Comparison of Computational Models for Damage-Induced Stress Softening

Z. Guo[1] and L.J. Sluys[1]

Summary

This paper is focused on the modeling of damage-induced *stress softening* in rubber-like materials. Miehe's discontinuous damage model and Ogden and Roxburgh's pseudo-elastic model are presented. Both models are implemented together with the Gao's material model in the finite element code DIANA. Homogeneous and inhomogenous analyses show that these two approaches are effective in three-dimensional analyses and present different characteristics and limitations due to their formulations.

Introduction

It has been known for a long time that the initial material properties of un-stretched (virgin) samples of rubber compounds are changed after the sample has been subjected to loading. This was observed by Mullins [1] and has subsequently become widely known as the *Mullins effect*. From purely phenomenological standpoint, Mullins effect is readily described in the context of an uniaxial deformation. Figure 1 shows the main features of the Mullins effect in simple tension in schematic form with the stress s plotted against stretch λ. The virgin undamaged material is first stretched as the extension ratio reaches

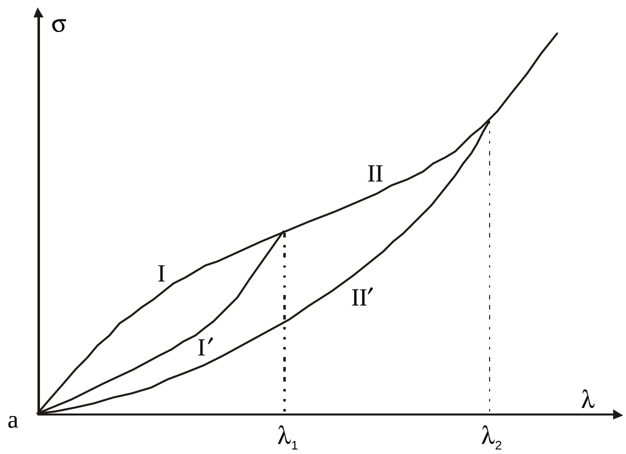


Figure 1. Schematic loading-unloading curves in simple tension (Mullins effect)

[1] Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2600GA Delft, the Netherlands

λ_1 and the stress follows path I. Then unloading taking place from λ_1 to 0 follows path I′. The second loading from 0 to λ_2 (>λ_1) first follows path I′ until $\lambda=\lambda_1$ then it follows path II. The second unloading starting from stretch ratio λ_2 to 0 follows path II′, which is different from path I′. At a given stretch, the stress on path II′ is less than the stress on path I′. This phenomenon is also called stress softening.

Different approaches [1-8] have been proposed to simulate stress softening. In this paper, both Miehe's discontinuous damage model [5] and Ogden and Roxburgh's pseudo-elastic model [7] have been presented. These two models have been implemented together with Gao's model in the finite element code DIANA [9, 10].

Phenomenological Models to Represent the Mullins Effect

Continuum damage mechanics has often been used to model the stress softening phenomenon. Miehe [5] considers that the cyclic loading processes consist of two phenomenological effects. Firstly, a Mullins-type discontinuous damage evolution is assumed where the damage accumulation occurs only within the first cycle of a strain-controlled loading process. Further, strain cycles below a maximum effective strain energy do not contribute to this type of damage. Secondly, a continuous damage accumulation within the whole strain history of the deformation process, which is also governed by the local effective strain energy, was taken into account. The large-strain material behaviour is commonly described by an elastic strain energy density function W, which can be expressed following continuum damage mechanics.

$$W = (1-d)W_0\left(\mathbf{F}\right) \tag{1}$$

where $\mathbf{F}$ is the deformation gradient tensor relative to the undeformed configuration of the material, W_0 is an effective strain energy function, $d \in [0,1]$ is a scalar damage variable which describes an isotropic damage effect characterized by an elastic softening of the material. For Miehe's discontinuous damage evolution, which may be used to simulate stress softening, the scalar damage variable d can be written as:

$$d\left(a\right) = d_\infty\left[1-\exp\left(-\frac{a}{h}\right)\right] \tag{2}$$

where d_∞ and h are material parameters, α is the maximum thermodynamic force or effective strain energy.

