Form and structural analysis of high-rise pneumatic structures

Hiroyuki Obiya[1], Katsushi Ijima[1], Noriaki Kawasaki[1], Shin-ichi Iguchi[2]

Summary

The present study shows the analysis for pneumatic structures, which has high-rise and large volume for their floor space. An incremental technique of 'the simultaneous control' is applied to the computational program that is based on the tangent stiffness method. The method is a clear and strict analytical theory to be able to solve the problems with large deformational behavior. Therefore, it makes possible to find initial shapes for the pneumatic structures as the precise equilibrium solutions. Moreover, 'the simultaneous control' gives the shapes of curved surfaces with large volume. Results are presented as the computational example in which the behavior of high-rise pneumatic structures becomes evident.

Introduction

The development of the finite element method made it possible to overcome some problems when designing pneumatic structures, and a number of papers have proposed about that. Pneumatic structures have been used for large span structures, for example, air domes for ballparks, gymnasiums, stadiums, and so on. However, as for such large pneumatic structures, there remained some problems of maintenances and its running costs. Therefore, it becomes rare that such a large-scale pneumatic structure is built recently. On the other hand, the pneumatic structures have been also used for temporary and/or smaller structures. It may be demanded that these structures have high-rise for small floor area depending on a purpose of use. In this case, some other problems may exist because of its surface shape. One is how to find the isotonic shape with such large volume, and another is how to keep its elastic stability against the external forces.

The authors' former paper has concluded that it is very rational to use both of the tangent stiffness method and the simultaneous control for the form finding of high-rise pneumatic structure. The simultaneous control is an incremental technique that can find solutions even in the state of post-buckling under distributed load. By the technique, one has to calculate the average of converted inner pressures of all nodes, and adopt it as the load intensity for next iteration step. Moreover, the converted inner pressure is to be balanced against the element edge forces at every node. As a result, the technique provides sure convergence and precise equilibrium solutions, even when the surface has so large volume.

[1] Saga University, 1 Honjo, Saga, 840-8502, Japan
[2] Forum8 Inc., 2-1-1 Gakuenkibanadai-nishi, Miyazaki, 889-2155, Japan

In those cases where the aim is to design the pneumatic structures with high-rise for their small floor space, another problem is their elastic stability. The form finding by the soap film elements, essentially, is to be used for determination of initial form without stress concentrations. The soap film element, however, has no stiffness to the tangent direction of its surface, so the real membrane elements replaced on the high-rise pneumatic structure may have the stress difference. Furthermore, the high-rise pneumatic structure may have compression stress, in some conditions, because of its larger self-weight comparing with its inner pressure.

This study verifies the stress state of the high-rise pneumatic structure whose shape is determined by the simultaneous control. The numerical example shows that the determined shape has enough rationality to be applied to real structures.

The tangent stiffness method for pneumatic structures

The tangent stiffness method uses common tangent stiffness matrix in both cases of form finding analysis and real membrane analysis. Therefore, it makes so easy to deliver numerical data from the form finding to the deformational analysis. Here the tangent stiffness equation is expressed when the method applied to 3-D finite element structures consistent of the triangle membrane elements.

Let the vector of the element edge forces independent of each other be indicated by **N**, and let matrix of equilibrium which relates **N** to the general coordinate system be **J**. Then the nodal forces **U** expressed in the general coordinate follow the equation:

$$\mathbf{U} = \mathbf{J}\mathbf{N} \tag{1}$$

The tangent stiffness equation is expressed as the deferential of Eq.(1), as follow.

