A Two-Surface Plasticity Model For Sands With Variable Degree of Non-Linearity

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Summary

In this paper, a deviatoric two-surface plasticity model with variable non-linear hardening modulus is formulated. This model inherits the simplicity of classical bounding surface model formulated as a special case of general plasticity theory. The model adopts a kinematic hardening circular cone as the yield surface and two non-circular conical surfaces corresponding to the deviatoric stress ratios at peak strength and start of dilation stress ratio. The shape of the non-circular surfaces is formulated in accordance with the experimentally established failure criteria and their sizes are related to the classical definition of 3D deviatoric stress invariants. A parametric study is performed to check the representation of the nonlinearity in the model.

Introduction

In pressure dependent bounding surface models [1], the nonlinearity of the cyclic loading curves are assigned by the definition of the hardening function, which is formulated with the scalar quantities, derived from the deviatoric stresses. Due to this inherent definition[1], nonlinearity of the stress-strain curves, generated by the cyclic loading is also fixed for certain stress paths. In this paper, the definition of hardening function is changed in such a way that, any degree of nonlinear curve can be generated by cyclic loading. In doing so, two-surface plasticity model is considered as the special case of generalized plasticity theory[2]. The formulation used here is basically a purely kinematically hardening model without any isotropic hardening. The elastic zone is a circular cone, moving around the outer bounding surface.

Model Description

The additive decomposition of total strain increment is used here as follows;

$$de_{ij} = de_{ij}^{e} + de_{ij}^{p} \quad (1)$$

Elastic part of the model is described in this subsection. As the first approximation, the shear modulus elastic deformations is formulated as a unique

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function of the mean principal stress σ_m, but independent of stress ratio while ignoring both inherent and stress-induced cross-anisotropy. The formulation described herein is based on the core idea that dynamic and statically determined values of shear modulus are essentially the same. The main formulation came from the work of Iwasaki el al.[4], in which a formulation of dynamic shear modulus was given based on the results of resonant column tests as follows,

$$G_e = G_0\left(\frac{(2.17-e)^2}{1+e}\right)(s_m)^m \tag{2}$$

where G_0 is a constant to be determined from relevant experimental result, e is void ratio, m is an exponent to be determined, which is 0.4 for Toyoura sand (Iwasaki et al.,[4]) and σ_m is the mean stress (= $s_1 + \sigma_2 + s_3$)/3. A total bulk modulus was formulated with a non-linear equation, which is similar to the equation of shear modulus, as follows,

$$K^t = K_0\left(\frac{(2.17-e)^2}{1+e}\right)(s_m)^m \tag{3}$$

The reason of formulating a total bulk modulus is the inability of a single yield model (a stress-dilatancy relation for shearing only) to capture the plastic volumetric strain characteristics for a wide range of stress path. By this way, the plastic volumetric strain can be captured to a significant proportion as shown in [3].

The plastic part of this model consists of a yield function, bounding and dilatancy surface, flow rule and a hardening law. There are some changes in the definition of bounding, dilatancy surfaces compared to the ref.1. These are shown in subsequent sections.

The yield function is of kinematic type. So it is a circular cone in the stress space with its apex at the origin, given by the equation

$$f = [(s_{ij} - pa_{ij}):(s_{ij} - pa_{ij})]^{1/2} - \sqrt{(2/3)}mp \tag{4}$$

where the back-stress ratio deviatoric tensor α_{ij} determines the position of the axis of the cone and the stress ratio scalar variable 'm' the 'size' of the cone. It is noteworthy that the evolution of α_{ij} and 'm' controls the kinematic and isotropic hardening respectively. In this research purely kinematic hardening is considered. Bounding and dilatancy surfaces will be similar in shape but sizes will differ. In this paper, generalized Mohr-Coulomb failure criterion in the invariant space is chosen for both bounding and dilatancy surfaces. The bounding/dilatancy surface in 3D stress-space can be written as;

$$f_{b,d} = h I_1 + \frac{1}{g(q,c)}\sqrt{J_2} = 0 \qquad (5)$$

where I_1=first stress-invariant, J_2= second invariant of deviatoric stress, f=yield function, h = deviatoric stress on the p -plane at Lode angle $q = 0^0$ depends on the angle of friction (for bounding function), and $g(q,c)$ = Lode angle (q) function.

