A new model for porous viscoplastic solids incorporating void shape effects

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Summary

The aim of this paper is to propose a new model for porous viscoplastic solids incorporating void shape effects. The sound matrix is assumed to obey a simple Norton law with exponent n. Several previous well-accepted models pertaining to various special cases are used as references. These models include : (i) that of Gologanu, Leblond and Devaux, for arbitrary spheroidal voids but an (ideal-)plastic matrix only ($n = \infty$) ; (ii) that of Leblond, Perrin and Suquet, for spherical or cylindrical voids only, but arbitrary n ; (iii) that of Ponte-Castaneda and Zaidman, for spherical voids but a linearly viscous matrix only ($n = 1$). Use is also made of the nonlinear Hashin-Shtrikman bound for voids of arbitrary shape and arbitrary n, but low triaxialities. The approach used basically consists of looking for a suitable heuristic expression for the "gauge surface" of the voided material, which is required to reduce to those corresponding to the reference models in the relevant special cases. The validity of the model is assessed through comparison between the analytical, approximate "gauge surface" proposed and that determined by considering a spheroidal RVE and performing some numerical minimizations of the viscoplastic potential over a large number of trial velocity fields. Finally, an equation for the void shape evolution is proposed to complete the constitutive model.

Introduction

An impressive number of models, which cannot all be cited here, have been proposed for the overall behaviour of porous viscoplastic materials, in order to predict ductile rupture of metals at high temperatures. In almost all models, the behaviour of the sound matrix is assumed to be governed by a simple Norton law with exponent n, connecting the local strain rate $\mathbf{d}$ and the local stress $\boldsymbol{\sigma}$. This constitutive law includes linearly viscous and (ideal-)plastic materials as special cases, for $n = 1$ and $n = \infty$ respectively. The problem is to define the resulting relation, which must necessarily involve the void volume fraction, between the overall strain rate $\mathbf{D}$ and the overall stress $\boldsymbol{\Sigma}$.

Only three models, however, try to account for the influence of *void shape*. The first one is due to Ponte-Castaneda and Zaidman (PCZ) [1]. It relies on the use of the nonlinear Hashin-Shtrikman (HS) bound (based on consideration of some "linear comparison material") for the overall viscoplastic potential, adopted as an *approximate* value instead of a *limiting* one. The PCZ model is very comprehensive, especially in later versions also incorporating the influence of the spatial distribution of voids. It nevertheless suffers from the fact that the HS nonlinear bound, although fully rigorous as a *bound*, is known to provide a poor *estimate* of the overall behavior for high values of the triaxiality (ratio of the overall mean stress over the overall equivalent stress), when the Norton exponent n is itself

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Proceedings of the 2004 International Conference on Computational & Experimental Engineering & Science 26-29 July, 2004, Madeira, Portugal

high. The PCZ model can therefore be considered as a good reference model only for $n = 1$ (linearly viscous material) or low triaxialities.

The second model incorporating void shape effects is due to Garajeu, Michel and Suquet (GMS) [2]. The GMS model relies on direct homogenization of some viscoplastic voided RVE. It undoubtedly yields better predictions than the PCZ model, except for values of n close to 1, but nevertheless suffers from several drawbacks. First, in the plastic case ($n = \infty$), the GMS model bears a close resemblance to the model of Gologanu, Leblond and Devaux (GLD) [3], but unfortunately rather in its early version than its final one. The GLD model, which has gained relatively wide acceptance and applies to spheroidal voids but plastic matrices only, indeed exists in two versions, the second one [3] being much better than the first one, as shown by Sovik [4]. Second, the GMS model considers only prolate cavities, no proposal being made for oblate ones. Finally, the expression proposed for the necessary evolution equation of the void shape is very inaccurate for deviatoric stress states, as is evident from reference [3] in the plastic case.

The third model accounting for the influence of void shape is due to Klocker and Tvergaard (KT) [5]. The KT model uses two “reference” models in some special cases, namely the GLD model and that of Leblond, Perrin and Suquet (LPS) [6] which applies to viscoplastic matrices but spherical or cylindrical voids only. In general, the KT model indisputably gives better estimates than the GMS and PCZ models for both prolate and oblate voids. Nevertheless, it still suffers from some drawbacks : it does not reproduce the good approximations provided by the HS nonlinear bound at low triaxialities and the PCZ model for $n = 1$.

