Constitutive Modelling of Elastoplastic Response of Sand

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Summary

In this contribution, a constitutive model is presented to simulate the elastoplastic characterization of sand. The model is general and sufficiently simplified in terms of number of material parameters and every parameter has a clear physical meaning. The emphasis has been placed on simulation of soil hardening and softening response and the establishment of model nonassociativity. The constitutive model is calibrated on the basis of standard triaxial test results. Comparisons of model predictions and laboratory measurements for various stress paths are presented.

Introduction

In the finite element analysis of geotechnical problems, the choice of an appropriate constitutive model may have a significant influence on the numerical results. The constitutive model should be able to capture the main features of the mechanical behaviour of geomaterials under complex states of stress. In recent years, various types of constitutive models have been developed. Some of them appear to be rather complicated and some model parameters are difficult to be obtained from standard laboratory tests.

In this contribution, on the basis of the hierarchical approach proposed by Desai [1], a constitutive model is presented for simulation of the elastoplastic response of sand. The model is general and sufficiently simplified in terms of the number of material parameters and every parameter has a clear physical meaning. In general, the proposed model is applicable for any frictional material. However, in this contribution, only sand is considered. Emphasis is placed on the presentation of the model characteristics in both the hardening and the softening ranges of response, and the establishment of model nonassociativity. Comparisons of model predictions and laboratory measurements for various stress paths are presented.

Constitutive Model

In this contribution, on the basis of hierarchal approach proposed by Desai [1], a constitutive model is expressed in functional form as:

$$\mathrm{F} = \frac{\mathrm{J}_2}{\mathrm{p}_a^2} - \left[-\alpha \cdot \left(\frac{\mathrm{I}_1 + \mathrm{R}}{\mathrm{p}_a} \right)^n + \gamma \cdot \left(\frac{\mathrm{I}_1 + \mathrm{R}}{\mathrm{p}_a} \right)^2 \right] \cdot \mathrm{F}_s = 0 \tag{1}$$

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where I_1 and J_2 are the first and second stress invariants respectively, p_a is the atmospheric pressure. Parameter n is related to the state of stress at which the material response changes from compaction to dilation. Parameter γ is related to the ultimate strength of the material. Parameter R represents the triaxial strength in tension, F_s is the function related to the shape of the flow surface in the octahedral plane,

$$F_s = (1 - \beta \cdot \cos 3\theta)^m \tag{2}$$

where $\cos 3\theta = \dfrac{3\sqrt{3}}{2} \cdot \dfrac{J_3}{J_2^{3/2}}$ and J_3 is the third invariant of the deviatoric stress and θ is the Lode angle.

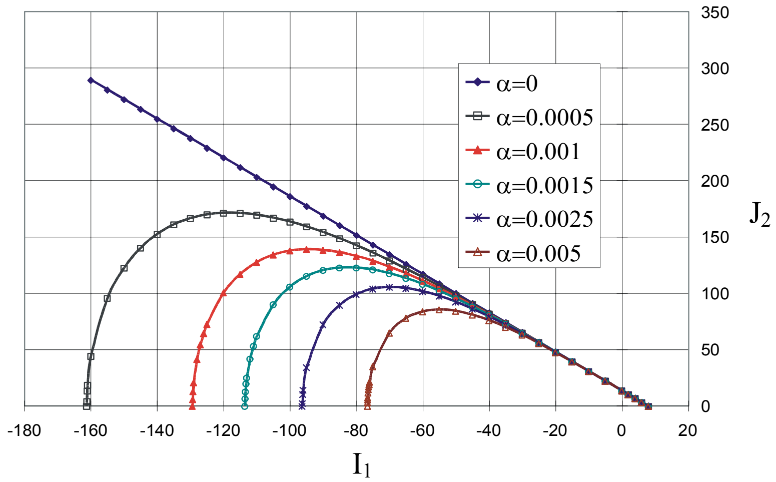

Fig. 1 The influence of α on yield surface F

The values of α control the size of the flow surface. It is typically defined as a function of deformation history. As α decreases, the size of the flow surface increases, Fig.1. When $\alpha = 0$, the ultimate stress response surface of the material is attained. β is related to the trace of the flow surface on the octahedral plane. Based on laboratory observations for various stress paths, the hardening response of sand is influenced both by the coupled and uncoupled actions from volumetric and deviatoric plastic deformations. In order to take these observations into account, in the framework of this investigation, parameter α is expressed as a function of both volumetric and deviatoric hardening components α_V and α_D:

