A Crack Along the Interface Between Two Transversely Isotropic Materials – the $\pm 45^\circ$ Pair

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Summary

In this study, the first term of the asymptotic displacement field is determined analytically for a crack between two transversely isotropic materials. The axial direction of the upper material is rotated with respect to the x_2-axis by $+45^\circ$ and that of the lower material by -45°. This field is used as the auxiliary solution in a three-dimensional M-integral for determining stress intensity factors. The stress intensity factors are obtained for one test case.

Introduction

In recent years, much research has been directed toward understanding interface fracture. Investigators have concentrated on the problem of an interface between two isotropic, homogeneous bodies. Another area of interest is the delamination of laminate composites. It is this latter subject which is considered in this study.

A fiber reinforced composite material with a $0^\circ/90^\circ$ interface was studied previously [2]. In that investigation, the effective mechanical properties were employed in determining the first term of the asymptotic fields in the neighborhood of a crack tip along the interface between these two materials. Since the in-plane and out-of-plane fields decouple, two-dimensional plane strain conditions were assumed. In this study, these layers are rotated to form a $\pm 45^\circ$ interface. Except near the crack tip, the fields are coupled and this problem is treated three-dimensionally.

These fields are employed as the auxiliary solution in a three-dimensional $M-$integral. This integral was first presented in [11] for in-plane mixed modes in isotropic, homogeneous material. It was extended to cracks along the interface of two isotropic materials in [9]. Banks-Sills and Boniface (2000) [2] adapted it for the $0^\circ/90^\circ$-interface in transversely isotropic material. It was extended for three dimensions in [5]. It is employed in the three-dimensional form here for the material studied.

Finally, a three-dimensional edge cracked slab is analyzed. The first term of the asymptotic expression for the displacement is prescribed on the outer surfaces of the body. One case is considered here in which $K_1 = 1$, $K_{II} = K_3 = 0$. The results expected are these prescribed values. In this way, accuracy of the method is examined for this case.

Asymptotic Displacement Field

The displacement field for a bimaterial body in which the upper layer contains fibers in the $+45^\circ-$direction and the lower layer contains fibers in the $-45^\circ-$direction are determined by means of the Stroh formalism as described in [8]. It should be noted that for both

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layers, $x_2 = 0$ is a plane of symmetry, and an asymptotic plane strain field is assumed near the crack tip. The interface crack is shown in Fig. 1

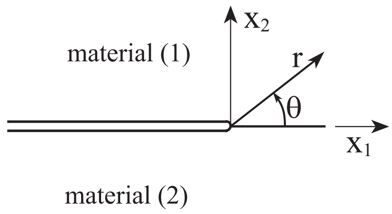

Figure 1: Crack tip coordinates.

The displacement field is given by

$$u_1^{(k)} = \frac{1}{\sqrt{1+4\varepsilon^2}\cosh\pi\varepsilon}\sqrt{\frac{r}{2\pi}}\left\{\Re\left(Kr^{i\varepsilon}\right)R_1^{(k)}(\theta)+\Im\left(Kr^{i\varepsilon}\right)Q_1^{(k)}(\theta)\right\} + \sqrt{\frac{r}{2\pi}}K_{II}S_{11}^{(k)}(\theta) \tag{1}$$

$$u_2^{(k)} = \frac{1}{\sqrt{1+4\varepsilon^2}\cosh\pi\varepsilon}\sqrt{\frac{r}{2\pi}}\left\{\Re\left(Kr^{i\varepsilon}\right)R_2^{(k)}(\theta)+\Im\left(Kr^{i\varepsilon}\right)Q_2^{(k)}(\theta)\right\} + \sqrt{\frac{r}{2\pi}}K_{II}S_{21}^{(k)}(\theta) \tag{2}$$

$$u_3^{(k)} = \frac{1}{\sqrt{1+4\varepsilon^2}\cosh\pi\varepsilon}\sqrt{\frac{r}{2\pi}}\left\{\Re\left(Kr^{i\varepsilon}\right)R_3^{(k)}(\theta)+\Im\left(Kr^{i\varepsilon}\right)Q_3^{(k)}(\theta)\right\} + \sqrt{\frac{r}{2\pi}}K_{II}S_{31}^{(k)}(\theta) \tag{3}$$

where the superscript $k = 1,2$ represents the upper and lower material, respectively, $i = \sqrt{-1}$, r and θ are polar coordinates illustrated in Fig. 1, $\Re$ and $\Im$ represent the real and imaginary part of a complex expression and the complex stress intensity factor

$$K = K_1 + iK_3\,. \tag{4}$$

In (1) through (3), following [8]