We look here for an improved model for porous viscoplastic materials incorporating void shape effects. The approach used is basically similar to that which serves for the derivation of the LPS model [6]. Instead of looking for an approximate analytic expression of the overall plastic potential, which is a very difficult task, one looks for an expression of the overall “gauge function” (as defined below). Several classical models pertaining to various special cases are used as references : (i) the GLD model, applicable to plastic materials ($n = \infty$) only but arbitrary void shapes ; (ii) the LPS model, applicable to viscoplastic materials (n arbitrary) but spherical or cylindrical voids only ; (iii) the PCZ model, applicable to linearly viscous materials ($n = 1$) only but arbitrary void shapes. The analytic expression of the gauge function looked for is required to match the expressions corresponding to these models in the relevant special cases. A further requirement is that it must match the value of the overall viscoplastic potential provided by the HS nonlinear bound for arbitrary n but low triaxialities.
It must be noted that this approach based on “reference” models is quite similar to that used by Klocker and Tvergaard [5] ; the main novelty is that we require our model to meet two additional references (the PCZ model for $n = 1$ and the HS nonlinear bound).

The GLD, LPS and PCZ models

We briefly present here the three reference models. The void volume fraction (porosity) is denoted f. The voids are assumed to be spheroidal, with axis of symmetry parallel to the diraction z. The semi-axis of the voids along this direction is denoted a, and the common semi-axes along the perpendicular directions x and y, b. The void shape is thus characterized by the single parameter a/b; prolate voids have $a/b > 1$ and oblate ones $a/b < 1$. Only axisymmetric loadings with the same axis of symmetry as that of the voids are considered : the sole non-zero components of the overall stress tensor are thus $\Sigma_{xx} = \Sigma_{yy}$ and Σ_{zz}.

The GLD model is applicable to voids of arbitrary shape, but the material is assumed to be (ideal-)plastic. The yield stress in simple tension is denoted σ_0. The expression of the overall yield function, which extends that of Gurson applicable to spherical voids only, reads :

$$\begin{aligned} F(\mathbf{\Sigma}, f, \frac{a}{b}) &\equiv \frac{C}{\sigma_0^2}(\Sigma_d + \eta\Sigma_h)^2 + 2(g+1)(g+f)\cosh(\kappa\frac{\Sigma_h}{\sigma_0}) \\ &\quad -(g+1)^2 - (g+f)^2 = 0, \\ \Sigma_d \equiv \Sigma_{zz} - \Sigma_{xx},\ &\Sigma_h \equiv 2\alpha_2\Sigma_{xx} + (1-2\alpha_2)\Sigma_{zz} \end{aligned} \tag{1}$$

(Σ_d is the deviatoric part of $\mathbf{\Sigma}$). In this expression, C, η, g, κ and α_2 are parameters depending on f and a/b, the detailed expressions of which are given in reference [3]. The quantity g is of purely geometric nature and plays the role of a “second, fictitious porosity” ; in the special case of penny-shaped cracks ($a/b = 0$) for instance, it is identical to the porosity which would result from spherical voids with radius equal to that (b) of the cracks.

The LPS model applies to viscoplastic materials but spherical or cylindrical voids only. Instead of providing an approximate expression for the overall viscoplastic potential $\Psi(\mathbf{\Sigma}, f, n)$, it gives an approximation of the overall *gauge surface*, composed of those stress tensors $\mathbf{S}$ corresponding to some constant, specified value of Ψ. This value is chosen in such a way that in the plastic case ($n = \infty$), the gauge surface becomes identical (up to an unimportant factor) to the yield surface. The gauge surface is described by some equation $F(\mathbf{S}, f, n) = 0$ where F is the *gauge function*. Prescribing $F(\mathbf{S}, f, n)$ is sufficient to prescribe $\Psi(\mathbf{\Sigma}, f, n)$, since Ψ is a positively homogeneous function of degree $n+1$ of $\mathbf{\Sigma}$. For spherical voids, the equation of the LPS gauge surface reads :

$$\begin{aligned} F(\mathbf{S}, f, n) &\equiv S_d^2 + f\left(H(S_m) + \frac{n-1}{n+1}\frac{1}{H(S_m)}\right) - 1 - \frac{n-1}{n+1}f^2 = 0, \\ H(S_m) &\equiv \left[1 + \frac{1}{n}\left(\frac{3}{2}|S_m|\right)^{\frac{n+1}{n}}\right]^n \end{aligned} \tag{2}$$

where $S_d \equiv S_{zz} - S_{xx}$ and $S_m \equiv \frac{1}{3}\mathrm{tr}\,\mathbf{S}$ denote the deviatoric and mean parts of $\mathbf{S}$. The function H is chosen in such a way that the model reproduces the exact solution of the problem of a hollow viscoplastic sphere loaded hydrostatically. A similar expression is proposed for cylindrical voids in reference [6].