$$\alpha = \eta_h \cdot \alpha_V + \left(1 - \eta_h\right) \cdot \alpha_D \tag{3}$$

with

$$\alpha_V = a_1 \cdot e^{b_1 \cdot \xi_v} \tag{4}$$

$$\alpha_D = a_1 \cdot \left[1 - \left(\frac{\xi_d}{c_1 + \xi_d}\right)^2\right] \tag{5}$$

$$\eta_h = \frac{\xi_v}{\xi_v + \xi_d} \tag{6}$$

ξ_V and ξ_d are the volumetric and deviatoric components of the effective plastic strain which are defined on the basis of plastic strain increments $d\varepsilon_{ij}^p$. η_h denotes the contribution of volumetric hardening to the overall material hardening response.

Coefficients a_1, b_1, c_1 in Eq.(4) and (5) are the model hardening parameters which can be determined from experiments. The expressions of the individual hardening parameters are:

$$a_1 = \gamma \cdot \left(\frac{3p_c}{p_a}\right)^{(2-n)} \tag{7}$$

$$b_1 = e^{\frac{\sqrt{3}\cdot(1+e_0)(2-n)}{\lambda-\kappa}} \tag{8}$$

$$c_1 = \sqrt{\frac{2}{3}} \cdot h_2 \cdot \left(\frac{p_c}{p_a}\right)^{h_1} \tag{9}$$

where p_c is the soil pre-consolidation pressure, e_0 is the void ratio at p_c, h_1 and h_2 are material constants which can be obtained by comparing the results of the tests with different confining pressures, λ is the slope of the isotropic normal compression line (iso-ncl) and κ is the slope of the unloading-reloading line (url) in a triaxial test.

An isotropic measure of response flow surface degradation has been introduced into the model to simulate the softening process. This adaptation of the model is achieved by means of specifying the variation of parameter α, as function of equivalent post fracture plastic strain ξ_{pf}:

$$\alpha = \alpha_R + \eta_s \cdot (\alpha_u - \alpha_R) \tag{10}$$

in which $\eta_s = e^{-\kappa_1 \cdot \xi_{pf}}$, α_u and α_R are the values of α corresponding to material ultimate stress response and residual stress state respectively. κ_1 is a material parameter that determines the material degradation rate.

For some frictional and cohesionless soils, material models incorporating the associated flow rule usually exhibit plastic dilation that is larger than the one that is observed in laboratory testing. In this case, it is necessary to employ a nonassociative flow rule for plasticity modelling.

In the hierarchical approach, a potential function Q is obtained by applying a correction/modification to the yield function as:

$$Q = F + h(I_1, J_i, \xi) \tag{11}$$

The size of Q is controlled by a hardening/softening parameter α_Q defined as:

$$\alpha_Q = \alpha + \alpha_c \tag{12}$$

in which α_c is a correction function expressed as:

$$\alpha_c = \kappa_c \left(\alpha_0 - \alpha\right)\left(1 - \chi_v\right) \tag{13}$$

The parameter α_0 in Eq. (13) is the value of α at the initiation of nonassociativeness. The parameter χ_v controls the contribution of volumetric plastic deformation to the expansion of the potential surface and is defined by:

$$\chi_v = \frac{\xi_v}{\xi} \tag{14}$$

The κ_c in Eq. (13) is the only extra material parameter that needs to be determined to capture material nonassociative behaviour. Experimentally κ_c can be obtained on the basis of the material response at ultimate state and determined by:

$$\kappa_c = \frac{\alpha_Q}{\alpha_0 \cdot \left(1 - \chi_v\right)} \tag{15}$$

The general procedures for the determination of the material parameters of the constitutive model can be found in Liu [2].

Model Verification and Application

In order to verify the proposed constitutive model, Eastern Scheldt sand was chosen as the test material. The tests were carried out by using triaxial test apparatus. Three different stress paths, i.e. CTC, TC and RTC under two initial consolidation pressures (150kPa and 400kPa) were investigated experimentally.

Fig. 2 and Fig.3 present the comparisons of the numerical predictions with the experimental results in terms of stress-strain curves and volumetric response. It can be observed that the model based on a nonassociative flow rule is more appropriate for description of the actual material behaviour and the numerical prediction shows good agreement with the experimental results.

