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Comparison of impact between experimental and simulation results in flexible elements

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Summary

Contacts from impacts are problems of great interest in different engineering areas, such as in the car industry, as well as in the nuclear industry; that between steel cantilever beams and steel balls and from both a theoretical and an experimental aeronautical industry. In this paper, we discuss the problems of impact perspective, using Hertz's contact theory with a non-linear beam model. We compare the results with Goldsmith's analytical theory also, in order to obtain a fuller understanding of the theory-numerical results. Experimental results are obtained from a ball striking against a cantilever beam.

Introduction

Contacts by impact are extremely important in many areas of engineering, such as: the car industry, the aeronautics industry, and the mining industry. One of the objectives of this work is to establish what happens at the moment of impact. It will be used to accomplish the integration of time to produce transients from impact forces. In this way, we can establish the forces, deformations, stresses and time of contact between bodies at the moment of impact. The impact between rigid elements was studied in [1]. For our impact study in flexible structures [2], we begin with Hertz' contact theory, and the focus will be from the point of view of Goldsmith's *local analysis* [3]. Combining these with the beam movement equation, we can establish a way of calculating the magnitude, and duration of the contact forces, for either contact or impact. In order to establish a more general method of solving impact problems, the non-linear beams model can be used, which is based on the finite element method suggested by Barrientos [4] using Matlab, in which, impact problems are generalized for different cases. Using this program, we can discover all the variables of interest for a mechanistic study. To verify the theoretical results, comparative analysis should be carried out between both methods, and a critical analysis of the results should be made through different measurements.

Analytic Method.

We will describe the equation for the movement of bodies, for which, Goldsmith [2] uses the equation of movement for forced vibration from the

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Euler-Bernoulli beam model with an impact at the point *x=L*. The movement equation of the cantilever beam striking against a rigid ball, is given by:

$$\omega_1(x,n\Delta\tau)=\frac{1}{\rho A}\sum_{j=1}^{n-1}F_j\sum_{i=1}^{\infty}\frac{\chi_i(x)\chi_i(L)}{\omega_i\int_0^L\chi_i^2dx}\frac{\cos\omega_i(n-j)\Delta\tau-\cos\omega_i(n-j+1)\Delta\tau}{\omega_i} \tag{1}$$

$$\omega_2(n\Delta\tau)=\omega_2((n-1)\Delta\tau)+v_2(n-1)\Delta\tau+\frac{F_{n-1}\Delta\tau^2}{2m_2} \tag{2}$$

From (1) (displacement of ball) and (2) (displacement of beam), we can calculate numerically the penetration α between the bodies and the impact force F, in the case where plasticity is not reached.

$$\alpha(\Delta\tau)=\omega_1(c,n\Delta\tau)+\omega_2(n\Delta\tau)=\left[\frac{F}{k_2}\right]^{\frac{2}{3}} \tag{3}$$

k_2 is a parameter that depends on the characteristics of the material and is used in the Hertz contact law. Integration of Equation (3) gives a penetration theory between the bodies and the impact force in a direct way.

Hertz's contact law.

Hertz's contact law permits us to determine the contact force between two bodies by means of the distribution of stress through the contact surfaces. This distribution will depend on the geometry of the contact surfaces. There is a known case of a contact between a face and a surface, (the beam and striking ball), and we can follow the stress distribution along the contact radius for an elastic contact, as shown by Goldsmith (Figure 1):.

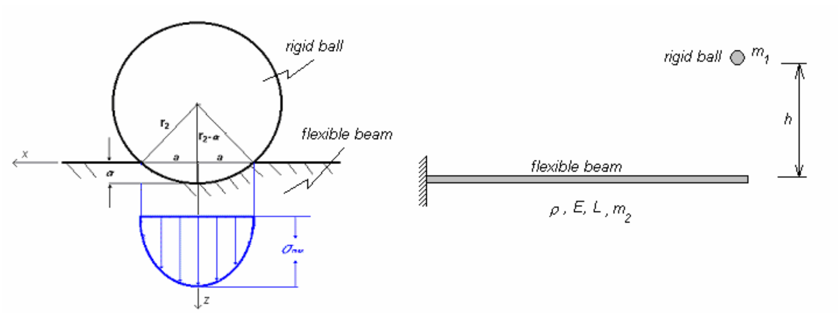

figure 1. (a) Stress distribution in the impact zone, (b) real problem.

In Hertz's contact law [2] the relationship between a force, F, and the penetration α, among the bodies is given by:

$$F = k_2 \alpha^{3/2} \tag{4}$$

Bodies make contact when their relative distance becomes zero, and as a consequence, a contact force appears, which accelerates the bodies in a direction opposite to the penetration. This contact force varies during the variation of penetration. We must use Equations (3) and (4) to obtain the contact force produced by an impact in the case of local analysis. Equation (1), (2) y (4) can be solved numerically. The force is calculated numerically, and is composed of the movement equation of the striking mass, (ω_1), and relating the penetration with the force, through Hertz's contact law (4).

