KNEE LIGAMENTS CONSTITUTIVE MODEL AND COMPUTATIONAL STRESS ANALYSIS DURING WALKING AND CYCLING

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Summary

The purpose of this paper is to present a new hyper-visco-elastic constitutive model for the human knee ligaments and results obtained by numerical simulation of stress and strain fields in the knee ligaments under normal walking and cycling. We used publicly available image data of the knee joint, an experimental kinematics data set and CAD and FE software to develop a 3D finite element model of the knee joint. Numerical results compare well with similar simulation published data.

Introduction

Knee ligament injury is a frequent clinical problem in many activities. The main functions of ligaments are, for the collaterals (MCL – medial and LCL – lateral), to give lateral stability to the knee during extension and to limit the external rotation, and for the cruciates (ACL- anterior and PCL – posterior) to stabilize the antero-posterior motion, to keep the articular areas in contact during flexions and to limit the internal rotation.

The main objectives of this study were to develop a realistic constitutive model of the knee ligaments and to simulate the distribution of the stresses and strains in the knee ligaments during normal walking and cycling. To model the complex mechanical response under physiological or extreme loads on the knee structure, a dynamic nonlinear approach was considered. This approach consisted of an accurate modeling of the joint geometry, kinematics data according to a particular type of movement and a constitutive model to describe the mechanical behavior of the ligaments tissue.

Knee joint computational model

For the 3D model, the serial 1mm cross cryosections of the knee joint were obtained from the *Visible Human* database of the *U.S. Library of Medicine*.

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The process of three-dimensional reconstruction started with image interpretation and labeling of structures to identify the organ contours, (i.e. the femoral condyles, tibial plateau surfaces, articular cartilage, menisci and insertion points of ligaments). The contours were made by b-splines using a computer aided design software package, Pro/Engineer. These curves were used to interpolate surfaces that defined the exterior faces of the 3D solid models of the knee joints.

The whole joint model was used for the kinematics computation. Anatomical landmarks were positioned on the knee 3D model as described in the VAKHUM Project [1]. Using these landmarks, an individual reference frame of the computational knee model, S_0, was created according to the standards of the International Society of Biomechanics. The same category of landmarks was identified on a human volunteer knee that was video recorded using two high speed video cameras. This analysis of the video data was performed using SIMI Motion software (SIMI) for every landmark on the physical model and a new reference frame was produced.

The displacement and rotation vectors for the relocation of the computational model in the global reference frame of the physical model, S_V were obtained using a numerical optimization procedure. Before optimization, a scale factor was considered for a preliminary resizing of the model. Then a better fit of landmarks coordinates was obtained according to:

$$s = \sum_i w_i d_i^O \Big/ \sum_j w_j d_j^V \tag{1}$$

where d_i^0 and d_j^V are distances from anatomical landmarks to their reference frame origins for computational and physical model, and w are weighting factors selected based on distance to articular surfaces. The coordinates of the landmarks on the scaled model were used in an error minimization function:

$$\min \Theta = \sum_i w_i \left(\sum_{j=1}^{3} \left(x_{i,j}^O - x_{i,j}^V \right)^2 \right)^{1/2} \tag{2}$$

where $x_{i,j}^O = f(\theta, \gamma)$ are the landmark coordinates of the knee model in S_V, function of displacement γ and rotation θ vector components between S_O and S_V frames, and $x_{i,j}^V$ are the landmarks coordinates of physical model. During a determined movement (i.e. normal walking), several coordinate transformations were applied to obtain different positions of the femoral segment in reference to the global axes system, while keeping tibia fixed. For the initial reference position, the points defining the femur were computed in the tibial reference system using the transformation:

$$\mathbf{A}_F^{ref} = \mathbf{R}_{SH}^T \cdot \mathbf{R}_{TH} \cdot \mathbf{A}_F + \mathbf{R}_{SH}^T \cdot \left(\mathbf{d}_{TH} - \mathbf{d}_{SH}\right) \quad (3)$$