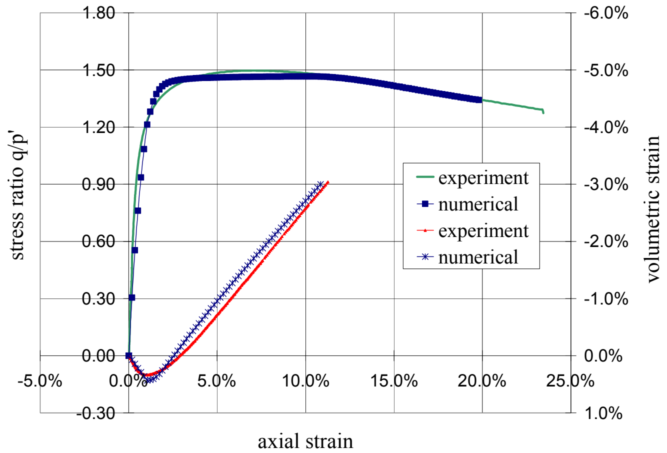

Fig. 2 Comparisons of stress-strain and volumetric response of CTC test with nonassociative flow rules (p_c = 400kPa)

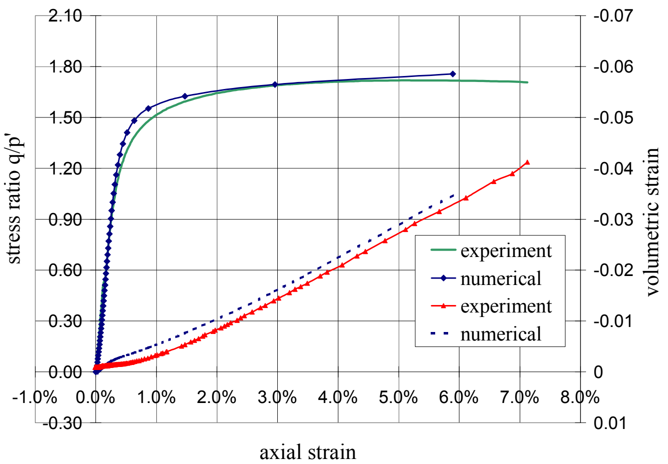

Fig. 3 Comparisons of stress-strain and volumetric response of RTC test with nonassociative flow rule (p_c = 400kPa)

The plot of the volumetric response calculated with different flow rules, Fig. 4, shows that the model based on the associative plasticity does not show good comparison with the actual observed behavior. By utilizing an associative flow rule, the model produces an overestimation of the plastic volume strain, especially at stress levels close to the ultimate stress state where excessive dilation can be observed. By correcting the yield function to account for nonassociative plasticity, the model shows good comparison with the observed behavior in terms of both, stress-strain and volumetric response

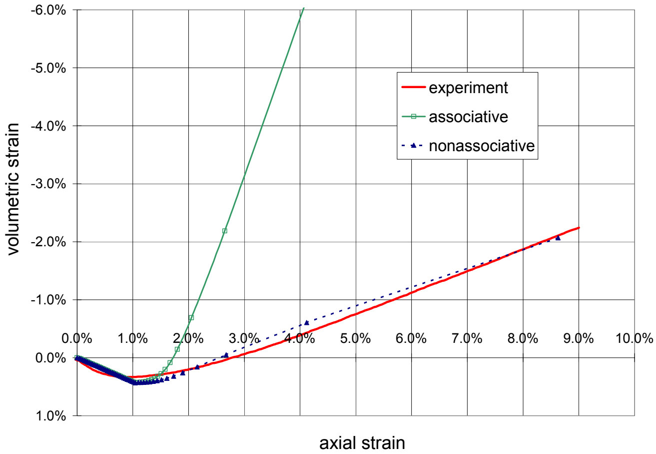

Fig. 4 Comparisons of volumetric response of CTC test with different flow rules ($p_c = 400$kPa)

Conclusions

The numerical results indicate that the constitutive model is capable of describing the response of sand in both the hardening and softening regimes. The nonassociative flow rule is more appropriate for description of the volumetric response of the actual material.

References

1. Desai, C.S. (1980): “A general basis for yield, failure and potential functions in plasticity”, *Int. J. Num. Analyt. Meth. in Geomech.*, Vol. 4, pp.361-375.

2. Liu. X. (2003): *Numerical Modelling of porous Media Response under Static and dynamic Load Conditions*. Ph.D. Thesis, Faculty of Civil Engineering, Delft University of Technology, Netherlands.