Now, from the above equation, non-dimensional form of bounding/dilatancy size can be determined as follows:

$$a_q^b = \frac{\sqrt{J_2}}{I_i} = h_b g(q); a_q^d = \frac{\sqrt{J_2}}{I_i} = h_d g(q); \qquad (6)$$

where a_q^b and a_q^d are the coefficient of the bounding/dilatancy surfaces, h_b and h_d are controlling the size of the above described bounding and dilatancy functions, which are described as follows:

$$h_b = \frac{2\sin j_{peak}}{\sqrt{3}\left(3 - \sin j_{peak}\right)}; h_d = \frac{2\sin j_{res}}{\sqrt{3}\left(3 - \sin j_{res}\right)} \qquad (7)$$

$g(q,c)$ is defined as follows:

$$g(q,c) = \frac{2c}{\left((1+c) - (1-c)\cos(3q)\right)}, \text{ where } \quad c = \frac{\left(3 - \sin j_{peak}\right)}{\left(3 + \sin j_{peak}\right)} \qquad (8)$$

Here, in this research, a non-associated flow rule is used in order to accommodate the dilating behaviour of dense sand. It is noteworthy to say that a non-associated flow rule does not violate the Drucker's stability postulate in incremental sense. So, unique solutions were obtained with this formulation.

The definition of the kinematic hardening law is the core of the two-surface model. It is assumed here that $\dot{a}_{ij}$ is directed towards the bounding image stress ratio, a_q^b , defined by equation (6) and that it depends on the distance $b_{ij} = \left(a_{ij}\right)_q^b - a_{ij}$. So the following equation can be written

$$\dot{a}_{ij} = < L > h\left(a_{ij}\right)_q^b - a_{ij} \qquad (9)$$

where 'h' is a positive scalar valued hardening function. A form of 'h' which is used in many bounding/two-surface models is given by-

$$h = h_0 \frac{\left|b_{ij} n_{ij}\right|}{b_{ref} - \left|b_{ij} n_{ij}\right|} \tag{10}$$

where h_0 is a positive constant and $b_{ref} = \sqrt{2/3}\left(a_q^b + a_{q+p}^b\right)$ a reference distance > $\left|b_{ij} n_{ij}\right|$.

In this paper, the definition of 'h' is modified to the following function:

$$h = \left|b_{ij} n_{ij}\right|_0 \cdot \frac{\partial}{\partial k}\left(\left(\frac{2.0\sqrt{ke_f}}{k + e_f}\right)^m\right) \tag{11}$$

where $\left|b_{ij} n_{ij}\right|_0$ is the distance to the image stress on the bounding surface at the start of loading or unloading. k is the effective plastic strain increasing monotonically with loading or unloading, initialized to zero at the start of each loading or unloading. The function's nonlinearity can be modified by changing the value of the exponent m and the constant e_f. The function has some essential property like, when $k = e_f$, $h = 0.0$.

Parametric Study

A parametric study is carried out to see the capability of this model in capturing the essential features of cyclic loading for $j_{peak} = 50^0, j_{res} = 44^0$. In this paper, two cases are studied. First case is $m = 0.8$, $k_f = 0.1$ and the second case is $m = 0.2$, $k_f = 0.1$. It has been found that the results are much encouraging for further study in this field. A 10 cm x 10 cm element is taken used with a 100 kPa confining stress and loaded by vertical displacement only (Fig. 1).

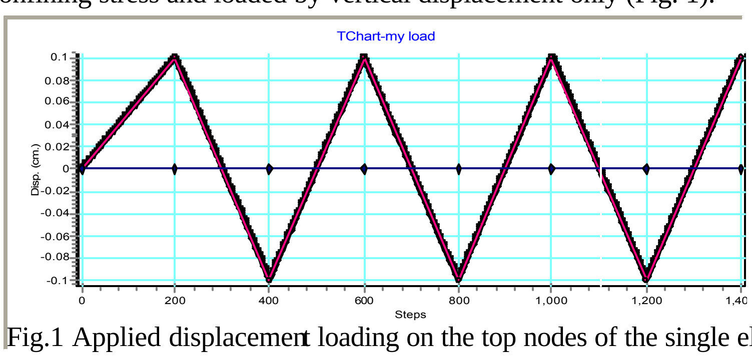

Fig.1 Applied displacement loading on the top nodes of the single element

Results and Discussion

Figs. 2 and 3 show the stress ratio vs. shear strain of the cyclic loading simulation for the first and second cases respectively. From the figures, the effect of change in the nonlinearity of the backbone and the reloading curves are very clear.

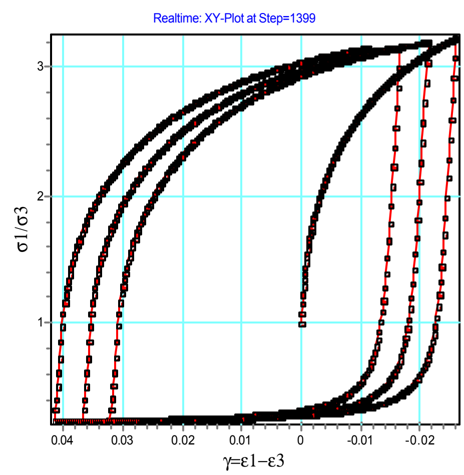

Fig. 2 Stress ratio versus shear strain for the first case ($m = 0.8, k_f = 0.1$)

