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Dynamic Crack Interaction in 2-D Piezoelectrics

F. García-Sánchez[1], A. Sáez[2], J. Domínguez[2]

Summary

In this paper the dynamic behavior of interacting cracks in piezoelectric materials under time-harmonic inplane electromechanical loading is investigated. By means of a hypersingular boundary element approach results are obtained for the scattering of normally incident longitudinal waves by two parallel cracks and, alternatively, two collinear cracks in a piezoelectric PZT-5H ceramic.

Introduction

Piezoelectric materials produce an electric field when deformed, and undergo deformation when subjected to an electric field. By virtue of this intrinsic coupling they have found extensive usage in smart devices as electromechanical actuators, sensors and transducers. However, when loaded in service these piezoelectric materials may fail prematurely due to their brittleness and presence of defects or flaws produced during their manufacturing process. Thus Fracture Mechanics plays an important role in the design of such piezoelectric devices. Analytical solutions for some simple crack geometries under dynamic loading have been presented by several authors (see, e.g., [1]). For more general conditions numerical methods need to be used. The Boundary Element Method (BEM) is particularly well suited to cases where stress concentrations, like the ones induced by the existence of cracks, are present. Recently, Zhang and coworkers have presented BEM formulations that makes use of a Laplace-domain fundamental solution to study problems in the time domain involving cracks in infinite solids subjected to both anti-plane [2] and in-plane [3] loading.

In this paper, a hypersingular frequency domain BEM for the analysis of cracked 2-D piezoelectric media is implemented and applied to the study of wave scattering by parallel and collinear cracks configurations. The time-harmonic fundamental solution derived by Denda et al. [4] for 2-D piezoelectrics is considered. The strongly singular and hypersingular kernels arising in the tractions boundary integral equation are analytically transformed into weakly singular and regular integrals prior to any numerical evaluation. This is achieved by the simple election of an integration variable consistent with the material characteristic parameters [5]. Discontinuous quarter-point elements are used next to the crack tip. Stress (SIF) and electric displacement intensity (EDIF) factors are computed directly from nodal values of the crack opening displacements (COD) and the

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[1]Department of Civil Engineering, University of Malaga, Plaza El Ejido s/n, 29013-Málaga, Spain.
[2]Department of Continuum Mechanics and Structural Analysis, University of Seville, Camino de los Descubrimientos s/n, 41092-Sevilla, Spain.

electric potential jump at the quarter-point element. The method is completely general and thus applicable to any crack configuration in both finite and infinite solids.

Formulation and Numerical Solution Procedure

Following Barnett and Lothe [6] the piezoelectric problem may be formulated in an elastic-like fashion by using a displacement vector extended with the electric potential and a stress tensor extended with the electric displacement components. Then, the mixed formulation of the BEM is expressed in terms of both the displacement and the traction boundary integral equations as

$$c_{IJ}u_J + \int_\Gamma p^*_{IJ}u_J d\Gamma = \int_\Gamma u^*_{IJ}p_J d\Gamma \quad ; \quad c_{IJ}p_J + N_r\int_\Gamma s^*_{rIJ}u_J d\Gamma = N_r\int_\Gamma d^*_{rIJ}p_J d\Gamma \tag{1}$$

where N_r is the outward unit normal at the collocation point, lowercase (elastic) and uppercase (extended) subscripts take values 1, 2 and 1,2,3 respectively. p^*_{IJ} and u^*_{IJ} are the fundamental solution extended tractions and displacements, associated to a line force (I=1,2) or to a line charge (I=3), and s^*_{rIJ} and d^*_{rIJ} are obtained by differentiation as

$$s^*_{rIJ} = C_{rIMn}p^*_{MJ,n} \quad ; \quad d^*_{rIJ} = C_{rIMn}u^*_{MJ,n} \tag{2}$$

