Designing Optimal Task Schedule by Membership Functions Choice in Fuzzy Real-Time Scheduling

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Abstract

To take care of uncertainty in the parameters of real-time systems several researchers have considered the use of fuzzy models. When the processing times and deadlines are considered as fuzzy quantities, the schedule designers faced dilemma of how to choose the shapes of membership functions. In this paper the effect of various choices of membership functions on the real-time scheduling produced has been presented by taking simple numerical examples. This will allow the designers to make a proper choice depending on the application at hand and adjust the sensitivity of task priorities to change in the fuzzy parameters of the model.

Keywords: Fuzzy Number, Membership Function, Fuzzy Deadlines, Real-Time Systems, Task Scheduling

I. INTRODUCTION

Computing systems in which correctness of the result depends on the timely production of results in addition to the logical outcome of computation are termed Real-Time Systems [1]. Time is therefore a principal factor in Real-Time Systems. This type of computing systems is present in Telecommunication Systems, Defense Systems, Aircraft Flight Control systems, Air Traffic Control, Space Stations and Nuclear Power Plants etc. Tasks in Real-Time Systems have explicit timing requirements besides other characteristics of general systems. Predictability, i.e. the ability to determine whether a set of tasks can be scheduled to meet all the timing requirements, is therefore one of the most important criteria in Real-Time Systems. Hence, task scheduling plays a crucial role in real-time systems. The goal of task scheduling in real time systems is to devise a feasible schedule, subject to a given task set, task characteristics, timing constraints, resource constraints, precedence constraints etc. [2].

The timing constraints of a particular set of tasks comprises of task deadlines, processing times, task arrival or release times, intervals between subsequent invocations of tasks i.e. period etc. The *release time* of task is the time before which it can not start execution, where as the time within which a task must be completed after it is released is known as the *relative deadline* of that particular task. Release time plus the relative deadline gives the *absolute deadline* of a task. The time required by the processor to execute a particular task is known as the *processing time or execution time*. Tasks may be periodic, aperiodic or sporadic. When tasks are released periodically they are *periodic tasks*. An invocation of *sporadic tasks* happens in irregular intervals rather than periodically where as *aperiodic tasks* are not periodic nor carry any bound on invocation rate [3].

In the early phase of real time system design only an approximate idea of the tasks and their characteristics are available and as result some uncertainty or impreciseness are associated with their timing requirements. To cope with this uncertainty in the timing constraints of tasks in Real-Time Systems several modeling techniques considering uncertainty (viz. probability theory, fuzzy set theory etc.) have been proposed [4]. These techniques provide more realistic timing analysis in comparison to simple crisp timing constraints. Among them Fuzzy mathematics edges past probabilistic theory in so many ways as computations are simpler here and expert help can be taken easily for task modeling. Moreover, it is faster in computations and provides more flexibility in modeling because without any significant addition in complexity we can choose from a wide variety of fuzzy membership types for a particular timing parameter [5,6,7]. The scheduling of real-time tasks having fuzzy constraints can be termed as fuzzy real-time scheduling. Several reports are available where fuzzy approaches were introduced for various real time scheduling algorithms and different models for Real-Time Systems were proposed. Fuzzy due dates were considered first by H. Ishii et al. [8] for a general scheduling problem. An attempt has been made by F. Terrier et. al. [9] for applying fuzzy calculus in real-time task scheduling, who claimed that they considered the task execution time as fuzzy for the first time. J. Lee et. al. [10] proposed a model for fuzzy rule based scheduler for scheduling real time tasks. For the formulation of multi–objective fuzzy scheduling problems T. Murata et. al. [11] considered the importance of individual jobs with OWA (ordered weighted averaging) operator. Most excellent work on fuzzy real time scheduling was done by Litoiu et. al. [12,13,14] in their proposed pessimistic model. They have introduced a cost function viz. satisfaction of schedulability, used before by H. Ishii et al. [8], and evaluated the satisfaction of each individual job having fuzzy deadlines. Then the problem is formulated as the maximization of the minimum value of the satisfaction function.