where $\mathbf{A}_F$ is the femoral landmark coordinates list obtained from the physical model, $\mathbf{R}_{SH}, \mathbf{R}_{TH}$ are the rotation vectors for shank and thigh and $\mathbf{d}_{TH}, \mathbf{d}_{SH}$ are the position vectors. For a particular position n of the joint, the femoral anatomical landmarks coordinate list $\mathbf{A}_F^n$ is applied according to the following transformation:

$$\mathbf{A}_F^n = \mathbf{R}_{SH\_n}^T \cdot \mathbf{R}_{TH\_n} \cdot \mathbf{A}_F^{ref} + R_{SH\_n}^T \cdot \left(\mathbf{d}_{TH\_n} - \mathbf{d}_{SH\_n}\right) \quad (4)$$

where $\mathbf{R}_{SH\_n}, \mathbf{R}_{TH\_n}$, and $\mathbf{d}_{TH\_n}, \mathbf{d}_{SH\_n}$ are the rotation and position vectors for shank and thigh extracted from the physical mode for the position n. From the processed kinematics data, successive relative positions of the femoral-tibial were computed. Internal-external and abduction-adduction rotations and anterior-posterior translation as function of flexion for the knee model were computed from volunteer kinematics data

The finite elements analysis was performed on a reduced model built from the ligaments and the bones insertion zones. Using the IGES standard, the model was exported from Pro/Engineer into Ansys (SAS IP, Inc), to generate a mapped mesh with 8-node continuum brick elements. A total of 848 elements were generated for the joint ligaments (Fig. 1).

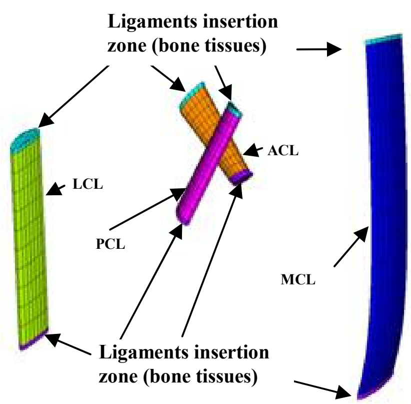

Figure 1. The finite element mesh representation of the articular ligaments with bone insertion zones meshed with rigid elements.

The bones were treated as rigid bodies [2]. The ligaments are considered in a pre-stress state: a baseline stress is present in the ligaments at all knee flexions. The precise experimental distribution of this stress field is not completely known. However, some experimental work suggested a resulting force of the ACL of 100N along the ligaments at full knee extension [3].

Ligaments constitutive models

Previously published constitutive models of the ligaments are based on various linear interpretations of material behavior [4, 5]. In this present study a non-linear constitutive model of ligaments was developed based on the hyper-visco-elastic hypothesis and on experimental data taken from literature [6]. The hyper-viscous-elastic constitutive model uses the following form for the strain energy potential of an isotropic material in terms of the strain invariants I_1, I_2, I_3:

$$W_e = \alpha\left(\beta\left(I_1 - 3\right) + \beta^2\left(I_1 - 3\right)^2\right) - \alpha\left(I_2 - 3\right) \tag{5}$$

where $\alpha = 0.660$ MPa and $\beta = 12.050$.

The elastic constitutive law was given by a partial derivative of the elastic potential with respect to incompressibility assumption:

$$\mathbf{S}\left(\mathbf{C}\right) = 2\frac{\partial \mathbf{W}_e}{\partial \mathbf{C}} = \alpha\left(\beta\mathbf{I} + 2\beta^2\left(I_1 - 3\right)\mathbf{I}\right) - \alpha\left(I_1\mathbf{I} - \mathbf{C}\right) - p\mathbf{C}^{-1} \tag{6}$$

where **p** is an undetermined hydrostatic pressure. The identification of the parameters α and β has been completed using a least square fit of the experimental stress-strain points:

$$E^2 = \sum_{i=1}^{n}\left(\sigma_i - \tilde{\sigma}_i\left(\alpha, \beta\right)\right)^2 \tag{7}$$