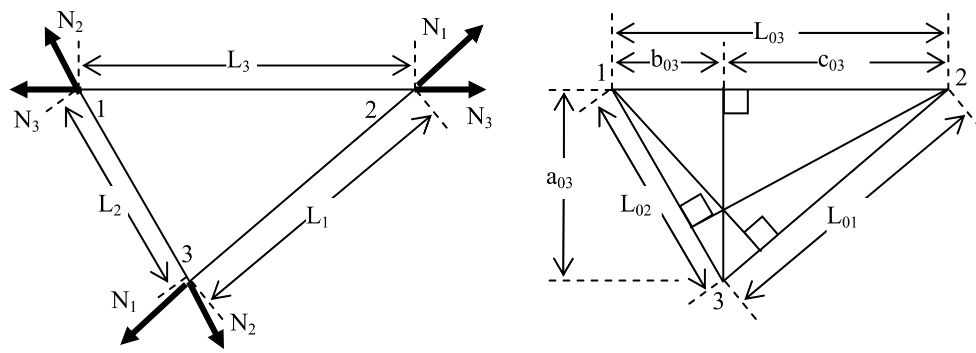
$$\delta\mathbf{U} = \mathbf{J}\delta\mathbf{N} + \delta\mathbf{J}\mathbf{N} = (\mathbf{K}_0 + \mathbf{K}_G)\delta\mathbf{u} \tag{2}$$

in which, $\mathbf{K_0}$ is the element stiffness which provide the element behavior in element (local) coordinate, and $\mathbf{K_G}$ is the tangent geometrical stiffness that can be commonly used both in the case of form finding and of the analyses for real membrane structures. In case of a triangular membrane element, the form of $\mathbf{K_G}$ becomes as follow:

$$\mathbf{k}_{\mathbf{G}i} = \frac{N_i}{L_i}\begin{pmatrix} 1-\alpha^2 & -\alpha\beta & -\alpha\gamma \\ & 1-\beta^2 & -\beta\gamma \\ Sym & & 1-\gamma^2 \end{pmatrix}_{i=1,2,3} \qquad \mathbf{K_G} = \begin{pmatrix} \mathbf{k_{G2}}+\mathbf{k_{G3}} & \mathbf{k_{G3}} & \mathbf{k_{G2}} \\ \mathbf{k_{G3}} & \mathbf{k_{G3}}+\mathbf{k_{G1}} & \mathbf{k_{G1}} \\ \mathbf{k_{G2}} & \mathbf{k_{G1}} & \mathbf{k_{G1}}+\mathbf{k_{G2}} \end{pmatrix}$$

(3),(4)

in which, *N*i is the element edge force whose direction is side of the triangle, and *L*i is the current side length. Further, α, β, and γ are the components of cosine vector along



(a) Element edge forces along side of a triangle (b) Non-stressed size of a triangular element

Fig. 1 Triangular element

side direction. (Fig. 1 shows the size of a triangular element.) Using this geometrical stiffness, the form finding analysis doesn't need the element stiffness of $\mathbf{K_0}$.

Moreover, after the shape has been determined, adding $\mathbf{K_0}$ (in Eq.(5)) to $\mathbf{K_G}$ makes it easy to switch to deformational analysis for the real membrane structures.

$$\mathbf{K_0} = \mathbf{B}^{\mathrm{T}}\mathbf{k_0}\mathbf{B} \tag{5}$$

$$\mathbf{k_0} = \frac{Et}{4A_0(1+\nu)}\left\{\frac{2}{A_0^{\ 2}(1-\nu)}\begin{pmatrix} S_1^2 & S_1S_2 & S_1S_3 \\ & S_2^2 & S_2S_3 \\ Sym & & S_3^2 \end{pmatrix} + \begin{pmatrix} L_{01}^2 & -L_{01}L_{02} & -L_{01}L_{03} \\ & L_{02}^2 & -L_{02}L_{03} \\ Sym & & L_{03}^2 \end{pmatrix}\right\} \tag{6}$$

E: Young's module, t: thickness of membrane ν: Poisson's ratio

$$S_i = (a_{0i}^2 - b_{0i}c_{0i})/L_{0i}\,, \quad i = 1,2,3 \tag{7}$$

in which, **B** is the compatible matrix between the each side length and the global coordinate. Before switching to the real membrane analyses, the non-stressed sizes should be converted as follows:

$$L_{0i} = \left\{1 - \frac{\sigma_s}{E}(1-\nu)\right\}L_i\,, \quad i = 1,2,3 \tag{8}$$

in which, σ_s is tensile of the soap film. The a_{0i}, b_{0i}, c_{0i}, and A_0 are obtained form L_{0i}.