This paper extends the model proposed by Litoiu et al. [13] for fuzzy numbers with various types of Membership Function Shapes. Mathematical models are developed and used in several test cases using the Cheddar Real Time Simulator [15]. The effects of the choice of different types of membership functions and their fuzzy parameter on the satisfaction intervals, the satisfaction of schedulability and the tasks priorities of set of real time tasks are studied and reported with examples for demonstration. The rest of the paper is organized as follows. Section-II introduces the terminology and formulation of the fuzzy real time scheduling problem. Section-III gives detailed analysis for triangular type membership functions for fuzzy deadlines. Finally the paper concludes in the section-IV with the experimental results and discussion.

II. FUZZY REAL-TIME SCHEDULING

A real-time system is considered to have general model as per following:

$T = \{T_i \mid i = 1,2,3,.............n\}$ is a set of n tasks such that

(1) Period of task T_i is P_i (2) Execution time of task T_i is e_i

(3) Deadline of task T_i is d_i (4) Release time of task T_i is r_i

(5) Phasing of task T_i is I_i (i.e. at jth invocation period of task T_i begins at $I_{ij} = I_i + (j-1)P_i$ and completes at $d_{ij} = d_i + I_{ij} = d_i + I_i + (j-1)P_i$, j = 1,2,..

(6) At the j-th invocation, the completion time for task T_i is C_{ij}

We consider the deadline d_{ij} of the task T_{ij} as fuzzy number. And our interest here is to see differences in the schedulability analysis of a set of real time tasks with fuzzy deadlines, when the shapes of the membership functions vary. To model the fuzzy deadline of a real time task T_{ij}, we now consider a general bell shaped membership function, shown in the Figure-1, with $[a_{ij}, b_{ij}]$ as it's 0-cut. The satisfaction of schedulability (S_d) that expresses the value of the compliance of the deadlines over all the periods can then be given by:

$$S_{d_i}(C_{ij}) = \begin{cases} 1 & \text{if } C_{ij} < a_{ij} \\ 1 - \dfrac{\int_{a_{ij}}^{C_{ij}} \mu(d_{ij})\, d(d_{ij})}{\int_{a_{ij}}^{b_{ij}} \mu(d_{ij})\, d(d_{ij})} & \text{if } a_{ij} \le C_{ij} \le b_{ij} \\ 0 & \text{if } C_{ij} > b_{ij} \end{cases}$$

Here, μ is the membership function for the fuzzy deadline d_{ij} and C_{ij} as the crisp execution time of the task T_{ij}. The quantity $\frac{\int_{a_{ij}}^{C_{ij}} \mu(d_{ij})d(d_{ij})}{\int_{a_{ij}}^{b_{ij}} \mu(d_{ij})d(d_{ij})}$ is actually the measure of dissatisfaction, how far the task completion time is missing the deadline. If we see the Figure-1 below, then the denominator is the total area under the curve, whereas the numerator is the shaded area.

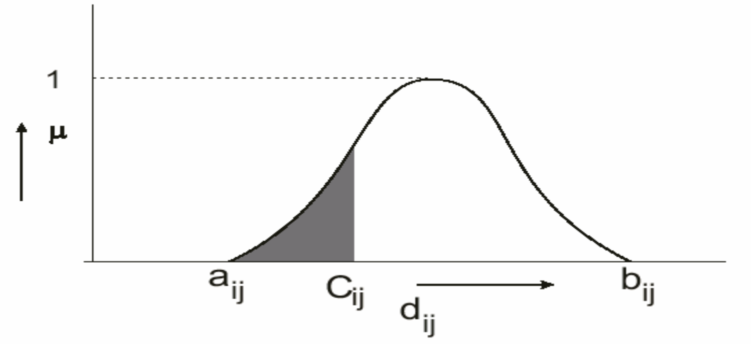


Figure-1: General Fuzzy Deadline

In the expression for S_{d_i}, the integrals exists for obvious reasons and in the intervals $[a_{ij}, b_{ij}]$ the functions S_d is strictly increasing and continuous. Therefore considering the deadlines as fuzzy numbers the scheduling problem for the real time task set may be summarized as

