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Computation of the Dynamical Behaviour of Faulty Rotor Systems

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Summary

Models and methods for evaluating the dynamical behavior of faulty rotor systems are briefly described.

Introduction

The simulation by means of models of the dynamical behaviour of rotor systems composed by rotating shafts, flexible or rigid couplings, oil film bearings or rolling element bearings and relevant supporting structures, such as turbogenerator groups in power plants or other rotating machinery, and the evaluation of the vibrations due to a developing fault, which could be measured in some measuring point of the system, are a great help for researchers and plant engineers for understanding the running condition of the machine, and in discovering possible malfunctions or faults at an early stage.

Models of the rotor systems

A finite beam element model is assumed for the rotating shaft, with a number of degrees of freedom per node ranging from 1 to 6 depending on the type of problem which has to be analysed [1].

Oil film bearings are generally represented in a linearized approach by 4×4 stiffness and damping matrices which are obtained by integrating Reynolds equations for the oil film, with given oil characteristics, bearing geometry, load and rotating speed. The position of the journal in the bearing has to be found in a non linear approach iteratively. In this position stiffness and damping coefficients are evaluated by applying Taylor series and by calculating numerically the derivatives of the forces with respect to journal displacement (for the stiffness) and to journal velocity (for the damping).

Sometimes the linearized approach is not accurate enough; in this case also non linear calculations can be performed, evaluating instead of the stiffness and damping coefficients, the oil film forces in each position of the journal in its orbit inside the bearing clearance. The supporting structure can be represented by pedestals (lumped mass, damping and stiffness systems which are not speed dependent) or, more conveniently, by a modal model of the complete structure condensed at the connecting nodes between the rotating shaft and the supporting structure, which are generally only the bearings when seals between rotor and stator can be neglected. The matrix equation obtained with the described model is given in eq. (1), where **M**, **C**, **K** and **Gyr** are the

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mass, damping, stiffness and gyroscopic matrices of the complete system, $\mathbf{F}(t)$ the external force vector and $\mathbf{x}$ is the vector of displacements of all nodes of the system.

$$[\mathbf{M}]\ddot{\mathbf{x}} + [\mathbf{C} + \mathbf{Gyr}]\dot{\mathbf{x}} + [\mathbf{K}]\mathbf{x} = \mathbf{F}(t) \tag{1}$$

Main concern is generally about lateral bending vibrations, which could be measured in correspondence of the bearings, where also alarm and trip levels are established. For calculating these vibrations generally models with 4 d.o.f.s per node are used, and 2 d.o.f.s for the bearings, since only radial displacements of the journal with respect to the bearing shell are taken into account and rotations are neglected. Similarly only 2 d.o.f.s are considered for the pedestals or for the condensed modal model of the supporting structure in correspondence of the bearings.

Since the torsional vibration is decoupled from bending vibrations, except some cases, the torsion behaviour can be analysed with f.e. models with only 1 d.o.f. per node. The presence of a gear, with a set of toothed wheels, generates coupling effects between torsional vibrations and lateral bending vibrations through the teeth contact forces, as described e.g. in [2]. Also the presence of a transverse crack in the rotating shaft, which destroys the axial/polar symmetry of the shaft, generates some coupling between lateral bending vibrations and torsional vibrations (as well as axial vibrations), as described in [3]. Such problems can be analysed only with models with 6 d.o.f.s.

Fault models

Most faults can be represented by a system of equivalent forces acting either directly on the rotor or as an action-reaction pair between rotor and stator. The faults which will be considered are: i) different types of unbalance; ii) thermal or permanent bow; iii) rigid coupling misalignment; iv) flexible coupling misalignment; v) bearing misalignment; vi) axial rotor asymmetries; vii) transverse cracks; viii) rotor to stator rubs; ix) instabilities; x) electrical faults.

Let **M**, **C** and **K** be the usual mass, internal damping and stiffness matrices of the (linear) rotor alone. Following equation holds:

$$[\mathbf{M}]\ddot{\mathbf{x}} + [\mathbf{C}]\dot{\mathbf{x}} + [\mathbf{K}]\mathbf{x} = \mathbf{F}_e + \mathbf{R} \tag{2}$$

where $\mathbf{x}, \dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the vibration displacement, velocity and acceleration vectors $\mathbf{F}_e$ is the external force vector, which includes the weight **W** and the residual unbalance rotating vector $Ue^{i\Omega t}$, Ω is the rotating speed and **R** the oil film force vector of the bearings. If a fault appears, vibration **x** will change by amount $\Delta\mathbf{x}$ (and similarly its derivatives) and this may be due to change $\Delta\mathbf{F}_e$ in the external exciting force system (as in case of blade loss) or to a change $\Delta\mathbf{R}$ of the oil film forces (as in case of bearing alignment change, or in case of oil film instability) or to a change $\Delta\mathbf{K}$ of the rotor