Rate effects were taken into account through linear viscoelasticity by a convolution integral of the form:

$$S_{ij} = \int_0^t G_{ijkl}\left(t - \tau\right)\frac{\partial E_{kl}}{\partial \tau}d\tau \tag{8}$$

where $G_{ijkl}\left(t - \tau\right)$ is the relaxation functions, S_{ij} is the second Piola-Kirchhoff stress tensor and E_{kl} is the Green's strain tensor. This stress was added to the stress tensor determined from strain energy functional. Only simple rate effects were included. The relaxation function was represented by the first six terms from the Prony series:

$$G(t) = \sum_{i=1}^{n} G_i e^{-\beta_i t} \tag{9}$$

characterized by input shear moduli G_i and decay constants β_i:

$$\begin{aligned} G_1 &= 0.2820, & G_2 &= 0.12934, & G_3 &= 0.6912 \\ \beta_1 &= 0.202, & \beta_2 &= 0.0005, & \beta_3 &= 0.0004 \end{aligned} \tag{10}$$

All parameters were determined by fitting the experimental stress-strain points using a least square procedure. Constitutive equation was implemented in the MAT_HYPERELASTIC_RUBBER material model in the Ls-Dyna software (Livermore Software Technology Corporation).

Numerical simulation results

The finite element programs Ansys and LS-DYNA were used to obtained approximate solutions to the problem of stress/strain pattern determination of the knee ligaments. An explicit dynamic analysis was performed, where the tibia bone tissues were constrained and the femoral bone parts were moved according to computed kinematics data for normal walking and cycling. The analysis shows (Fig. 2) the most engaged zone in the joint stability of ACL. For all ligaments, 4 bundles (anterior, posterior, lateral and medial) were considered and the stress and strain distribution along them were computed (Fig. 3).

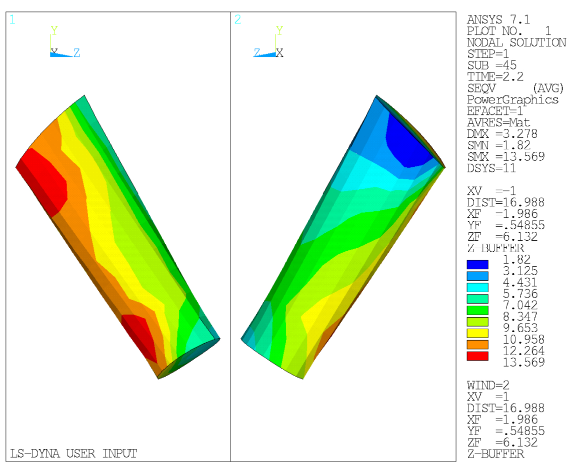

Figure 2. Stress distribution (daN/mm^2) over ACL ligament for a particular position at 20 degrees flexion angle during normal walking.

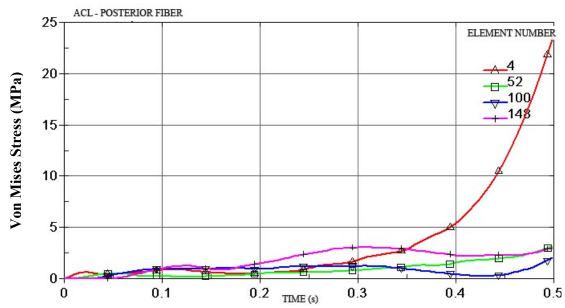

Figure 3. An example of stress distribution (MPa) over ACL, for posterior bundle, during cycling, function of time (s), for a flexion angle from 0 to 20^0.

These above results show a good integration of the new constitutive model in the computational simulation. The model in combination with experimental data will be used in future studies to identify the joint material properties, geometry, and boundary conditions that are important determinants of joint behavior during athletic activities which involve extensive use of the knee joint. Clinical implications of the present biomechanics model arise from identifying the movement categories that should be avoided after ligament injuries or planning exercise regimens following ligament repair. Another important application is in ligament reconstruction.

Reference

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