$$Maximize \;\; S = \min_{i,j} S_{d_i}(C_{ij})$$

finding an assignment of priorities, where $i = 1,......,n$ and $j = 1,.......,P/P_i$. We now consider, $\min_{i,j} S_{d_i}(C_{ij}) = t$. Thus, $S_{d_i}(C_{ij}) \geq t \quad \forall i = 1,......n$ and $j = 1,.....P/P_i$. Hence, from the continuity of the satisfaction function S_d and it's inverse function, we can write: $C_{ij} \le I_i + (j-1)P_i + d_i^/(t)$, where, $i = 1,.....n$ and $j = 1,....P/P_i$. Here, the quantity $d_i^/(t)$ is a crisp quantity that depends on the minimum satisfaction (t) and is termed as the modified deadlines and the optimal priority assignment of the real time tasks are according to the increasing order of this quantity. Now our interest is to find those values of t ($0 \le t \le 1$) for which modified deadlines of two tasks become equal i.e. those t_{ij} that satisfy:

$$\{t_{ij} \,|\, d_i^/(t) = d_j^/(t),\ j = 1,...,n,\ i = 1,.....,n,\quad 0 \le t_{ij} \le 1\}$$

Now since the modified deadline of different tasks changes at these t_{ij}, the priorities of the tasks changes only at these points. Thus we check the priorities of the tasks over various intervals $[t_m, t_{m+1}]$, obtained by sequentially placing the quantities t_{ij} in the increasing order. Now we use successive search to identify the interval in which satisfaction is maximum. Then highest satisfaction interval is identified for the worst case completion time, $W_i(C_i)$ and the task priorities in that interval are determined. Here, $W_i(C_i) = \sum_{j=1}^{i} e_j \lceil C_i / P_j \rceil + B_{L_i}$, B_{L_i} being the time for which a high priority task is blocked by a low priority one. Finally, the value of S_{d_i} for various tasks are calculated, the minimum of which is the satisfaction of the schedulability.

III. SATISFACTION S_d AND SATISFACTION INTERVALS $[t_m, t_{m+1}]$ FOR TRIANGULAR FUZZY DEADLINES

A fuzzy triangular membership function, say $Tri,(Triangular\ (x; a_{ij}, b_{ij}^/, b_{ij})$, is specified by three parameters $\{a_{ij},\ b_{ij}^/,\ b_{ij}\}$, as given below:

$$Tri = \begin{cases} 0 & if \quad x < a_{ij} \\ \dfrac{(x-a_{ij})}{(b_{ij}^{/}-a_{ij})} & if \quad a_{ij} \le x < b_{ij}^{/} \\ \dfrac{(b_{ij}-x)}{(b_{ij}-b_{ij}^{/})} & if \quad b_{ij}^{/} \le x < b_{ij} \\ 0 & if \quad x \ge b_{ij} \end{cases}$$

Considering triangular type fuzzy deadline d_{ij} for task T_{ij}, as shown in Figure-2, the satisfaction function S_{d_i} can be expressed as:

$$S_{d\,i} = \begin{cases} 1 & if \quad C_{ij} < a_{ij} \\ 1-\dfrac{(C_{ij}-a_{ij})^2}{(b_{ij}-a_{ij})(b_{ij}^{/}-a_{ij})} & if \quad a_{ij} \le C_{ij} \le b_{ij}^{/} \\ \dfrac{(b_{ij}-C_{ij})^2}{(b_{ij}-a_{ij})(b_{ij}-b_{ij}^{/})} & if \quad b_{ij}^{/} \le C_{ij} \le b_{ij} \\ 0 & if \quad C_{ij} > b_{ij} \end{cases}$$

For task T_i, the modified deadline is given by:

$$d_i^{/} = \begin{cases} a_i + \sqrt{(1-t)X_i} & if \quad t > L_i \\ b_i - \sqrt{(tY_i} & if \quad t \le L_i \end{cases}$$

Here, $L_i = (b_i - b_i^{/})/(b_i - a_i)$; $X_i = (b_i - a_i)(b_i^{/} - a_i)$; $Y_i = (b_i - a_i)(b_i - b_i^{/})$.