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stiffness matrix, due to some structural defect (such as a crack, f.i.). Let's consider this last case and splitting the total displacement $\mathbf{x}$ in its statical component $\mathbf{x}_0$ and in its dynamical component $\bar{\mathbf{x}}$, eq. (3) is obtained:

$$[\mathbf{M}]\ddot{\bar{\mathbf{x}}} + [\mathbf{C}]\dot{\bar{\mathbf{x}}} + [\mathbf{K}]\bar{\mathbf{x}} = Ue^{i\Omega t} + \bar{\mathbf{R}} + [\Delta\mathbf{K}](\mathbf{x}_0 + \bar{\mathbf{x}}) \tag{3}$$

where $\bar{\mathbf{R}}$ is the dynamical component of $\mathbf{R}$. The last term in equation (3) represents the equivalent 8-force system due to the structural defect, which, depending on $\bar{\mathbf{x}}$, has to be calculated iteratively. Similarly, also a bow or a rigid coupling misalignment can be modelled by a set of equivalent external forces according to eq. (4), where X_S is the statical rotating bow generated by the coupling misalignment or by an internal unsymmetrical stress distribution and $\mathbf{x}_T$ is the total vibration.

$$[\mathbf{M}]\ddot{\mathbf{x}}_T + [\mathbf{C}]\dot{\mathbf{x}}_T + [\mathbf{K}]\mathbf{x}_T = [\mathbf{K}]\mathbf{x}_S = [\mathbf{K}]X_S e^{i\Omega t} \tag{4}$$

Computational methods - Free vibration

Natural frequencies and damping ratios can be found by analysing the free vibrations using eq. (1) with no loads, evaluating the complex eigenvalues: the imaginary parts are the natural frequencies, the real parts are proportional to damping ratios. Both values depend on the rotating speed (bearing stiffness and damping matrices and the gyroscopic matrix depend on the rotating speed). The diagrams of the natural frequencies and damping ratios as function of rotating speed are called Campbell diagram and stability map respectively. An example is shown in figure 1 for a high speed vacuum pump.

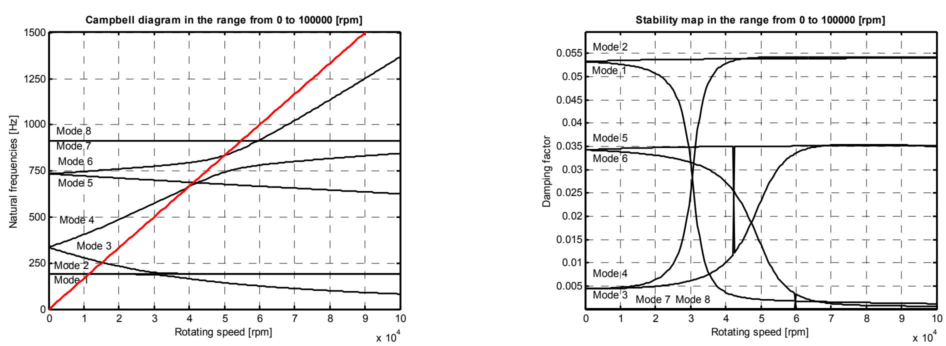

Figure 1. Campbell diagram and stability map.

Computational methods - Steady state forced vibrations

If a steady state solution, at a given rotating speed, is analysed, and if the stiffness is constant and the load is 1X, then the solution can be found in the frequency domain, according to eq. (5).

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$$\left[-(n\Omega)^2\mathbf{M}+in\Omega\mathbf{C}+\mathbf{K}\right]\mathbf{X}_n=\mathbf{F}_{f_n} \tag{5}$$

This can be repeated changing step by step the rotating speed. The resulting diagram of vibration amplitude as function of rotating speed (Bode diagram shown in figure 2) represents the dynamical behaviour of the system during a slowly changing rotating speed transient, which can be considered a succession of steady state conditions because the free vibration during the transient is weakly excited and completely damped out.

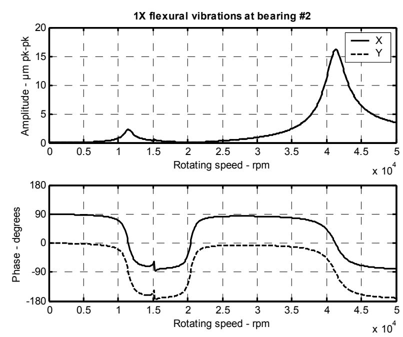

Figure 2. Unbalance excited frequency response in brg. #2 of a vacuum pump.

If the load is constant the constant deflection can be found by eq. (5) with Ω=0. If the load has other harmonic components (2X or 3X) again eq. (5) is used with 2Ω or 3Ω instead of Ω. If the load is random with known PSD, again eq. (5) can be used for calculating the response for the single spectral component.

If the stiffness is not constant but depends on the rotation angle Ω*t*, **K**(Ω*t*), as it occurs when the shaft is not polarly symmetrical or cracked (see eq. (3)), then the steady state solution can be found by an iterative procedure: an harmonic balance approach can be used in the frequency domain. An example of the computed vibrations of a cracked generator is given in figure 3 taken from ref. [4].

