Elastic-plastic stress waves in ductile-hardening materials using finite element techniques and a split-hopkinson pressure bar

F. J. Ferreira[1], M. A. P. Vaz[2], F. J. Q. de Melo[3] and J. F. S. Gomes[2]

Summary

High strain rates in metallic materials are present in some engineering applications and designs subjected to sharply variable forces. In such situations, the load intensity may determine a stress state in excess to limit values. In non-linear analysis a constitutive material model is used as a simple approach an equivalent post-yield Young modulus. A simple and straightforward numerical model, based on finite element techniques was developed. The constitutive model deals with a smooth high order function to approach the evolution of the material behaviour. The numerical data was compared with the corresponding one from experimental tests carried out in a Split Hopkinson Pressure Bar (SHPB), an important experimental research tool in the assessment of the deformation rate of materials submitted to high speed impact loads. The main objective of this work was to simulate the wave propagation phenomena in the understanding of the experimental data.

Introduction

The fast rate deformation resulting from impulsive loads meet important applications in the design of metallic or advanced composite structural parts, where, the assessment of their behaviour can be investigated both with numerical [1] and experimental tools. A highly used experimental technique is based on the Hopkinson apparatus. This device has simple and straightforward features; however, the numerical modelling of its physical operating principle is very useful to understand the material behaviour during test. From simplest designs of the Hopkinson apparatus until more sophisticated set-ups, as the system used in the *Large Dynamic Testing Facility* (LDTF) of the *Joint Research Centre* (JRC) of Ispra (VA), Italy, worthy contributions for automotive structure design have been reported [2] using a large sized Hopkinson bar in the LDTF to evaluate the energy absorbing capacity of automobile sub-structure bodies under impact loads. This represented a considerable economy in the simulation of a crash test of the complete structure body. Another example of investigation on material behaviour under fast rate dynamic loads refers to the work of [3] in the assessment of concrete test-specimens under axisymmetric loads in a split-Hopkinson pressure bar. To obtain the material properties under high strain rates the simplest procedure generally consists on unidirectional tensile or compression tests. In this work an optimised design of a split Hopkinson pressure bar developed in LOME facilities was used to perform the experimental tests. Also the

[1] -Polytechnique Institute of Porto, IPP
[2] -Dep. of Mechanical Engineering and Industrial Management- University of Porto
[3] -Dep. of Mechanical Engineering, University of Aveiro

stress wave propagation phenomenon is investigated with the use of special rod finite elements. The constitutive law for the material behaviour is assumed according to a gradual evolution from linear elastic to a post yield hardening.

The modelling of stress wave propagation with FEM - Linear elastic materials

The stress wave propagation along variable section rods is discussed using simple two-nodes finite rod elements. These elements allow a reasonably accurate stress wave simulation, even with plastic deformations. This numerical technique has received refinements to include contact interaction between two rods for linear elastic problems [4]. In the present work the material behaviour includes cases as the purely linear elastic (with a relative interest) and analyses involving a bilinear material behaviour. The formulation of the axial wave propagation phenomenon along straight homogeneous rods can be approached with a finite element model form the generalized dynamic equilibrium equation:

$$[\mathrm{M}]\ddot{\mathbf{X}} + [\mathrm{C}]\dot{\mathbf{X}} + [\mathrm{K}]\mathbf{X} = \mathbf{F}(\mathrm{t}) \tag{1}$$

where [M], [C] and [K] are, respectively, the mass, the damping and the stiffness matrices of a bar structure integrating a set of rod elements. If an external time dependent force **F**(t) strikes the bar, its particles move with the displacement vector **X**(t), the velocity $\dot{\mathbf{X}}(\mathrm{t})$ and the acceleration $\ddot{\mathbf{X}}(\mathrm{t})$. For wave propagation analysis the damping effects are usually neglected. To define the numerical modelling of the propagation of axial waves along a bar this one is divided into a set of rod finite elements as the represented in Figure 1.