Form finding by the simultaneous control

When a soap film with a boundary is inflated under inner pressure, there is a maximum point on its pressure-volume path. The simultaneous control is effective to find shapes, which are over the maximum point and have high-rise. In this case, the inner pressure intensity is renewed every iteration step, and calculated as follows:

1) Calculation of the converted inner pressures on all of free nodes and the control point. The converted inner pressure should be balanced with the resultant of element edge forces at every node.

2) The average of the converted inner pressures is adopted as external forces for next iteration step.

In the paper [3], details around the simultaneous control have been described.

Numerical examples

Fig. 2 shows the flow chart of the program developed. The program uses several common algorithms for both of the form finding and the real membrane analyses, and realizes efficiency. For conditions of numerical example, Table1 shows the physical values, and Fig.3 shows hexagonal primary boundary shape and its mesh division pattern.

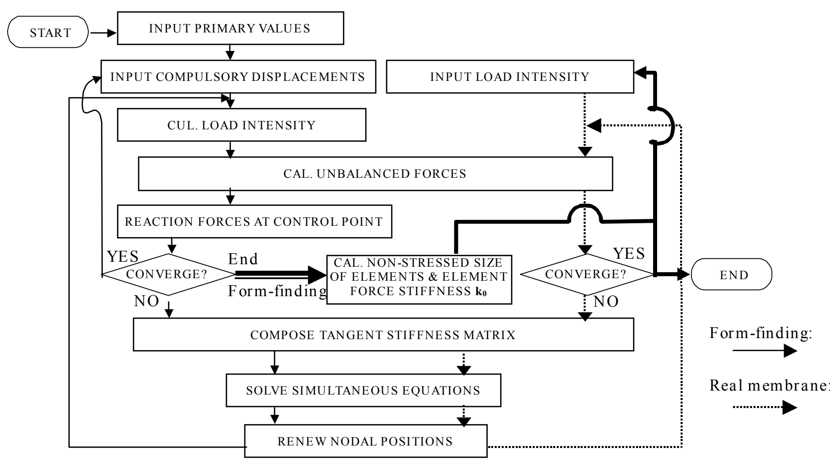


Fig. 2 Flow chart of the program for both of form finding and real membrane

Table 1 Physical values for example

FORM FINDING	
Tensile of soap film:	3.0kN/m
Control point:	center of hexagon
REAL MEMBRANE	
Young's modules*thickness:	170.0kN/m
Poisson's ratio:	0.4
Self-weight of membrane:	15.0N/m^2

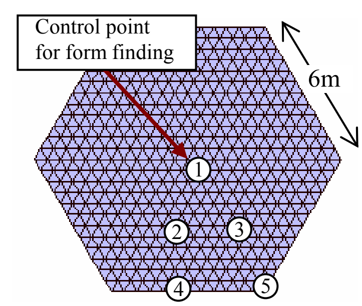


Fig.3 Primary boundary shape and its mesh division of example

The calculation program starts the isotonic surface analysis by the simultaneous control, in which the control point is set at the center node in Fig.3. Fig.4 is the Inner pressure-Volume curve, and there is a peak of inner pressure. In this analysis, four solutions of 'Shape A' to 'Shape D' are chosen as the initial shapes for pneumatic structures. After that, the stiffness of real membrane, in which defined in Eq.(5)~(8), is added to all the elements. The unbalanced forces, remained only to the tangent directions, are absorbed by the real membrane stiffness. Therefore, the perfect equilibrium solutions

