Identification of Equivalent Material Constants of a Printed Circuit Board

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Summary

The purpose of this work is to introduce a procedure, exploiting the concept of model updating, to obtain the effective material constants of a printed circuit board using experimental modal testing data. The proposed method employs a sequential quadratic programming algorithm in the optimization phase, and uses natural frequency sensitivities to direct the search path. The finite element analyses of the PCB were performed using a commercial general-purpose code, MSC/NASTRAN.

Introduction

The reliability of an electronic system is crucially related to the dynamic response of the electronic packaging of the system. In order to achieve a good estimate of the dynamic response of a printed circuit board (PCB) by the finite element method (FEM), it is necessary to know the material constants of the PCB as accurately as possible. A common PCB, which is primarily made of fiberglass known as FR-4 and thin copper layers, is decorated with holes for interconnection between layers and with small bits and pieces of other metals. A detailed finite element (FE) modeling of such a complex structure constitutes a very difficult task, and the computational time required to solve this FE model can also be formidable. However, these difficulties can be resolved, if effective (or equivalent) material constants of the PCB are obtained and thus the FE model is simplified while the simplified model can still accurately predict the dynamic response of the structure.

The smearing techniques [1,2] "smear" the material and geometric properties of a PCB and the components on it to determine the effective homogenized properties in an effort to reduce the complexity of the model. Ong and Lim [3] employed the finite element model updating procedure [4] to tune the support conditions of a bare PCB while treating the PCB as made of an isotropic material. Wang and Lai [5] also utilized a similar procedure to establish the equivalent material constants of perforated plates. Combining both FE analysis results and experimental data and tuning certain parameters of the FE model by optimization techniques, finite element model updating has become a feasible approach to

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improve the predictions for the dynamic response of a structure. Model updating can also be a useful tool for design optimization, structural modification, and non-destructive diagnosis [6].

The main goal of this work is to introduce a procedure, exploiting the concept of model updating, to obtain the effective material constants of a PCB using experimental modal testing data. The proposed method employs a sequential quadratic programming (SQP) algorithm in the optimization phase, and uses natural frequency sensitivities to direct the search path. The effective material constants are obtained by minimizing the squared norm of an error vector defined as the normalized difference between the FE analysis and experimental modal data. The FE analyses of the PCB were performed using a commercial general-purpose FE code, MSC/NASTRAN.

General Formulation of the Method

In structural optimization terminology, the weight, static deflections, stresses, natural frequencies, mode shapes, or time responses, etc., of a structure may be defined as structural responses. An identification technique for equivalent material constants of a structure, utilizing sensitivity information, can be developed based on the first order approximation of the structural responses, which can be expressed as

$$\mathbf{f} = \mathbf{f}^0 + \mathbf{S}\,\Delta\mathbf{p} \tag{1}$$

where $\mathbf{f}$ is an m by 1 vector containing structural responses, $\mathbf{f^0}$ is the vector of structural responses evaluated using the current design $\mathbf{p^0}$, $\mathbf{S}$ is the sensitivity matrix defined as the partial derivatives of the responses with respect to the design parameters $\mathbf{p}$, which is an n by 1 vector, evaluated at $\mathbf{p^0}$,$\mathbf{\Delta p}$ is the difference vector of $\mathbf{p}$ and $\mathbf{p^0}$, and m and n are the numbers of structural responses and design parameters, respectively. The design parameters can be a selected set of finite element input parameters such as the Young's modulus, Poisson's ratio, and density, etc. Subtracting both sides of Eq. (1) by observed (or experimental) responses and non-dimensionalizing the result to yield

$$\mathbf{e} = \mathbf{e}^0 + \overline{\mathbf{S}}\,\Delta\overline{\mathbf{p}} \tag{2}$$

where $\mathbf{e}$ is an error vector which defines the distance between $\mathbf{f}$ and its experimental counterpart $\mathbf{f^e}$ and $\overline{\mathbf{S}}$ is the m by n modified sensitivity matrix. The squared norm of the error vector can be written as

$$\|\mathbf{e}\|^2 = (\Delta\overline{\mathbf{p}})^T\,\overline{\mathbf{S}}^T\,\overline{\mathbf{S}}\,\Delta\overline{\mathbf{p}} + 2\left(\mathbf{e}^0\right)^T\overline{\mathbf{S}}\,\Delta\overline{\mathbf{p}} + \left(\mathbf{e}^0\right)^T\mathbf{e}^0 \tag{3}$$

Since constant terms in an object function will not affect the outcome, the optimization problem seeking to minimize the squared norm of the difference between two paired vectors, one analytical and the other experimental, can be defined as follows.