Ogden & Roxburgh [7] have proposed a theory of pseudo-elasticity to describe the damage-induced stress softening effect in rubber-like solids. The essence of the theory is that material behaviour in loading path is described by a common elastic strain energy density function and in unloading path by a different strain energy density function. The switch between strain energy functions is affected by incorporation of a damage variable

h into the strain energy function, which is then referred to as a pseudo-elastic energy function, written as $W(\mathbf{F}, h)$ Thus,

$$W(\mathbf{F},h) = hW_0(\mathbf{F}) + f(h) \tag{3}$$

in which $\phi(h)$ refers to a damage function, which determines the damage parameter h. Ogden & Roxburgh choose f to be such that

$$-f'(h) = m \times erf^{-1}\left(r\left(h-1\right)\right) + W_m \tag{4}$$

where m and r are positive parameters and erf $^{-1}$ ( ) is the inverse of the error function.

Finite Element Implementation Formulae

For the purpose of finite element implementation, afore-introduced elastic damage formulation is necessary for a combination with a hyper-elastic material model. Gao proposed an elastic law that separately considers the resistance of materials to tension and compression in 1997 [9, 10]. Hence,

$$W = a(I_1^n + I_{-1}^n) \tag{5}$$

a and n are material parameters. I $_{-1}$ is a strain invariant defined by

$$I_{-1} = \frac{I_2^*}{I_3^*} = \frac{3(I_1^2 - I_2)}{I_1^3 - 3I_1I_2 + 2I_3} \tag{6}$$

I_1, I_2 and I_3 are strain invariants. Constitutive relations for rubber-like materials are initially described via the invariants of the Right Cauchy-Green stretch tensor C as the basic set of parameters. Gao's model can cover a large range of deformations by properly selecting the parameters a and n.

It is necessary to determine the stress-strain relation and the incremental stress-strain relation for finite element implementation. For Miehe's model, the second Piola-Kirchhoff stress σ can be obtained via

$$s = (1-d)\, s_0 \tag{7}$$

in which σ_0 is the effective second Piola-Kirchhoff stress. The incremental stress-strain relation

$$\frac{\Delta s}{\Delta e} = D = \begin{cases} (1-d)\, D_0 - \dfrac{\partial d(a)}{\partial a}(s_0 \times s_0) & \text{loading} \\ (1-d)\, D_0 & \text{unloading or reloading} \end{cases} \tag{8}$$

For Ogden & Roxburgh's model, the second Piola-Kirchhoff stress σ

$$s = \begin{cases} s_0 & \text{loading} \\ h\, s_0 & \text{unloading or reloading} \end{cases} \tag{9}$$

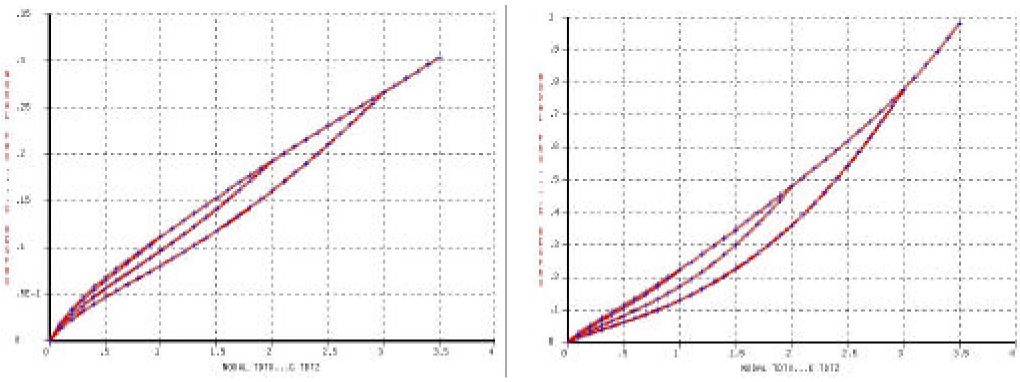
The incremental stress-strain relation is

$$D = \begin{cases} D_0 & \text{loading} \\ h\, D_0 + 4\dfrac{\partial W_0}{\partial C} \bullet \dfrac{\partial h}{\partial C} & \text{unloading or reloading} \end{cases} \tag{10}$$

These two energy-based damage models are distinct from the strain-based models since the extent of damage sustained by the material is controlled by the maximum energy state. Therefore, this model is readily applicable to three-dimensional states of deformation, especially, for computational purposes.