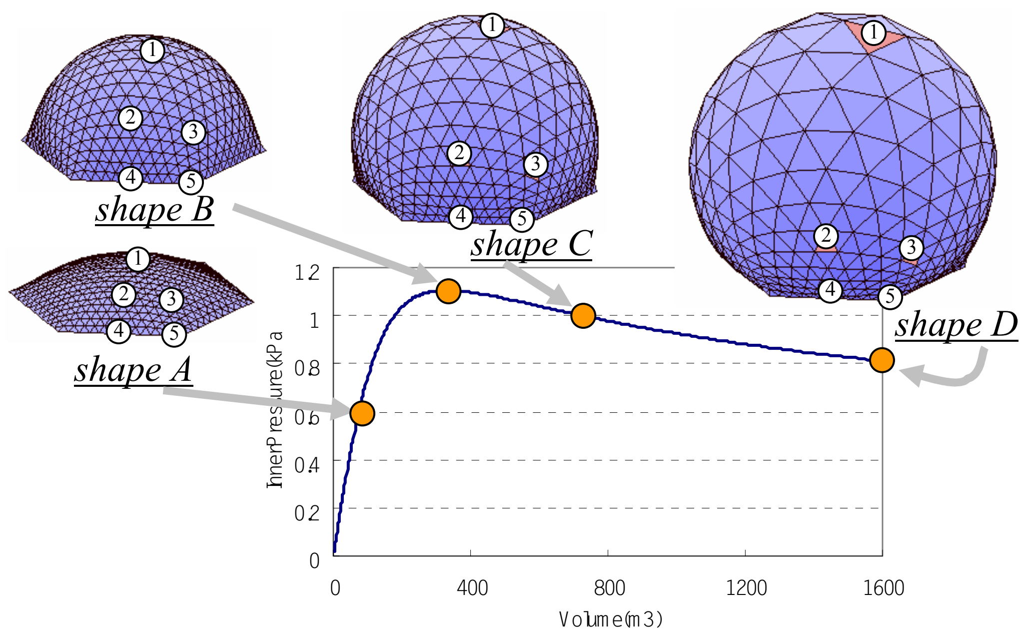
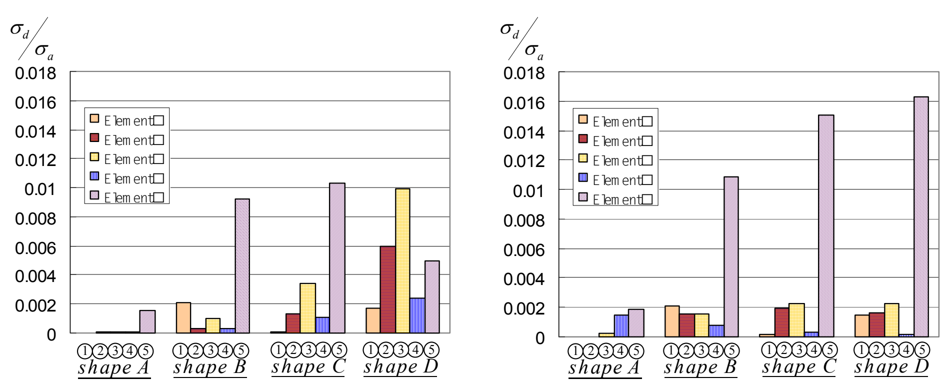


Fig.4 Inner Pressure-Volume curve and shapes (solutions) to be discussed



(a) Weightless equilibrium (b) Self-weight of membrane = 15N/m^2

Fig.5 Principal stress difference per mean principal stress are obtained after several times of iterations. Here, the stress conditions of five typical elements, indicated as □~□ in Fig.3, are discussed. Figures 5-(a)&(b) shows the ratios of principal stress difference σ_d for mean principal stress σ_a inside each element, under the conditions of weightless equilibrium(a) and under the 15kN/m^2 of self-weight(b). According to these figures, comparatively high stress is recognized in the elements where its curvature changes suddenly (for example □). However, the ratio of σ_d/σ_a is around 1.6% to the utmost, even the structure has so high-rise. Furthermore, there is no compression stress under dead load conditions.

Conclusions

In this study, a computational program for high-rise pneumatic structures has developed. The program can easily switch from the form finding analysis to the real membrane analysis because of using the common geometrical stiffness matrix in the both analyses. By the computational example, even if it was the high-rise shape which exceeded the extreme value of the pressure-volume curve, the equilibrium shape could be found in stability. Furthermore, the advantage of the isotonic surface as a prototype for pneumatic structures was recognized even if it had such a high-rise.

The problems to be investigated in future are as follows:

1) Application to the deflation analysis of high-rise pneumatic structures.
2) Verification about some live loads such as wind load.
3) Processing when compressive stress occurred, in conjunction with 1) and 2).

Some of above will be presented in the conference.

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