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1+\beta}{1-\beta}\right) \tag{5}$$

where

$$\beta = \left\{ -\frac{1}{2}\,\mathrm{tr}(\breve{\mathbf{S}})^2 \right\}^{1/2} . \tag{6}$$

The 3×3 matrix $\breve{\mathbf{S}}$ is given by

$$\breve{\mathbf{S}} = \mathbf{D}^{-1}\mathbf{W} , \tag{7}$$

$$\mathbf{D} = \mathbf{L}_1^{-1} + \mathbf{L}_2^{-1} \tag{8}$$

and

$$\mathbf{W} = \mathbf{S}_1\mathbf{L}_1^{-1} - \mathbf{S}_2\mathbf{L}_2^{-1} . \tag{9}$$

The subscripts 1 and 2 in (8) and (9) represent, respectively, the upper and lower material. Since $\mathbf{S}_k$ and $\mathbf{L}_k$ are real and

$$-\mathbf{A}_k\mathbf{B}_k^{-1} = \mathbf{S}_k\mathbf{L}_k^{-1} + i\mathbf{L}_k^{-1} , \tag{10}$$

knowledge of the left hand side of (10) is sufficient to determine (8) and (9). In (10), summation of indices is not indicated. The matrix $\mathbf{A}_k\mathbf{B}_k^{-1}$ for the upper and lower materials is found with the aid of work carried out in [10]. The functions $R_i^{(k)}(\theta)$, $Q_i^{(k)}(\theta)$ and $S_{i1}^{(k)}(\theta)$, $i = 1,2,3$, $k = 1,2$ are analytic and will be presented elsewhere.

It may be noted, for a crack along the interface between two transversely isotropic materials at the angles $\pm 45^\circ$, this is a coupled problem. That is, all modes are involved for all displacements as seen in eqs. (1) through (3). Thus, a body of this type must be analyzed as a three-dimensional problem.

J and M$-$Integral for the $\pm 45^\circ$ Interface Crack

In this section, the conservative M-integral is described for the material interface considered in this study, as well as three-dimensional analyses. This is an energy based method which is derived from the three-dimensional $J-$integral. It may be pointed out that the stress intensity factors may be separated by means of the $M-$integral.

First, Griffith's energy may be derived for this material pair as

$$G = \frac{\mathrm{sgn}\,(E_{22})\,|E_{22}|}{2\cosh^2 \pi\varepsilon}\left(K_1^2 + K_3^2\right) + \frac{\mathrm{sgn}\,(E_{11})\,|E_{11}|}{2} K_{II}^2 . \tag{11}$$

where

$$\mathbf{E} \equiv -\mathbf{A}\mathbf{B}^{-1}\,, \tag{12}$$

and E_{11} and E_{22} are identical for the upper and lower materials.

For a straight through crack which is treated in this study, the three-dimensional area $J-$integral is given by (see [4] and [7])

$$\int_0^L J(x_1)\,\delta\ell(x_1)\,dx_1 = \Delta a \lim_{\Gamma\to 0}\int_V \left[\sigma_{ij}\frac{\partial u_i}{\partial x_2} - W\delta_{2j}\right]\frac{\partial q_2}{\partial x_j}\,dV \tag{13}$$

where L is the length of the crack front and x_1 is the coordinate along the crack front (see Fig. 2a), $\delta\ell$ is the virtual crack extension, Δa its maximum extent, $W = 1/2\sigma_{ij}\varepsilon_{ij}$ is the strain energy density and δ_{ij} is the Kronecker delta. In (13), the surface Γ is shown in Fig. 2b in an x_2, x_3 plane. The limit is taken so that the volume V reaches from the crack tip to an outer surface S. Taking the limit to the crack tip ensures path independence. On S, q_2 is zero; it takes on the value $\delta\ell(x_1)$ along the crack front; it is continuously differentiable in V. The requirements for q_2 guarantee the validity of the right hand side of (13).