As explained above, the PCZ model was developed for arbitrary values of a/b and n, but in fact provides a good estimate in the whole stress space only for $n = 1$. The equation of the gauge surface proposed then reads :

$$
\begin{aligned}
F(\mathbf{S}, f, \frac{a}{b}) \quad \equiv \quad & S_d^2 + \frac{f}{(1-f)\left[1-(1-3\alpha_1)(1-3\alpha_1')\right]}\left(\frac{1-3\alpha_1'}{1-3\alpha_1}S_d^2\right. \\
& \left. +\frac{9}{4}S_m^2 + 3(1-3\alpha_1')S_d S_m\right) - 1 = 0
\end{aligned} \tag{3}
$$

where α_1 and α_1' are coefficients depending on f and a/b, given in reference [3].

The model proposed

The gauge surface proposed is deduced from the GLD yield function (1) in the same way as the LPS gauge function (2) from the Gurson yield function. Indeed the problem (extending an expression valid for $n = \infty$ to arbitrary n) is the same, the only difference being the void shape (spherical for the Gurson and LPS models, spheroidal for the GLD model and here). We thus replace the "cosh" function by $\frac{1}{2}(H + \frac{n-1}{n+1}\frac{1}{H})$ and introduce a factor of $\frac{n-1}{n+1}$ in the term $-(g+f)^2$ in the equation of the gauge surface :

$$
\begin{aligned}
F(\mathbf{S}, f, \frac{a}{b}) \quad \equiv \quad & C(S_d + \eta S_h)^2 + (g+1)(g+f)\left(H(S_h) + \frac{n-1}{n+1}\frac{1}{H(S_h)}\right) \\
& -(g+1)^2 - \frac{n-1}{n+1}(g+f)^2 = 0, \\
& S_h \equiv 2\alpha_2 S_{xx} + (1-2\alpha_2)S_{zz}, \;\; H(S_h) \equiv \left[1 + \frac{1}{n}(\kappa|S_h|)^{\frac{n+1}{n}}\right]^n
\end{aligned} \tag{4}
$$

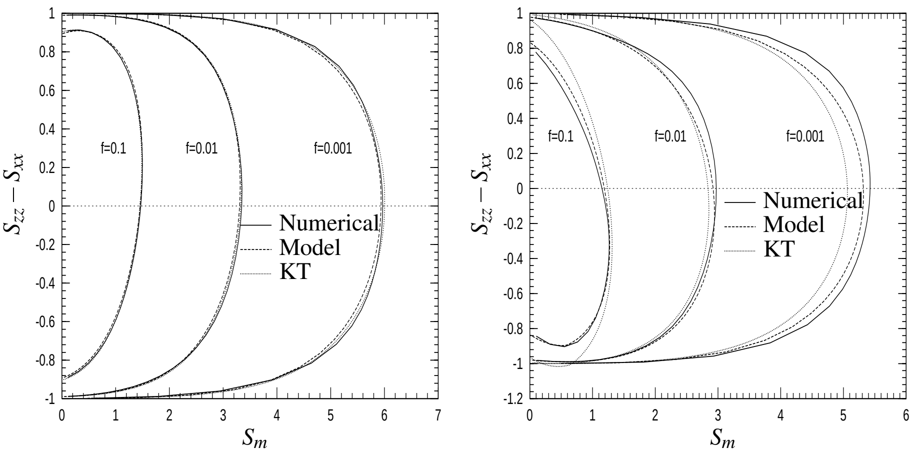
This equation matches the reference models in the relevant special cases provided that the coefficients take suitable values then. This and other considerations lead to general expressions for the coefficients g, α_2, C, η and κ, depending on f, a/b and n. Indeed, the parameter g being of purely geometric nature (see above), it is logical to ascribe it the same value as in the GLD model [3], independent of n. The parameter α_2 determines the weighting coefficients of the radial (S_{xx}) and axial (S_{zz}) stress components in the stress S_h, which is the essential parameter governing void growth. There is no clear reason either why it should depend on the matrix rheology. This suggests to also take α_2 as independent of n and given by the same value as in the GLD model [3]. Also, for $n = \infty$ and $n = 1$, the GLD and PCZ models give distinct values $\kappa^{(\infty)}$ and $\kappa^{(1)}$ for κ. The following "interpolation" formula for arbitrary n is then proposed :

$$
\kappa = \frac{\kappa^{(1)}}{n} + \frac{n-1}{n}\kappa^{(\infty)} \tag{5}
$$

Finally, since the nonlinear HS bound provides a good estimate of that zone of the gauge surface corresponding to low triaxialities, we enforce coincidence and tangency of the HS gauge surface and the present one at some typical point in this zone. We thus get complex expressions (depending on f, a/b and n) for the two remaining coefficients C and η.

