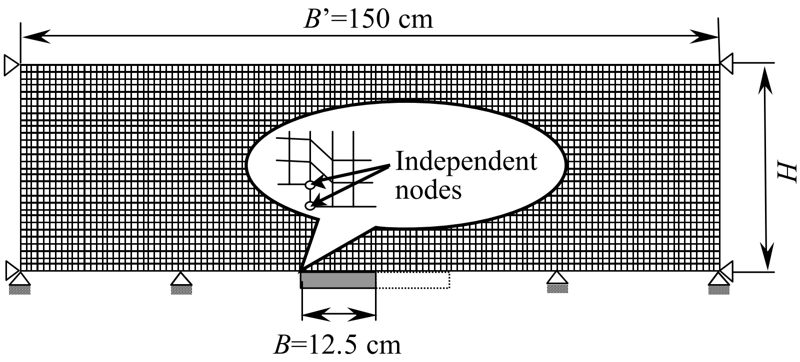


Fig. 1 Finite element mesh and boundary conditions.

Table 1 Physical properties of soil and model parameters.

(a) Physical properties of Toyoura sand

γ_g (kN/m^3)	ϕ_f (deg.)	c (kPa)	e	Water content (%)
1.51	40	0.0	0.74	0.3

(b) Material constants and initial values of the subloading surface model

Parameter	Value	Parameter	Value
F_0(kPa)	10.0	ϕ_{cr} (deg.)	27.0
ρ	0.008	ν	0.3
γ	0.0008	u	5.0

In order to reduce the locking of the shear deformation, the independent fixed nodes are arranged into both edges of the trapdoor. The lowering of the trapdoor is reproduced by the displacement increment control. The physical properties of Toyoura sand, material constants and the initial values of the subloading surface model are listed in Table 1. The initial stress $\boldsymbol{\sigma}_0$ is given from the overburden pressure, i.e., the vertical and horizontal initial stresses σ_z and σ_h are given by $-\gamma_g H$ and $K_0\sigma_z$ (K_0: coefficient of earth pressure at rest ($K_0 = 1.0$ in the simulation)). The ratio of the overburden depth to the trapdoor width (H/B), called the *overburden ratio*, is selected in three cases as H/B=1 (elements: 1200, nodes: 1334), 2 (2400, 2554) and 3 (3600, 3754).

Results and Discussions

Fig. 2 shows comparison of the relationship between the trapdoor displacement and stress ratio $\sigma_z / \gamma_g H$ (σ_z : mean vertical stress) acting on the trapdoor for various overburden ratios. The stress ratio decreases with increasing the trapdoor displacement and then approaches the constant value called the *steady stress ratio*. The steady stress ratio decreases with H/B but the degree of its reduction becomes smaller with the increase of H/B. The present FEM result totally agrees with the experimental result.

Fig. 3 shows the ultimate stress ratio and H/B relation. Here the experimental data is compared with the present FEM result and the solution of Terzaghi's loosening earth pressure theory [1], which is given by

$$\sigma_z = \frac{B\gamma_g - 2c}{2K_0 \tan\phi_f}\left\{1 - \exp\left(-\frac{2K_0 H \tan\phi_f}{B}\right)\right\}. \tag{3}$$

Terzaghi's solution tends to underestimate the ground strength in every overburden depths. In the meanwhile, FEM result can predict quantitatively well the experimental result.

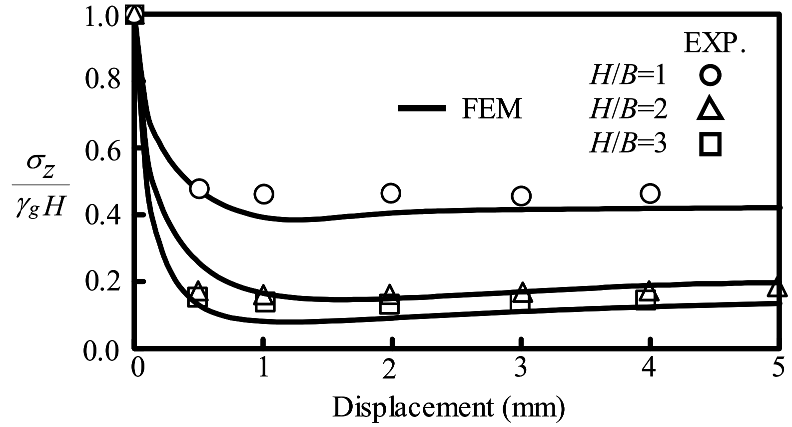


Fig. 2 Relationship between the trapdoor displacement and stress ratio $\sigma_z/(\gamma_g H)$.