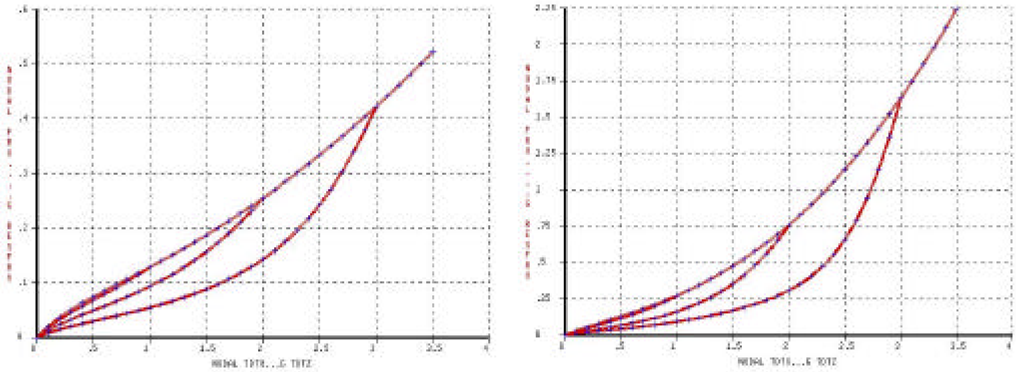
Numerical Results

In this section we first consider a cubic material element with edges of unit length under loading conditions of uniaxial tension and pure shear as homogenous examples. In the numerical calculations, we use the HX25L element, which is an eight-node iso-parametric solid brick element with a pressure degree-of-freedom by setting the material constants a=0.03 and n=1.5 (see Equation 5); for Miehe's damage model, d_∞=0.5 and h=2.0; for Ogden pseudo-elastic model, r=1.5 and m=2.0. From the numerical results of Figures 2 and 3, it clearly shows that Miehe's model and Ogden's model together with Gao's material are working well for three-dimensional homogenous analyses.



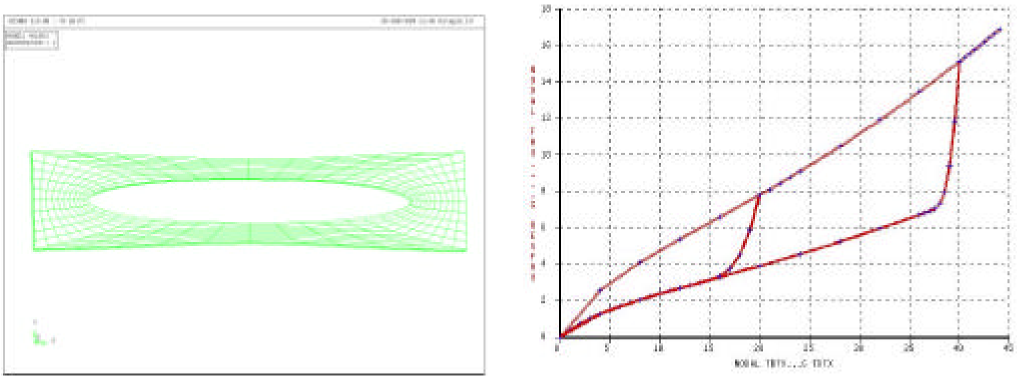
(a) (b)

Figure 2. Numerical curves of loading force against displacement at for the Miehe's model (a) uniaxial tension (b) pure shear



(a) (b)

Figure 3. Numerical curves of loading force against displacement for the Ogden & Roxburgh's model: (a) uniaxial tension (b) pure shear



(a) (b)

Figure 4. Strip with a hole: (a) 200% deformed mesh (b) load-displacement curve.

Furthermore, each model expresses different characteristics. In the Miehe's model the curves of loading force against displacement in first primary loading is lower than in Ogden's model. This is because Miehe's discontinuous damage model considers that damage starts from loading (equations 2, 7) and Ogden's pseudo-elastic model considers that damage starts from unloading (equations 4, 9). So, the loading curve will perfectly follow the elastic path in Ogden's model.

Secondly, we consider a strip with a circular hole as an inhomogenous example. Figures 4 show the results with Ogden's model. Figures 4a gives the deformed mesh at 200% deformation. Figure 4b illustrates the numerical curve of the loading force at one corner node against displacement. From the curve it shows that damage influences the rubber specimen properties and the simulation of whole loading-unloading-reloading process works well. But, we must pay attention to loading step size, otherwise divergence will occur.

Conclusion

Both Miehe's and Ogden & Roxburgh's models are capable to simulate the Mullins effect for rubber-like materials with their own characteristics. Since the extent of damage sustained by the material is controlled by the maximum energy state, these models are readily applicable to three-dimensional computational analyses.

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