$$\min \frac{1}{2}(\Delta\overline{\mathbf{p}})^T \mathbf{H}\,\Delta\overline{\mathbf{p}} + \mathbf{c}^T \Delta\overline{\mathbf{p}} \tag{4}$$

subject to $\mathbf{p}^l \leq \mathbf{p} \leq \mathbf{p}^u$

where $\mathbf{H} = \overline{\mathbf{S}}^T\,\overline{\mathbf{S}}$ and $\mathbf{c} = \overline{\mathbf{S}}\,\mathbf{e}^0$. The upper and lower bounds on the design parameters are set to ensure that the updated results are physically meaningful. Eq. (4) is a standard quadratic programming (QP) problem, which can certainly be solved by any QP routine. However, since the entire formulation is based on the first order approximation, Eq. (1), iteration is usually required, especially for highly non-linear functions like natural frequencies as the structural responses.

Now the identification procedure for the effective material constants of a PCB can be stated as follows:

1. Perform a pretest FE analysis on the structure with estimated material constants to give a general understanding about the mode shapes of the structure.
2. Decide appropriate accelerometer locations based on the information obtained in the first step, and conduct a modal testing on the structure to acquire natural frequencies and mode shapes.
3. Perform FE analyses and frequency sensitivity calculations evaluated at the current estimate of the material constants.
4. Match each pair of analytical and experimental frequencies by using modal assurance criterion (MAC) [4].
5. Use a QP routine to solve Eq. (4) for a set of updated material constants.
6. Check for convergence. If yes, output final updated results and stop. Otherwise, update material constants and go to step 3.

Finite Element Analysis and Modal Testing of the PCB

The PCB, a Micro-Star 845E Max mainboard, weighs 209.7 grams and has a dimension of 305 mm by 200 mm and a thickness between 1.5 mm and 1.55 mm. The variation in thickness is due to the fact that the surface of the composite plate is uneven. MSC/NASTRAN was employed to build up the FE model and perform the FE analyses and sensitivity calculations. The analytical model was built using shell elements and orthogonal materials (MAT8) under a free-free boundary condition. The material properties for the pretest, initial model were set as those of FR-4, which were the Young's moduli E_1=E_2=E=17 GPa, Poisson's ratioν_{12}=ν =0.12, shear moduli G_{12}=G=E/2(1+ν)=7.589 GPa, and densityρ=1870 kg/m^3 [3].

To execute the modal testing experiment, the PCB was suspended by two elastic bands, and then the plate was excited by a miniature impact hammer

(ENDEVCO Model 2301) and the vibration signal was recorded by a miniature accelerometer (ENDEVCO 2250A-10, weighed 0.4 grams). There were a total of 70 equally divided measurement points (10 by 7) on the PCB. Using the ultra lightweight hammer and transducer minimized the adversary effects of mass loading on the test structure. Both input and output signals were analyzed using B&K 3560C Pulse analyzer to create frequency response functions, which were subsequently evaluated by the Spectral Dynamics STAR 6.1 modal software to obtain the natural frequencies, damping, and mode shapes. Table 1 shows the resulted natural frequencies and damping. Table 2 compares the results from the experiment and from the analysis of the initial FE model. Since the damping values are so small, the experimental frequencies are compared directly to the undamped frequencies from the FE analysis without any conversion.

Table 1. The natural frequencies and damping of the PCB by modal testing

Mode no.	1	2	3	4	5	6	7	8	9	10
Frequency (Hz)	40.37	58.63	102.90	135.85	143.20	183.59	208.55	234.40	329.28	355.91
Damping (%)	0.9257	0.6263	0.5431	0.3683	0.3889	0.3839	0.4041	0.5952	0.3582	0.6098

Table 2. Comparison of the experimental and initial FE model modal properties

Experimental mode no.	Experimental frequency (Hz)	Finite element mode no.	Finite element frequency (Hz)	% Difference	MAC value
1	40.37	* 2	50.98	26.274	0.9829
2	58.63	* 1	50.02	-14.689	0.8612
3	102.90	3	114.49	11.269	0.9883
4	135.85	4	116.96	-13.906	0.9584
5	143.20	5	137.68	-3.853	0.6082
6	183.59	6	156.14	-14.952	0.6525
7	208.55	7	206.60	-0.934	0.9789
8	234.40	8	241.83	3.167	0.9638
9	329.28	9	272.21	-17.331	0.9499
10	355.91	10	321.85	-9.570	0.5438

* Matched FE modes are not in their original order.