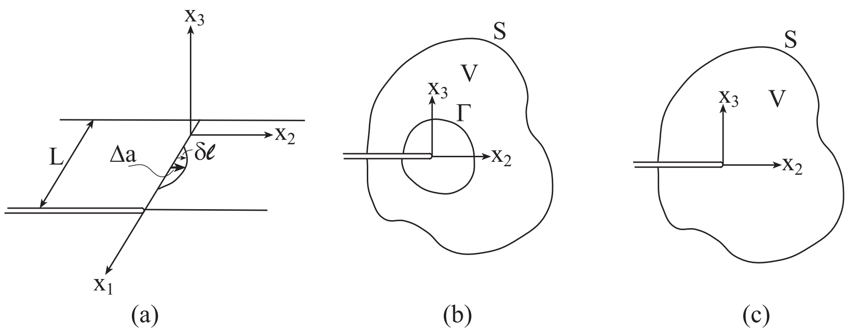

Figure 2: (a) Virtual crack extension $\delta\ell$ for through crack. (b) In-plane volume V, outer surface S and inner surface Γ. (c) Inner surface Γ collapses to crack front.

To obtain accuracy in the calculation from finite element results, the function q_2 is defined for twenty noded isoparametric elements as

$$q_2 = \sum_{m=1}^{20} N_m(\xi,\eta,\zeta)\,q_{2m} \tag{14}$$

where N_m are the finite element shape functions and ξ, η and ζ are the coordinates of the parent element (for further details, see [1]).

By the derivation of eq. (13), $G(x_1) = J(x_1)$. The values of J or G may be obtained accurately along the crack front by implementing eq. (13). This allows one to determine the combined value of the stress intensity factors given in eq. (11). In order to obtain the individual stress intensity factors, the $M-$integral must be extended for the $\pm 45^\circ$ pair. This derivation will be presented elsewhere. Once the first term of the asymptotic solution is known, the derivation follows that for the bimaterial isotropic case in [6].

Test Problem

An edge crack in a finite thickness slab is considered. The displacements given in eqs. (1) through (3) are prescribed on the outer boundaries of the body with $K_1 = 1$, $K_{II} = K_3 = 0$. Along the crack faces, traction free conditions are assumed. Fiber reinforced carbon/epoxy material (AS4-3502) was analyzed. Some material properties may be found in [2].

The program ADINA [3] was employed to carry out the finite element analyses. Twenty noded isoparametric brick elements were used; at the crack tip, they were distorted to quarter-point elements leading to a square-root singularity. It should be noted that the mode II stress is square-root singular, whereas the other stresses are square-root oscillatory. Thus, this element does not completely model the stress behavior. Two meshes were employed: one containing 34,641 nodal points and the other 55,181 nodal points. The mesh is focused at the crack front. Through the thickness, both meshes contained 15 elements.

Integration of the $M-$integral is carried out in volumes which are orthogonal to the crack front and are an element thickness in that depth. The volume extends away from the crack tip, but always includes the crack tip elements.

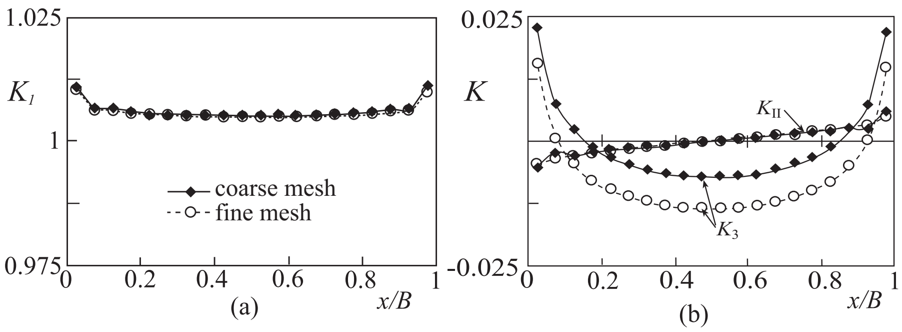

Figure 3: Stress intensity factors (a) K_1, (b) K_{II} and K_3.

Results for K_1, K_{II} and K_3 obtained by means of both coarse and fine meshes are exhibited in Fig. 3. It may be observed that while $K_1/K_{1theory}$ is approximately 1.005 and $|K_{II}|/K_{1theory}$ is less than about 0.005 for both meshes; $|K_3|/K_{1theory}$ is 0.007 for the coarse mesh and 0.013 for the fine mesh in the center of the body. The increase in error with the finer mesh probably occurs as a result of the element dimension ratio near the crack tip which increases from 1:1:4 to 1:1:7.

Although the $M-$integral is theoretically path independent, the region adjacent to the crack front is not as accurate as other domains. This is attributed to the near tip elements which do not model correctly the oscillatory stress behavior. Moreover, it appears that the error is concentrated within a small region near the crack tip. Unlike the two-dimensional $M-$integral, even a domain removed from the crack tip region, as in Fig. 2c, includes inaccurate results from the near tip elements. Therefore, the near tip error affects any domain taken.

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