Model for Cyclic Simple Shear Behavior of Sands Accounting for Principal Stress Rotation and Finite Strain Deformation

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Introduction

This paper presents further verification of the model proposed in (1) for the deformation of sand during rotation of principal stress directions. The model has been adapted for simple shear loading and has been used to simulate the results of simple shear tests on sand reported in (2). The emphasis of the verification is on the prediction of the rotation of principal stress directions in simple shear tests. As the formulation was developed to model the effects of principal stress rotation, it should be interesting to confirm if the model can predict the amount of rotation of principal stresses involved in simple shear tests.

Failure Surface

A two-dimensional representation will be assumed for simplicity. It is then sufficient to represent the state of stress in the stress space $p-S_1-S_2$ as shown Fig. 1, where $p=\left(\sigma_x+\sigma_y\right)/2$, $S_1=\left(\sigma_y-\sigma_x\right)/2$ and $S_2=\sigma_{xy}$. In this representation, a vector from the origin of the S_1-S_2 stress plane has a length equal to the shear stress $q=\sqrt{S_1^2+S_2^2}$ and makes an angle equal to 2α from the S_1 axis. α is the angle σ_1 makes from the *y*-axis. Assuming initially isotropic response, the failure criterion can be written as:

$$F=q-r_f\, p \tag{1}$$

where r_f is the stress ratio q/p at failure. The failure surface appears as a circular cone in the stress space, Fig. 1.

Flow Rule

Based on the results of a series of extensive tests (3), the flow of sand in the S_1-S_2 stress plane is represented by the plastic potential formulation shown in Fig. 2. In this figure, the plastic strain increment components $\left(\dot{\varepsilon}_y^p-\dot{\varepsilon}_x^p\right)$ and $2\dot{\varepsilon}_{xy}^p$ have been superimposed on the S_1-S_2 stress plane. The strain increment vector on this plane has a length equal to the plastic shear strain increment $\dot{\varepsilon}_s^p$ and makes an angle to 2β from the $\left(\dot{\varepsilon}_y^p-\dot{\varepsilon}_x^p\right)$ axis. β is the angle $\dot{\varepsilon}_1^p$ makes from the *y*-axis. This direction is evaluated as the

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normal to the failure surface at the conjugate point $\left(S_1^c, S_2^c\right)$ which is the intersection of the failure surface and the stress increment vector extended from the current stress point until it hits the failure surface. This flow rule is based on the experimental observation that flow on the $S_1 - S_2$ stress plane is dependent on the stress increment direction (3). From Fig. 2:

$$\dot{\varepsilon}_y^p - \dot{\varepsilon}_x^p = \lambda \left(\frac{\partial F}{\partial S_1} \right)_{S_1 - S_1^c, S_2 - S_2^c} = \lambda \left(\frac{S_1^c}{r_f \, p} \right) \quad , \quad \dot{\varepsilon}_{xy}^p = \lambda \left(\frac{\partial F}{\partial S_2} \right)_{S_1 - S_1^c, S_2 - S_2^c} = \lambda \left(\frac{S_2^c}{r_f \, p} \right) \tag{2}$$

where λ gives the magnitudes of the plastic strain increments, and can be calculated as:

$$\dot{\varepsilon}_s^p = \sqrt{\left(\dot{\varepsilon}_y^p - \dot{\varepsilon}_x^p\right)^2 + \left(2\dot{\varepsilon}_{xy}^p\right)^2} = \lambda \tag{3}$$

while the direction of $\dot{\varepsilon}_1^p$ measure from the *y*-axis can be solved as

$$\tan 2\beta = \frac{2\dot{\varepsilon}_{xy}^p}{\dot{\varepsilon}_y^p - \dot{\varepsilon}_x^p} = \frac{S_2^c}{S_1^c} \tag{4}$$

The plastic volumetric strain increment $\dot{\varepsilon}_{vd}^p$ is assumed to be due to dilatancy, and is determined from the following stress-dilatancy relation proposed by [x]:

$$\frac{\dot{\varepsilon}_v^p}{\dot{\varepsilon}_s^p} = r_c - \frac{q}{p} \cos 2\psi \tag{5}$$

where $\psi = |\alpha - \beta|$ is the angle of noncoaxiality [x] and r_c is the stress ratio at zero dilatancy or phase transformation. Elastic strains are given by the usual linear elasticity relations.