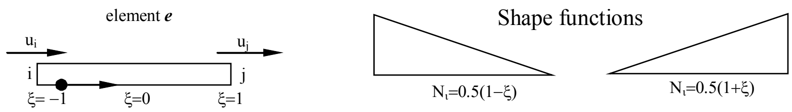


Figure 1: Rod finite element and shape functions

Considering negligible the effect of the Poisson ratio in the transverse section, the stiffness matrix of a rod finite element is:

$$[\mathrm{K}] = \frac{\mathrm{EA}}{\mathrm{L}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{2}$$

where E and A are the Young modulus of the bar material and the transverse area, while L is the element length as in Figure 1. The mass matrix considering a total mass lumping on both nodes of the element is:

$$[M] = \rho AL\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{3}$$

where ρ is the specific mass of the bar material. Equation (1) is integrated in time, giving the structure deformation at every time step. Matrices [K] and [M] in (1) refer now to the assembled stiffness and mass distributions. The use of a lumped mass distribution leads to relevant simplification in the time integration algorithm. The integration of equation (1) is efficiently carried out with the central difference method (CDM), better modelling the wave propagation phenomenon. The iterative algorithm is as follows:

$$\frac{1}{\Delta t^2} m_{kk}\,{}^{k}x_{i+1} = F_k - \frac{EA}{L}\left(-\,{}^{k-1}x_i + 2\,{}^{k}x_i - {}^{k+1}x_i\right) + \frac{2}{\Delta t^2} m_{kk}\,{}^{k}x_i - \frac{1}{\Delta t^2} m_{kk}\,{}^{k-1}x_i \tag{4}$$

where in (4), index k refers to the k^{th} diagonal elements of [K] or [M], or to the k^{th} element order in the displacement vector $\{X_i\}$. The operation with (4) needs that the time step Δt must observe values not exceeding a critical one, Δt_{crit} given by:

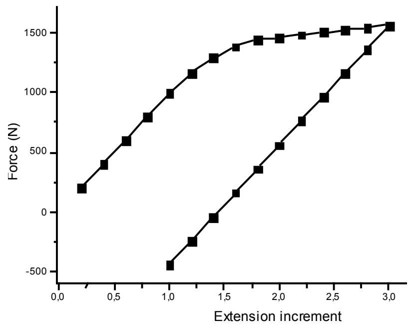
$$\Delta t_{crit} = \frac{L}{c}; \qquad c = \sqrt{\frac{E}{\rho}} \tag{5}$$

where c is the velocity of sound in the bar material.

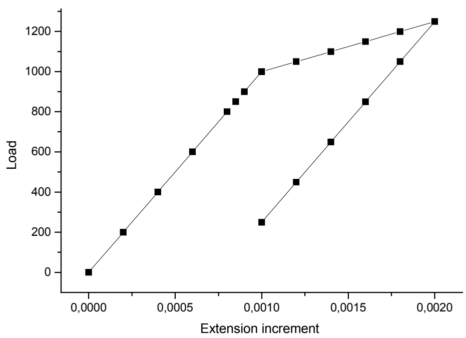
A hardening material with a bilinear equivalent behaviour

The material behaviour can extend from the elastic-perfectly plastic until a bilinear equivalent material. The transition between the two constitutive states can be assumed continuous, where a smooth transition curve between the two young modulus slope, as in Figure 2(a), or joining two straight lines, as in Figure 2(b). To set-up a finite element solution dealing with a material non-linear problem, the following assumptions are useful:

- The deformation field is based on small first order displacements;
- The non-linear conditions for the solution come from the material change characteristics during the stress-deformation evolution;
- The stresses are evaluated considering nominal values for the transverse section. This assumption is consistent with other, where the Poisson ratio of the material was considered ν=0, giving a negligible contribution to the transverse expansion.



(a)



(b)

Elastic Young modulus $E_1=10^6$ N/mm^2; Post-yield hardening Young modulus $E_2= 0.25\times10^6$ N/mm^2; Yield load: 1000 N

Figure 2: Rod under an increasing tensile load followed of unloading (a) with smooth transition past elastic limit load and (b) with sharp bilinear behaviour.

The general iterative solution (4) must undergo modifications, once the material behaviour depends on each actual deformation state. For any node in the structure (exception made for the first and the last nodes in the bar arrangement), algorithm (4) is rewritten as follows:

$$\frac{1}{\Delta t^2} m_{kk}\ {}^{k}x_{i+1} = F_k - ({}^{k}R_{i\,left} + {}^{k}R_{i\,right}) + \frac{2}{\Delta t^2} m_{kk}\ {}^{k}x_i - \frac{1}{\Delta t^2} m_{kk}\ {}^{k-1}x_i \tag{6}$$

In previous equation, with the structural contribution from a node k at time step *i*, the internal reaction vectors are identified as follows:

${}^{k}R_{i\,left}$:replacing $\frac{EA}{L}({}^{k}x_i - {}^{k-1}x_i)$ for internal reaction in element *k-1* at left of node *k*;

${}^{k}R_{i\,right}$:replacing $\frac{-EA}{L}({}^{k+1}x_i - {}^{k}x_i)$ for internal reaction in element *k* at right of node *k*

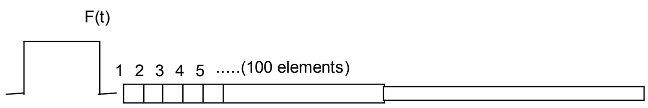
Both internal reactions are algebraically added, for the joint contribution of elements converging at node k, excluding the first and the last node. The Young modulus is replaced for an equivalent constitutive function F, characterizing the internal reaction R for the material. This vector is evaluated in an incremental procedure, updating their values at each time step with use of algorithm (6). At each time step *i*, a new displacement vector is calculated and the internal reaction at each structure element *k* is updated after a deformation increment as follows:

$$\begin{aligned} &{}^{k}R_i = {}^{k}R_{i-1} + F({}^{k}\Delta x_i - {}^{k}\Delta x_{i-1}); \quad \textit{where}: \\ &\textit{extension}\ {}^{k}\Delta x_i = {}^{k+1}x_i - {}^{k}x_i \\ &\textit{extension}\ {}^{k}\Delta x_{i-1} = {}^{k+1}x_{i-1} - {}^{k}x_{i-1} \end{aligned} \tag{7}$$

F is a constitutive function defining the current material behaviour under deformation and it can involve geometric parameters (as the element length and transverse area) and equivalent Young modulus changing when a pre-defined stress state is reached.

Example I – Bar with two sections changing abruptly

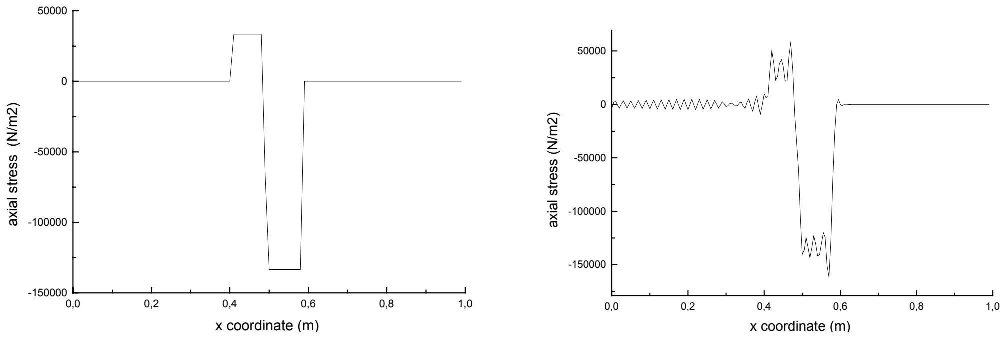
To analyse the stress waves propagation of along variable section bars, some examples with elasto-plastic deformation rod elements are carried out and discussed. Example 1 consists in a two section bar submitted to a step load of 1000N at the left end. This load holds for 10 times the critical time step (Δt_{crit} = 10μs).



Dimensions and mechanical properties: Length L=1m with section transition at 0.5 m; Transverse section: $A_{larger} = 0.01m^2$; $A_{smaller} = 0.005m^2$; Young modulus (homogeneous material) $E=1\times10^9\ N/m^2$; Specific mass ρ =1000Kg/m^3

Figure 3: A two sections bar submitted to a step load at left end

The stress wave distribution is solved using algorithm (4). Past a time of $60\times\Delta t_{crit}$ the stress distribution reveals a transmitted wave of 133kPa while a reflected wave of 33kPa is generated. These results are confirmed by a theoretical analysis [5]. With a time step just equal to the critical value, the stress wave presents distribution, as shown in Figure 4.



a) b)

Figure 4: Stress wave distribution, in example 1 bar after 1100 μs past the impact, a) Δt_{crit} b) time step=$0.995\times\Delta t_{crit}$.

This is, however, an ideal situation, once real bars present oscillations due to the wave dispersion phenomenon. An approach to this fact can be modelled if a time step Δt slightly smaller than the critical value is chosen; for example $\Delta t = 0.995\times\Delta t_{crit}$. Figure 5 presents the stress distribution for a case similar to the previous, but solved with a different time step. Here it is possible to obtain a distribution with some oscillations, better modelling the real behaviour of the structure.