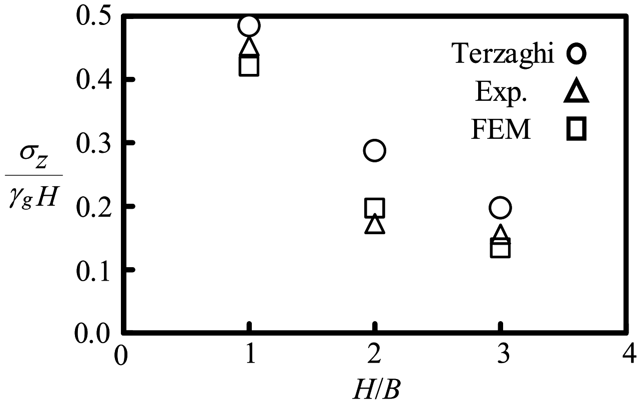
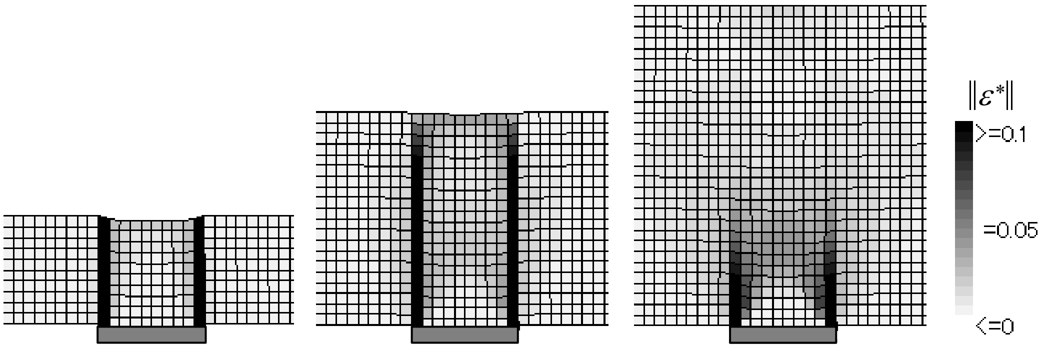


Fig. 3 Relationship between overburden ratio and $\sigma_z/(\gamma_g H)$ (at steady stress state).

Fig. 4 shows the distributions of the magnitude of deviatoric strain at the trapdoor displacement 5 mm. In *H*/*B*=1 and 2, the localized shear band develops vertically up to the ground surface although the arch action is observed slightly in *H*/*B*=2. On the other hand, the shear band does not develop up to the surface since the arch action is formed clearly around the trapdoor. The present FEM program can describe realistically the progressive failure phenomenon when the trapdoor is displaced downward.

Fig. 5 shows the settlement profile in the surface at the trapdoor displacement 5 mm. The settlement above the trapdoor occurs remarkably and its profile exhibits the convex curve concentrated into the width of the trapdoor in *H*/*B*=1 and 2 since the vertical shear band is formed clearly up to the surface as shown in Fig. 4. In this case, the soil mass above the trapdoor is discharged together with the trapdoor descent. On the other hand, in *H*/*B*=3, the settlement widely distributes over the surface as compared with *H*/*B*=1 and 2 although it becomes large above the trapdoor. This means that the progressive failure of the ground is inhibited by the formation of the arch action. Namely, in this case, it is considered that the elastic deformation induced by the stress redistribution due to the trapdoor descent mainly propagates over the ground.

Finally, the deformation behavior in the state that the two adjacent trapdoors are displaced downward is shown in Fig. 6. The localized shear band appears around the first trapdoor when it is displaced downward but develops up to the surface by the lowering of the second one. This progressive failure behavior has been reported in the various experiments up to the present (e.g. [6,7]).



(a) *H*/*B*=1 (b) *H*/*B*=2 (c) *H*/*B*=3

Fig. 4 Distributions of the magnitude of deviatoric strain (trapdoor displacement 5 mm).