Yield Surface and Hardening Rule

The yield surface is assumed to have a similar shape as the failure surface, Fig 1, and undergoes only kinematic hardening

$$f = \sqrt{\left(S_1 - c_1 p\right)^2 + \left(S_2 - c_2 p\right)^2} - r_e \, p \tag{11}$$

where c_1 and c_1 are the kinematic hardening parameters corresponding to location of the center of the yield surface in the S_1-S_2 stress plane while r_e is given a very small, finite value. When r_e is very small, $c_1 p$ and $c_2 p$ can be taken as equal to S_1 and S_2 respectively. Furthermore, the rotational kinematic hardening rule can now be simply given as:

$$\dot{c}_1 = \frac{\dot{S}_1}{p}; \ \dot{c}_2 = \frac{\dot{S}_2}{p} \tag{12}$$

The scalar quantity λ can be obtained from the consistency condition which gives:

$$\dot{\lambda} = \frac{1}{H_p} \frac{\partial f}{\partial \sigma_{kl}} \dot{\sigma}_{kl}$$

where H_p is the plastic hardening modulus. Since the elastic region in the S_1-S_2 stress plane is assumed to be very small, the loading direction as given by $\partial f / \partial \sigma_{ij}$ can not be determined. This is because $\partial f / \partial S_1$ and $\partial f / \partial S_2$ can not be evaluated when $r_e \to 0$. However, experiments indicate that flow in the S_1-S_2 stress plane can be taken as associative, i.e., the direction of flow is the same as the loading direction. This can be accomplished by replacing the derivatives $\partial f / \partial S_1$ and $\partial f / \partial S_2$ with the normals to the failure surface at the conjugate stress point.

$$\frac{\partial f}{\partial S_1} = \left.\frac{\partial F}{\partial S_1}\right|_{S_1 - S_1^c \cdot S_2 - S_2^c} = \frac{S_1^c}{r_j p} \quad , \quad \frac{\partial f}{\partial S_2} = \left.\frac{\partial F}{\partial S_2}\right|_{S_1 - S_1^c \cdot S_2 - S_2^c} = \frac{S_2^c}{2 r_j p} \tag{15}$$

Finally λ can now be calculated as:

$$\lambda = \frac{1}{H_p} \left\{ \left(-\frac{q}{p} - \frac{S_1^c}{r_j p} \right) \dot{\sigma}_x + \left(-\frac{q}{p} + \frac{S_1^c}{r_j p} \right) \dot{\sigma}_y + \left(\frac{S_2^c}{r_j p} \right) \dot{\sigma}_{xy} \right\} \tag{16}$$

Hardening Functions

To formulate the variation of the plastic hardening H_p during loading, the concept of a field of nesting contours of surfaces of equal plastic hardening modulus very similar to the nesting yield surfaces formulation of Mroz [5] is used. The H_p –surfaces after the K_o consolidation in simple shear apparatus. The movement of these surfaces is prescribed in a manner that insures their non-intersection. The plastic hardening modulus for a surface of radius r_i is calculated as:

$$H_{pi} = G_p p \left(1 - \frac{r_i}{r_j} \right)^2$$

where G_p is the initial plastic shear modulus.

Modifications for Finite Deformation

Comparison With Experimental Results

The above formulations are used to simulate the results of drained simple shear tests on Leighton Buzzard sand reported in [2]. The predicted and measured stress and strain curves are shown in Fig. 4 while the predicted and measured angles α, β and ξ are shown in Fig. 5. As can be seen, excellent agreement between predicted and experimental results are obtained. The predicted amounts of rotation of principal stress directions in tests of three densities of sand are compared to the measured results in Fig. 5. The figure illustrates that the model predicts the amount of principal stress rotation in simple shear tests very satisfactorily.

References

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2 Cole, E.R.L. (1967): ``The Behaviour of Soils in the Simple Shear Apparatus", *Ph.D thesis*, Cambridge University.