Numerical method

The model that the finite element used was that in which the shape of the beam can be subjected to great deformation with non -lineal conduct. This beam model considers shearing, extension, flexion, and torsion of the beam. This product has the capability of studying the cinematic of the dynamic of beams with linear and non-linear conduct, applied to systems of flexible multibodies. The beam element is discretized using the finite elements method, and temporal integration of the movement equation is made using an HHT algorithm [4]. The movement equation, written as a function of the finite elements, can be presented as

$$\left[\frac{1}{h^2\beta}M + (1+\alpha)\gamma h\beta D + (1+\alpha)K_T(\Phi_{n+1}^{(i)})\right]\Delta\Phi_{n+1}^{(i+1)} = (1+\alpha)F_{n+1} - M\ddot{\Phi}_{n+1}^{(i)} - (1+\alpha)D\dot{\Phi}_{n+1}^{(i)} - (1+\alpha)P(\Phi_{n+1}^{(i)}) + \alpha F_n + \alpha D\Phi_n + \alpha P(\Phi_n) \tag{9}$$

where M is the mass matrix, D is the damping matrix, P is the vector of internal forces, F_n represents the vector of external charges (impact force in this case obtained from Hertz's contact law), K_T is the tangent matrix that includes the material and geometric matrix. The sub-index represents the time and the super index represent the iteration number. The constants α and β and γ are parameters of integration of HHT algorithm, and h is a step in time.

Experimental results.

Previous results have an acceptable logic, from a mechanical point of view, but to prove the exactness of these results, we have performed experiments and obtained the necessary results, through measurements and theory. The experiment was performed in the vibration laboratory located at the University of Concepción, and is shown in Figure (2).

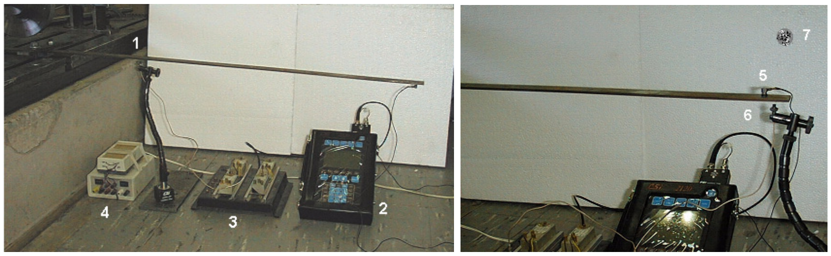

Figure 2. Experimental set up: 1 cantilever beam (had length L=1.15 m, width A=0.02 5m, and thickness E_s=*0.005 m*. The mechanical characteristics are, *E=2.1*105 MPa*, density ρ=*7850 kg/m*^{3}, and σ_0=*340 MPa)*., 2 analyzer, 3 amplifier, 4 source power, 5 accelerometer, 6 displacement sensor, 7 rigid ball.

To compare the results, we measured the maximum displacement of the beam at the impact point, as well as the print radius left on surface. As we do not have a system that could directly determine the speed of the striking mass, we used gravitational force g as the speed generator, with the mass falling from different heights, *h*. The impact speed was obtained from the following equation: $v=\sqrt{(2gh)}$.

Results. Comparison of different methods.

To verify the results between analytical methods (Goldsmith) and the finite element method, we analysed the solutions that were obtained for each method. A cantilever beam was used with the characteristics doing in figure 3. The striking ball had a radius of r_2=*0.005 m*, and mass m_2=*0.009 kg*.

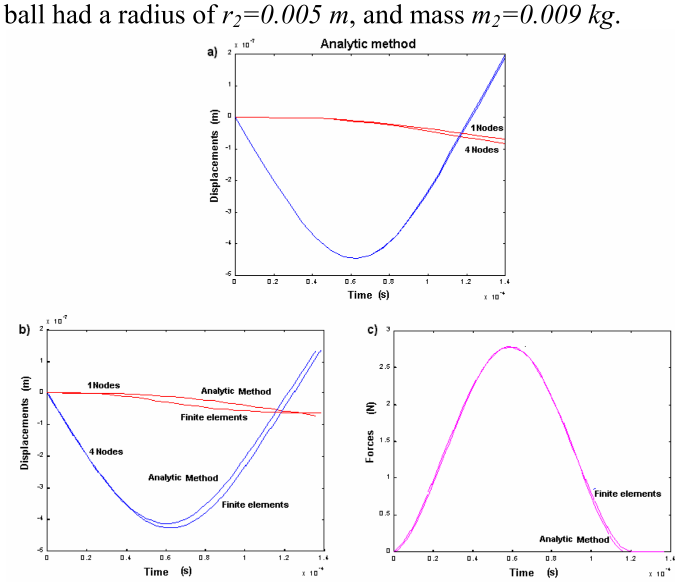

Figure 3, {a) Body displacements using different numbers of modes, b) body displacement using different methods, and c) shape of impact force.