The fundamental solution may be split into singular (static) plus regular (frequency dependent) terms [4]. For the singular part we consider the classical static solution [7]

$$u^{*S}_{IJ} = -\frac{1}{\pi}\mathrm{Re}\left[\sum_{R=1}^{3}A_{JR}H_{RI}\ln(z^x_R - z^\xi_R)\right] \tag{3}$$

where A_{JR} and H_{RI} are complex constants depending on the material properties, and the collocation point $\boldsymbol{\xi}$ and the observation point $\mathbf{x}$ are defined in the complex plane by

$$z^\xi_R = \xi_1 + \mu_R\xi_2 \quad ; \quad z^x_R = x_1 + \mu_R x_2 \quad ; \quad R = 1,2,3 \tag{4}$$

μ_R being the roots of the characteristic equation of the piezoelectric material.

We consider the regular part of the fundamental solution as given in [4]

$$u^{*R}_{JK}(\mathbf{x},\xi,\omega) = \frac{1}{8\pi^2}\int_{|\mathbf{n}|=1}\sum_{m=1}^{3}\frac{\varepsilon^m_{JK}}{\rho c^2_m E^m_{qq}}\Big(\psi(k_m|\mathbf{n}\bullet(\mathbf{x}-\xi)|) + 2\ln|\mathbf{n}\bullet(\mathbf{x}-\xi)|\Big)dS(\mathbf{n}) \tag{5}$$

where ε^m_{JK} and E^m_{qq} depend on the material properties, c_m and k_m are the phase velocity and the wave number, respectively, $\mathbf{n}$ is a unit 2-D vector and

$$\psi(s) = i\pi\, e^{is} - 2[\cos(s)\,\mathrm{ci}(s) + \sin(s)\,\mathrm{si}(s)] \tag{6}$$

is a singular function such that the kernel of the integral in (5) behaves regularly. In (7) si and ci stand for the sine and cosine integrals.

Considering the following change of variables

$$\chi_M = z_M^x - z_M^\xi = (x_1 - \xi_1) + \mu_M (x_2 - \xi_2) \quad ; \quad M = 1,2,3 \tag{7}$$

the strongly singular and hypersingular terms in (1) can be easily integrated following the regularization procedure described in reference [5] for static piezoelectricity.

The discretization approach follows [5]: discontinuous quadratic BE are used to mesh the crack, with the crack tip elements being straight line quarter-point (figure 1) and standard quadratic elements are used for the rest of the boundary.

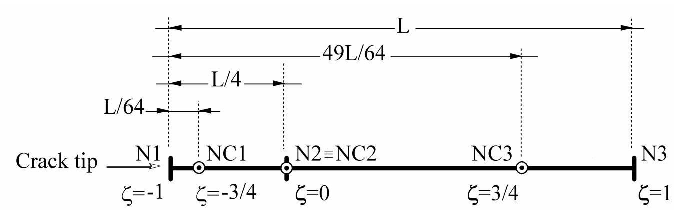

Figure 1. Discontinuous quarter-point element.

The SIF and EDIF can be directly computed form the nodal values of the COD and the electric potential discontinuity at the collocation node NC1 next to the crack tip [5]

$$\begin{bmatrix} K_{II} \\ K_I \\ K_{IV} \end{bmatrix} = 2\sqrt{\frac{2\pi}{L}} \left[\mathrm{Re}(\mathbf{B})\right]^{-1} \begin{bmatrix} \Delta u_1^{NC1} \\ \Delta u_2^{NC1} \\ \Delta \phi^{NC1} \end{bmatrix} \tag{8}$$

where **B** depends on the material properties.

Numerical Results and Discussion

To validate the formulation, the problem of a Griffith crack of length 2a in a piezoelectric ceramic PZT-6B (see table I for material properties) is first considered.