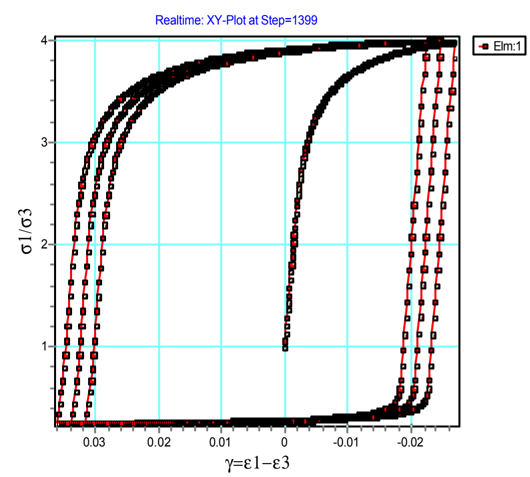

Fig. 3 Stress ratio versus shear strain for the first case ($m = 0.2, k_f = 0.1$)

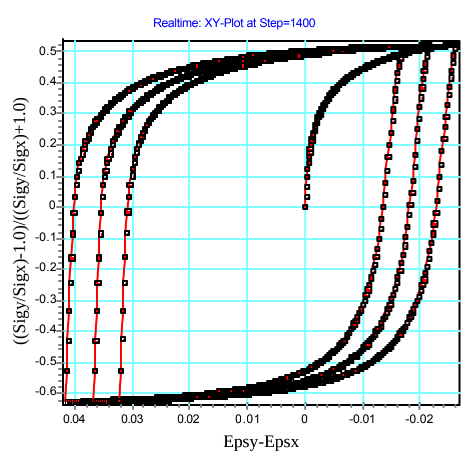

Fig. 4 $\sin(j)$ versus shear strain for the first case ($m = 0.8, k_f = 0.1$)

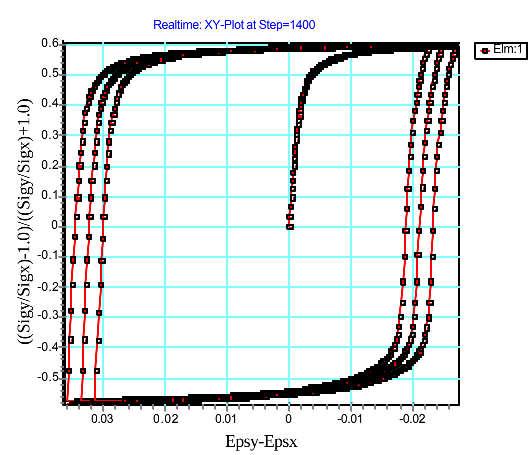

Fig. 5 $\sin(j)$ versus shear strain for the first case ($m = 0.2, k_f = 0.1$)

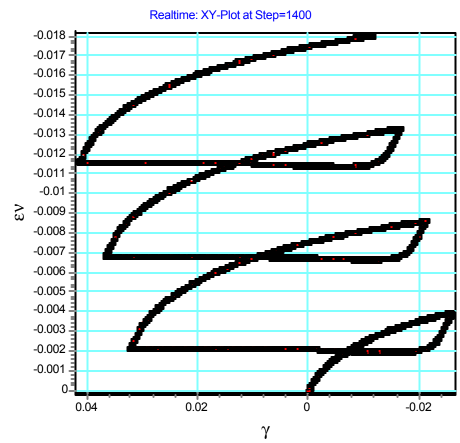

Fig. 6 Volumetric strain versus shear strain for the first case ($m = 0.8, k_f = 0.1$)

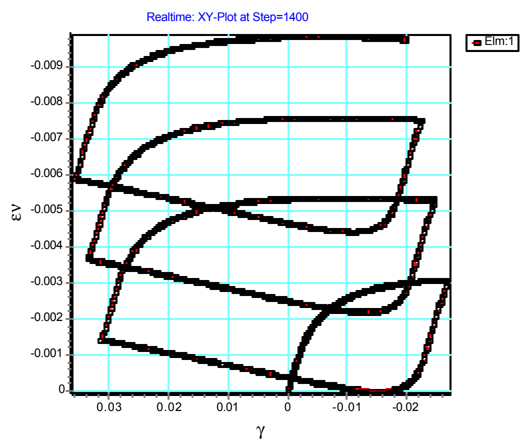

Fig. 7 Volumetric strain versus shear strain for the first case ($m = 0.2, k_f = 0.1$)

Figs. 4 and 5 show the $\sin(j)$ vs. shear strain of the cyclic loading simulation for the first and second cases respectively. From the figures, the effect of change in the nonlinearity of the backbone, the unloading and reloading curves are very clear. Figs.6 and 7 show the volumetric strain vs. shear strain of the cyclic loading simulation for the first and second cases respectively. The effect of change in the nonlinearity is very clear in these figures.

Conclusion

In this paper, a variable nonlinearity based hardening modulus is included in the classical two-surface model. It has been showed that the new approach enhances the nonlinear behaviour of the virgin, unloading and reloading curves very much.

Reference

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