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From Figure 3, we can appreciate how to obtain results from the various methods utilized. Figure 3a shows the solution, with the analytical method for different numbers of modes. Figures 3b y 3c compare the solution between analytical methods and numerical methods to establish that the results from both methods are similar, and the small differences (in displacement) arise from differences in the beam patterns. As both methods are related, adding the numbers of modes of vibration is considered to be an analytical method (4 modes). We need to consider the quality of the elements representing the beam (10) in the finite element method, even though this variation does not affect the calculation of the contact forces.(Figure 3c).

The effect of the different parameters that participate in the impact must be studied in order to be able to establish any variation in the impact phenomena. In particular, the *Coefficient of restitution* is defined as the number of striking masses between the initial and final speeds (Newton's law). The results are not shown in this work. The distribution of energy is also obtained with this program. The effect of time on the impact was analysed, and also the plastic deformation produced by various striking mass speeds (Figure 4).

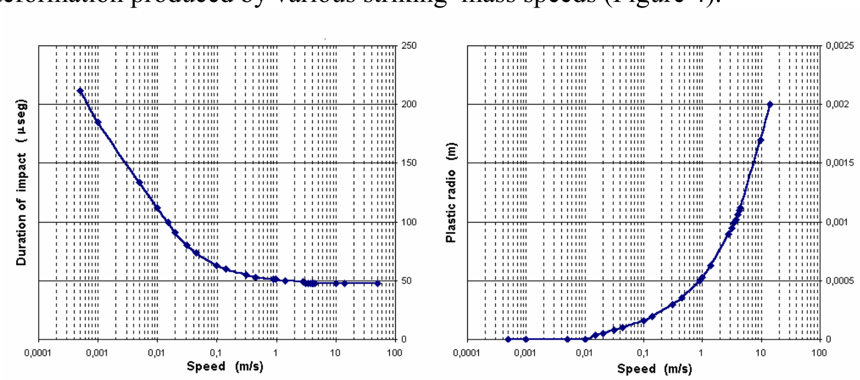

figure 4. {a) Duration of impact vs speed, and (b) dimension of plastic radius.

It must be noted that the time of impact diminishes with time, while the speed increases until a minimum is reached, when the time is no longer dependent on the impact speed. The plastic deformation is null at small speeds, and when the elastic limit is reached, the plastic radius increases with speed, reaching a maximum equal to the radius of the striking ball r_2. We analysed the variation of the maximum displacement of the beam produced by the impacts, in which a striking mass was allowed to fall from different heights. We then measured the maximum displacement of the beam (see Figures 5 and 6). To detect the increase in the force that acts over the beam, we calculated the static force equivalent, which is obtained by dividing the maximum displacement of the rigidity of beam (Figure 6). An increase in equivalent force was measured with an increase in the falling height, and therefore, the impact speed. The theory in this paper was developed according to these measurements. There was only an average difference of 4.3% when considering the effect of factors such as sound and the plastic component of the contact materials.

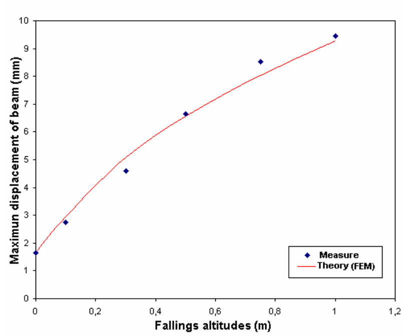

Figure 5. Maximum displacement of beam

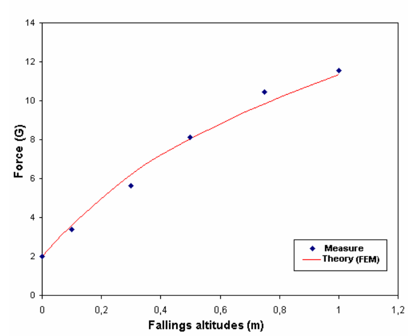

Figure 6. Equivalent force.

Conclusions.

A knowledge of impact phenomena is very important in different areas of engineering design, for which the known theories assist greatly in the comprehension and prediction of the consequences of these phenomena. The theories of Goldsmith are a good way to explain impact phenomena among flexible elements, but they are limited by the math management used in different cases. The finite element method shown in Simo's model using Hertz's contact theory, shows a promising approach to impact phenomena. It can be used to predict displacements and forces involved in the impact. It is necessary to improve these models of a perfect plastic substance, in order to predict with more accuracy, the effect on surfaces of contact. The effect of having plastic deformation in the impact zone helps to diminish the contact force at the time of impact. The duration of the time of impact decreases when plastic deformation occurs.

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