Comparison of approximate and “exact” gauge surfaces and directions of flow

For all values of f, a/b and n, a numerical, supposedly exact gauge surface can be obtained by considering a suitable RVE and performing some minimization of the average value of the local viscoplastic potential over a large number of trial velocity fields. In practice, we adopt a spheroidal RVE confocal with the void, and use the family of velocity fields proposed by Lee et Mear [7], especially adapted to the spheroidal geometry. Figure 1 shows the results obtained for a prolate and an oblate void, typical values of n and a/b, and several values of f, along with the predictions of our model and the KT model.



(a) prolate void : $a/b = 5$; $n = 5$ (b) oblate void : $a/b = 1/5$; $n = 5$

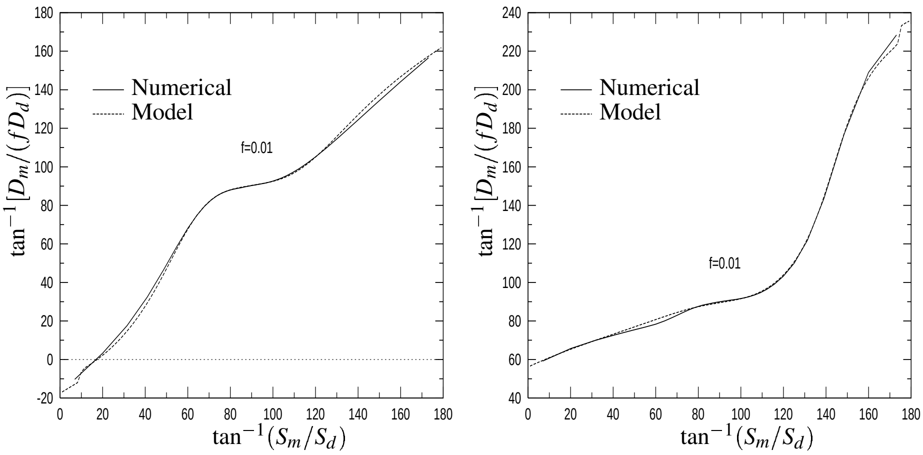
FIG. 1 – *Numerical and approximate gauge surfaces*

In order to preserve legibility, the PCZ and GMS gauge surfaces are not represented here. However, numerical calculations show that the GMS model (for prolate voids only) overestimates the value of S_m at high triaxialities, and that the PCZ model, although nearly exact for $n = 1$, is quite inaccurate for high values of n and the triaxiality, especially for low porosities.

The direction of the normal to the gauge surface governs, via the “normality rule”, that of the viscoplastic flow. Figure 2 shows the comparison, in a typical case, of the approximate and “exact” directions of this normal, for a prolate and an oblate void. The quantity on the horizontal axis is the angle $\tan^{-1}(S_m/S_d)$ (in degrees), characterizing position on the gauge surface, and that on the vertical one is the angle $\tan^{-1}[D_m/(fD_d)]$ (also in degrees) where $D_d \equiv D_{zz} - D_{xx}$ and $D_m \equiv \frac{1}{3}\mathrm{tr}\,\mathbf{D}$, characterizing the direction of the normal.

Void shape evolution

To complete the constitutive model, we propose evolution equations for the internal



(a) prolate void : $a/b = 5$; $n = 5$ (b) oblate void : $a/b = 5$; $n = 5$

FIG. 2 – *Directions of normals to the approximate and numerical gauge surfaces*

parameters f and $S \equiv \ln(a/b)$ (void shape parameter). The evolution equation of f is the usual one, deduced from incompressibility of the sound matrix. Also, using again the GLD model (for $n = \infty$) and the PCZ model (for $n = 1$) as references, we propose the following equation for the void shape evolution, where T stands for the triaxiality :

$$\begin{aligned} \dot{S} &= h(e,f,T,n)(D_{zz} - D_{xx}) + 3\frac{1-f}{f}\frac{(1-3\alpha_1)(1-3\alpha_1')}{1-3\alpha_1+3f(\alpha_1-\alpha_1')}D_m \\ h(e,f,T,n) &= 1 + h_f(f,n)\ h_e(e_1,n)\ h_T(T,n),\quad h_e(e_1,n) = \frac{3(\alpha_1-\alpha_1')}{1-3\alpha_1}\left(\frac{3}{2}\right)^{\frac{n-1}{n}} \end{aligned} \tag{6}$$

where the functions h_f and h_T are deduced from heuristic considerations and comparisons with numerical calculations, and the function h_e derives from Eshelby's famous solution.

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