If the breathing of the crack depends not only on the stationary loads such as the weight in horizontal axis machines, but also on the actual vibration of the rotor, as it can occur in vertical axis machines, then the stiffness matrix is function of the rotation angle Ω*t* and of the vibration **x**.

Eq. (1) becomes non linear: if the non linearity is not very high again the iterative procedure in the frequency domain can be used as in [5]. But if the non-linearity is rather high the iterative procedure does not converge, the solution becomes unstable and the eq. (1) has to be solved numerically in the time domain.

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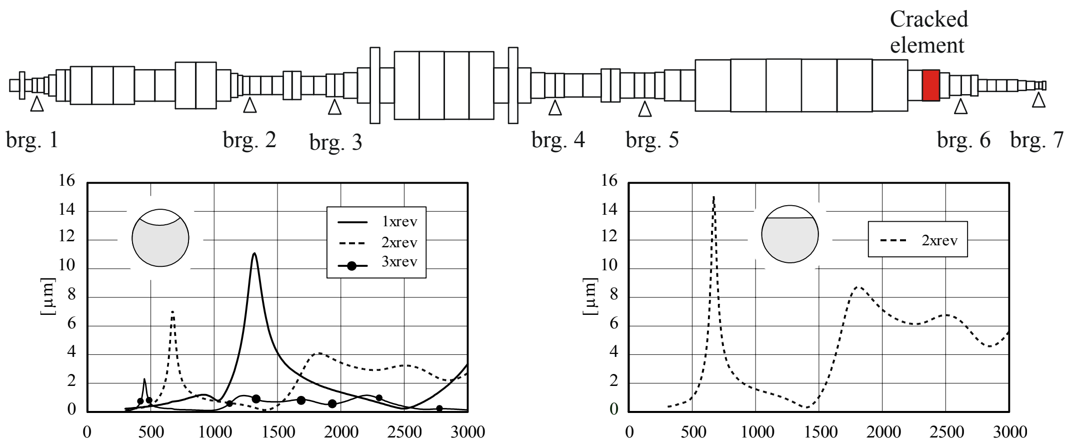

Figure 3. Vibrations in brg. 6 due to a breathing crack (left) and to an open crack/slot (right).

Time domain numerical integration must also be used when a broadband quasi random excitation is applied to a cracked shaft at a steady state rotating speed, as it may occur in a hydraulic machine. Different harmonic components can be recognized in the spectrum of the calculated time domain response. Figure 4 is related to the model of a cracked shaft with only the torsional degree of freedom, rotating at 1500 rpm (25 Hz). The shaft is a test-rig rotor (shown in figure 4) on which many studies have been performed especially on the effects of transverse cracks. The periodical torsional stiffness combined to the stationary torque generates torsional vibration components at 25, 50 75 Hz and higher frequencies, the torsional natural frequencies are also excited and, due to the combination of critical speed excitation and crack induced periodical stiffness variation, also sideband components appear.

Computational methods - Transient vibrations

Finally also transient behaviour calculations are required, especially when transients are sharp and short, as it occurs during start ups of smaller machines, and of electric motor driven machines.

Also behaviours during typical accidents such as blade loss or “bird intake” in aeronautical turbines, or short circuit in electrical machines, are analysed through transient vibration calculations.

A typical example with Runge-Kutta time step integration 3-5 is shown in figure 5 for the start up of the same test-rig rotor, with the start up transient torque given by reference [6]:

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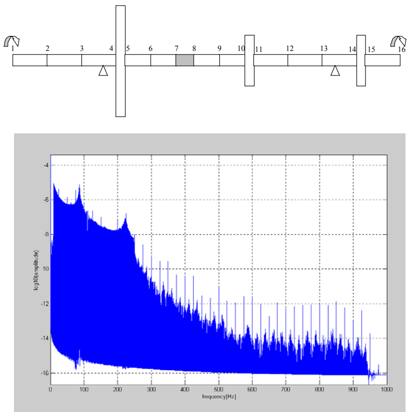

Figure 4. Response in node 15 caused by the broadband excitation as $\log_{10}$ scale, crack depth 60% in element 7.

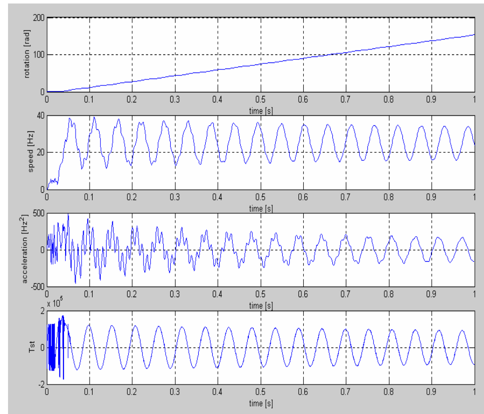

Figure 5. Rotation, speed, acceleration of node 9 and start-up torque.

Conclusions

Models of rotor systems in normal and faulty operating conditions are briefly presented and different methods that can be used for analyzing its different vibrational behaviours, in the frequency domain and in the time domain, are presented as well as some numerical results and examples.

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