Table I. Material properties

	Elastic (GPa)				Piezoelectric (C/m^2)			Dielectric (C/GVm)	
	C_{11}	C_{22}	C_{66}	C_{12}	e_{21}	e_{22}	e_{16}	ε_{11}	ε_{22}
PZT-6B	168	163	27.1	60	-0.9	7.1	4.6	3.6	3.4
PZT-5H	126	117	23	84.1	-6.5	23.3	17	15.04	13

Plane harmonic longitudinal waves, with an associated stress amplitude σ_0, impinging normally on the crack are considered as in Shindo et al. [1] for comparison purposes:

$$u = 0\ ;\quad v = v_0 \exp[i\omega(\frac{y}{C} + t)]\ ;\quad \phi = \phi_0 \exp[i\omega(\frac{y}{C} + t)] \tag{9}$$

Where $C = \sqrt{\left(C_{22} + e_{22}^2/\varepsilon_{22}\right)/\rho}$

Figure 2 shows the variation of the normalized mode I SIF versus the normalized frequency $\omega a/C_s$, C_s being $(C_{66}/\rho)^{1/2}$. Good agreement is observed with the analytical solution given in [1].

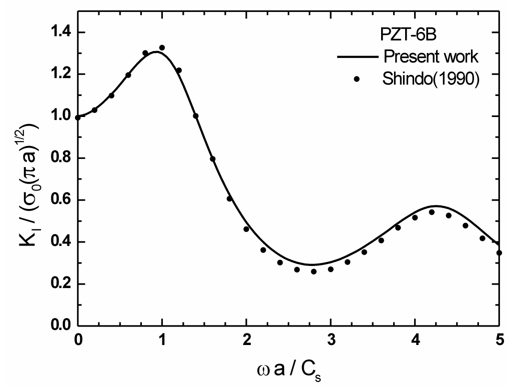

Figure 2. Dynamic SIF for a Griffith crack.

Next, the scattering of the same plane harmonic wave as defined in equation (9) by two collinear cracks (figure 3) in PZT-5H ceramic (Table I) is studied. Crack separations of h=a/2, a, 2a are considered and the obtained results are compared with those of the Griffith crack to illustrate the dynamic interaction effects. Numerical results for the SIF and the EDIF are shown in figures 4 and 5, respectively.

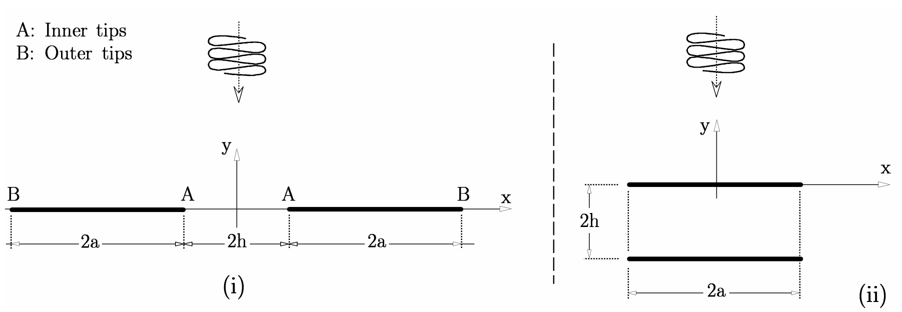

Figure 3. Problem geometry: (i) Two collinear cracks; and (ii) Two parallel cracks.

Finally, the scattering of the plane wave defined in (9) by two parallel cracks (figure 3) in an unbounded PZT-5H ceramic is considered. Normalized values of the SIF and the EDIF at the upper crack are shown in figures 6 and 7, respectively. Crack separations of h=0 (Griffith crack), a/2, a, 2a are studied. Note that the interaction effects are much larger for this configuration.

The BE mesh used for all the examples consists of ten discontinuous quadratic elements per crack, the crack tip elements being quarter-point elements.