Updating Results and Discussion

A main portion of the updating procedure was programmed in a Fortran code, including mode matching and optimization. Three cases were studied to obtain the effective material constants for the PCB using the model updating technique. The first, second, and third cases use the first one, three, and five experimental frequencies, respectively, to update the FE model. The modal properties of the updated FE model for each case are then compared with the experimental results. A comparison of modes not included in the updating process reflects the quality of the estimated effective material constants of the updated model. Table 3 through 5 shows such a comparison. The termination criterion of the updating process for all three cases is that the frequency differences between the experimental and analytical results are within 1% for those modes included during updating. Table 3, 4, and 5 clearly state that as more modes are included, the updated FE model can produce more desirable results. The updated effective material constants for the three cases are shown in Table 6. Figure 1 depicts the iteration history of the 10 frequency differences for the third case.

Table 3. Comparison of the experimental modal properties and the FE analysis results updated by using only the first one experimental mode

Experimental mode no.	Experimental frequency (Hz)	Updated FE mode no.	Updated FE frequency (Hz)	% Difference	MAC value
1	40.37	1	40.34	-0.093	0.9830
2	58.63	2	42.84	-26.920	0.8603
3	102.90	* 4	91.96	-10.627	0.9884
4	135.85	* 3	90.48	-33.395	0.9575
5	143.20	5	117.03	-18.277	0.8900
6	183.59	6	122.71	-33.159	0.8684
7	208.55	7	169.29	-18.824	0.9788
8	234.40	8	190.91	-18.553	0.9540
9	329.28	9	232.65	-29.348	0.9395
10	355.91	*12	281.70	-20.849	0.4690

* Matched FE modes are not in their original order.

Table 4. Comparison of the experimental modal properties and the FE analysis results updated by using only the first three experimental modes

Experimental mode no.	Experimental frequency (Hz)	Updated FE mode no.	Updated FE frequency (Hz)	% Difference	MAC value
1	40.37	1	40.63	0.626	0.9812
2	58.63	2	58.82	0.325	0.8726
3	102.90	3	101.88	-0.991	0.9870
4	135.85	4	135.34	-0.375	0.9684
5	143.20	5	151.35	5.690	0.9629
6	183.59	6	170.90	-6.910	0.9529
7	208.55	7	205.15	-1.628	0.9828
8	234.40	8	224.20	-4.354	0.9602
9	329.28	9	324.09	-1.576	0.9544
10	355.91	*11	354.91	-0.280	0.8919

* Matched FE modes are not in their original order.

Table 5. Comparison of the experimental modal properties and the FE analysis results updated by using only the first five experimental modes

Experimental mode no.	Experimental frequency (Hz)	Updated FE mode no.	Updated FE frequency (Hz)	% Difference	MAC value
1	40.37	1	40.46	0.207	0.9809
2	58.63	2	58.59	-0.059	0.8844
3	102.90	4	102.39	-0.493	0.9871
4	135.85	3	135.62	-0.168	0.9731
5	143.20	5	143.94	0.516	0.9593
6	183.59	6	182.58	-0.549	0.9501
7	208.55	7	207.36	-0.569	0.9839
8	234.40	8	226.76	-3.259	0.9719
9	329.28	9	338.85	1.081	0.9533
10	355.91	*11	348.95	-1.956	0.9185

* Matched FE modes are not in their original order.

Table 6. Updated effective material constants for the three cases using one, three, and five experimental modes, respectively, during updating

No. of modes used	E_1 (GPa)	E_2 (GPa)	ν_{12}	G_{12} (GPa)	ρ(kg/m^3)
1	15.147	12.393	0.107	5.734	2268.90
3	28.511	26.834	0.232	5.558	2268.90
5	28.214	24.119	0.456	5.486	2268.90

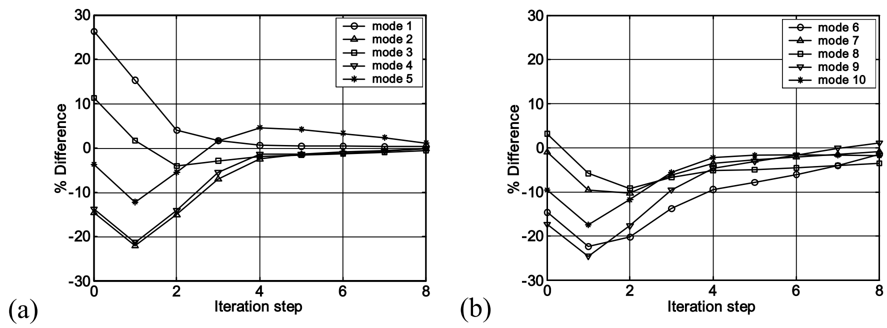

Figure 1. Iteration history of the frequency differences for the third case: (a) first five matched modes and (b) last five matched modes.

Acknowledgements

This research work was supported by the National Science Council, Taiwan, R.O.C. under Grant No. NSC 91-2212-E-236-001. Their support is gratefully appreciated.

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