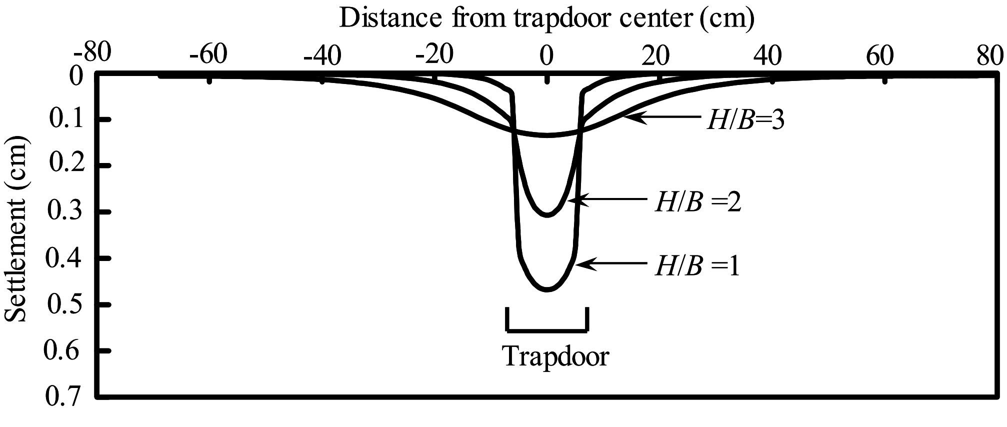
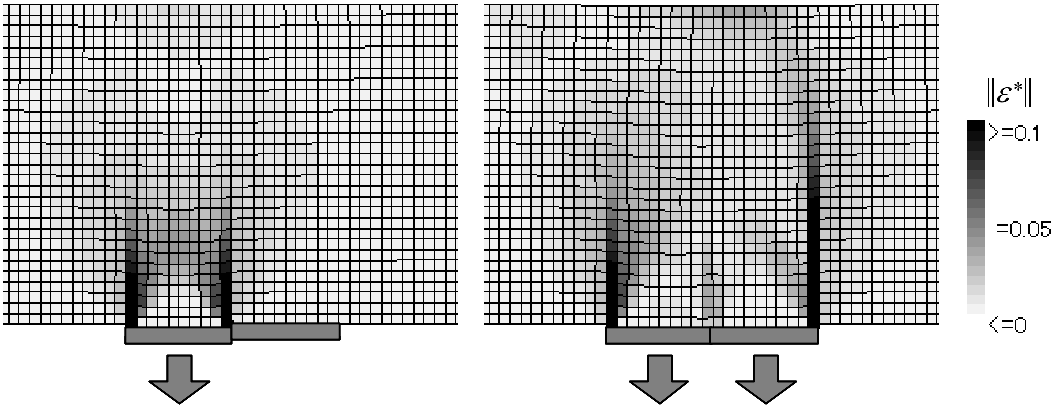


Fig. 5 Settlement profile in the ground surface (trapdoor displacement 5 mm).



(a) First lowering stage (b) Second lowering stage

Fig. 6 Deformation behavior due to lowering of two adjacent trapdoors.

Reference

1 Terzaghi, K. (1976) (Copyright 1943):“Theoretical Soil Mechanics”*, John Wiley & Sons*, pp. 194-198.

2 Vardoulakis, I., Graf, B. and Gudehus, G.(1981): “Trap-Door Problem with Dry Sand: A Statical Approach based upon Model Test Kinematics”, *Int. J. Numer. Anal. Meth. Geomech.*. Vol.5, pp.57-78.

3 Nagatani, H., Otani, J. and Takano, D. (2003): “Visualization of the Mechanism of Tunnel Face Failure”, *Proc. International Workshop on X-Ray CT for Geomaterials,* pp.169-174.

4 Hashiguchi, K. and Ueno, M. (1977): “Elastoplastic Constitutive Laws of Granular Materials”, *Constitutive Equations of Soils (Proc. 9th Int. Conf. Soil Mech. Found. Eng., Spec. Sess. 9, Tokyo)*, *JSSMFE*, pp.73-82.

5 Hashiguchi, K., Saitoh, K., Okayasu, T. and Tsutsumi, S. (2002): “Evaluation of Typical Conventional and Unconventional Plasticity Models for Prediction of Softening Behaviour of Soils”, *Geotechnique*, Vol. 52, No. 8, pp.561-578, 2002.

6 Noguchi, T. and Tarumi, H.(1996): “Study on Vertical Earth Pressure Distribution in Sand by Trap Door Test”, *J. JSCE*, **□-30**, No. 534, pp. 77-85 (in Japanese).

7 Noguchi, T. and Tarumi, H.(2001): “Study on Ground Deformation Analysis during Shield Tunneling by Nonassociated Flow Rule-based FEM”, *J. JSCE*, **□-50**, No. 672, pp. 133-144 (in Japanese).