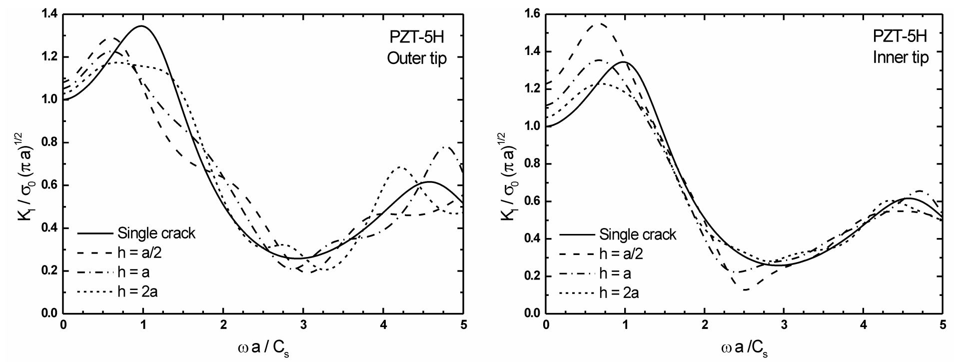

Figure 4. Normalized SIF for the two collinear cracks configuration.

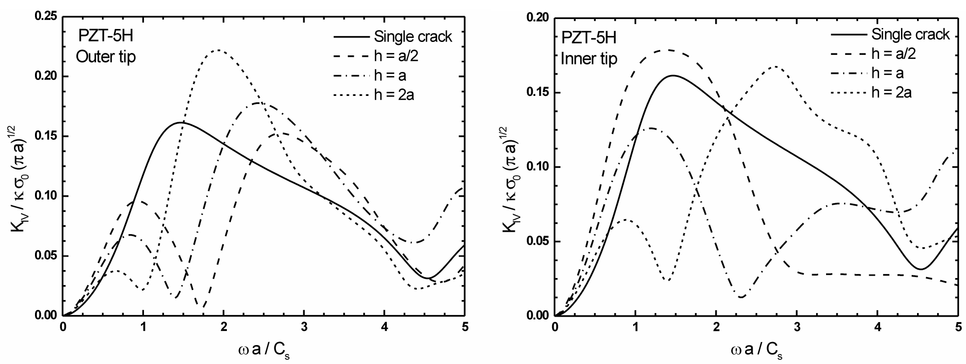

Figure 5. Normalized EDIF for the two collinear cracks configuration ($\kappa = \varepsilon_{22}/e_{22}$).

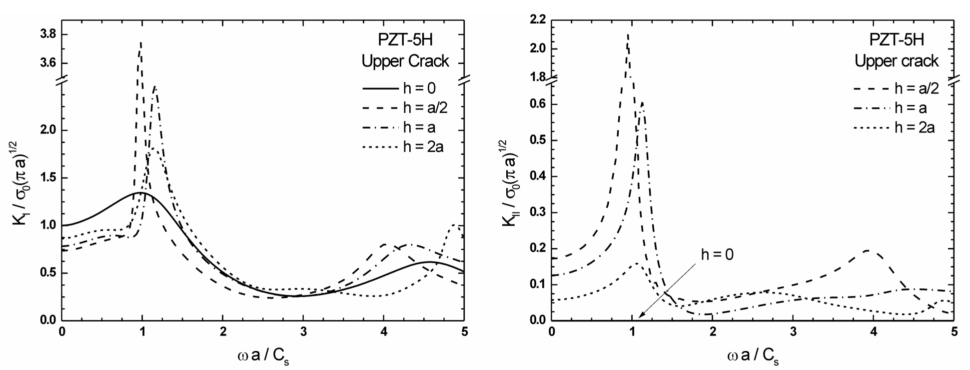

Figure 6. Normalized SIF for the two parallel cracks configuration. Upper crack.

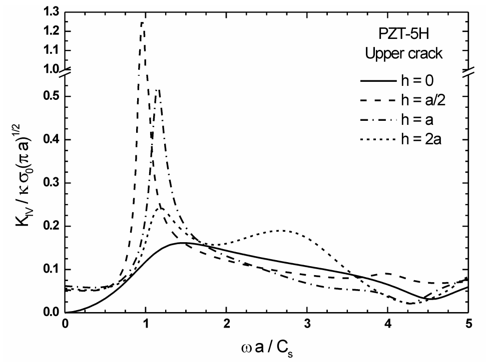

Figure 7. Normalized EDIF for the two parallel cracks configuration. Upper crack.

Acknowledgements

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