Example 2 – Numerical model of the Split-Hopkinson Pressure Bar (SHPB)

This example models the set-up commonly known as SHPB. Essentially, two long bars contact the test specimen, as in Figure 6. The test specimen receives an incident wave; transmitting and reflecting it.

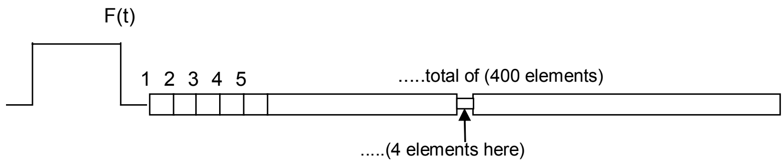


Figure 6: Simplified model of the Split Hopkinson Pressure Bar

The structure model has 400 equal length elements, with the test specimen divided in four. The step load is applied on the left end and the resulting stress wave is obtained in an element in input bar at equal distance from the ends. Specifically the element number 100 is selected to model a strain gauge, where the transmitted and reflected stresses are recorded. The geometric data and mechanical properties of the set-up are as follows: Length of input and output bars 0,5 m; transverse sections $4.9.\times10^{-4}$ m^2 and 1.225×10^{-4} m^2, respectively (rod diameters of 25mm and 12.5mm for the test specimen). Material properties: E_{steel} = 210GPa and E_{alum}=70 GPa. The post yield equivalent Young modulus is 1.4 GPa. Figure 7 presents the graphical output for the axial stress wave through element nº 100.

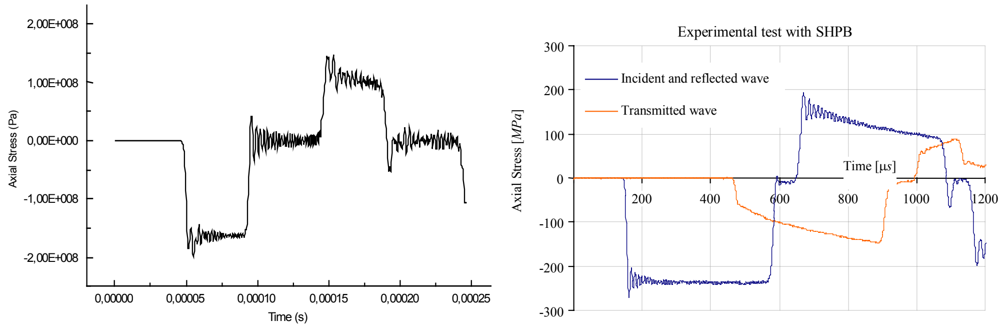


Figure 7: Example 2 - Stress wave at the element nº 100, a) in finite element analysis b) Experimental results with SHPB test,

As can be seen from previous Figures, results are very similar despite a larger impact time in the experimental test. In the numerical analysis only the incident and reflected waves were presented wile this analysis is completed with the inclusion of the transmitted wave in the experimental procedure.

Discussion and conclusions

The performance of a developed finite element solution for the propagation of elastic-plastic waves along straight bars was analysed. The comparison between the finite element and the experimental procedure with SHPB has shown a good agreement in elasto-plastic wave propagation. The experimental results could open way to a better understanding of the behaviour of other kind of materials.

References

1 Espinosa, H. D.; Lu, H-C.; Zavattieri, P. D. and Dwivedi, S. (2001). “A 3-D Finite Deformation Anisotropic Visco-Plasticity Model for Fiber Composites”, *J. of Composite Materials*, Vol. 35, nº 5, Feb. 2001, pp 369.

2 Albertini, C.; Cadoni, E. and Labibes, K. (1997) “Precision Measurements of Vehicle Crashworthiness by Means of a Large Hopkinson Bar”, EURODYMAT’97, Toledo, Spain

3 Albertini, C. and Montagnani, M. (1994) “Study of the true tensile stress-strain diagram of plain concrete with real size aggregate; need for and design of a large Hopkinson bar bundle”, EURODYMAT’94, Oxford, U.K.

4 Hughes, T. (1992), “The finite element method”, McGraw-Hill Co.; NY, 2nd Ed.

5 Johnson, W. (1973) “Impact Strength of Materials”, Edward Arnold (Publishers) Limited, London.