The union of K_1, K_2, K_3, K_4 given below, for which modified deadlines of two tasks are equal (such that $0 \le t_{ij} \le 1$). The quantities t_{ij} can be found from the union of the following K_1, K_2, K_3 and K_4, given below.

$$K_1 = \{t_{ij} \mid t_{ij} = \frac{(a_i - a_j)^2}{(\sqrt{X_i} - \sqrt{X_j})^2}\} \text{ for } t_{ij} > L_i,\ t_{ij} > L_j$$

$$K_2=\{t_{ij} \mid t_{ij}=\frac{(b_i-b_j)^2}{(\sqrt{Y_i}-\sqrt{Y_j})^2}\} \text{ for } t_{ij}\le L_i,\ t_{ij}\le L_j$$

$$K_3=\{t_{ij} \mid t_{ij}=\frac{2X_iY_j-(X_i-Y_j)(Z_{ij}-X_i)\pm 2\sqrt{Z_{ij}X_iY_j(X_i+Y_j-Z_{ij})}}{(X_i+Y_j)^2}\} \text{ for } t_{ij}>L_i,\ t_{ij}\le L_j$$

$$K_4=\{t_{ij} \mid t_{ij}=\frac{2X_jY_i-(X_j-Y_i)(Q_{ij}-X_j)\pm 2\sqrt{Q_{ij}X_jY_i(X_j+Y_i-Q_{ij})}}{(X_j+Y_i)^2}\} \text{ for } t_{ij}>L_i,\ t_{ij}\le L_j$$

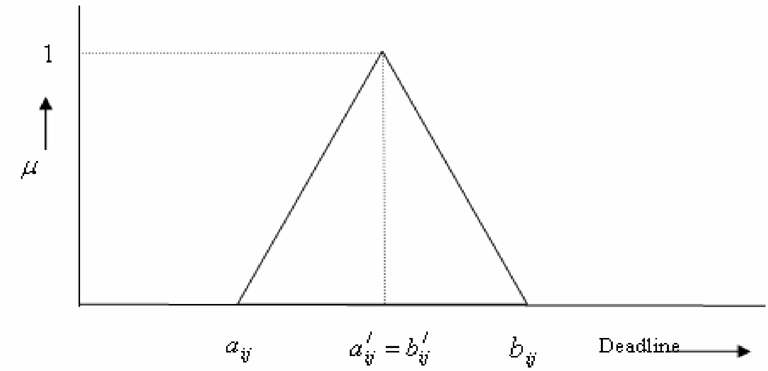


Figure-2: Triangular Fuzzy Membership Function

where $Z_{ij}=(b_j-a_i)^2$ and $Q_{ij}=(b_i-a_j)^2$. We are avoiding here detail mathematical model for other membership functions (trapezoidal, semi-elliptical etc) due to restrictions on paper length. Those are already developed and reported for publication.

IV. EXPERIMENTAL RESULTS

To assess the relative performance of fuzzy real time scheduling using different membership functions to model fuzzy uncertainty in execution times and deadlines we conducted several simulation experiments using our own C software for computing the modified deadlines and then using it to run Cheddar real time simulator [15]. We have done a number of simulation experiments. We are considering a task set comprising three tasks with characteristics, given in Table-1, to show our findings. Here, we have considered three different membership functions shapes for task deadlines (Triangular MF (I), Triangular MF (II) and Trapezoidal MF).

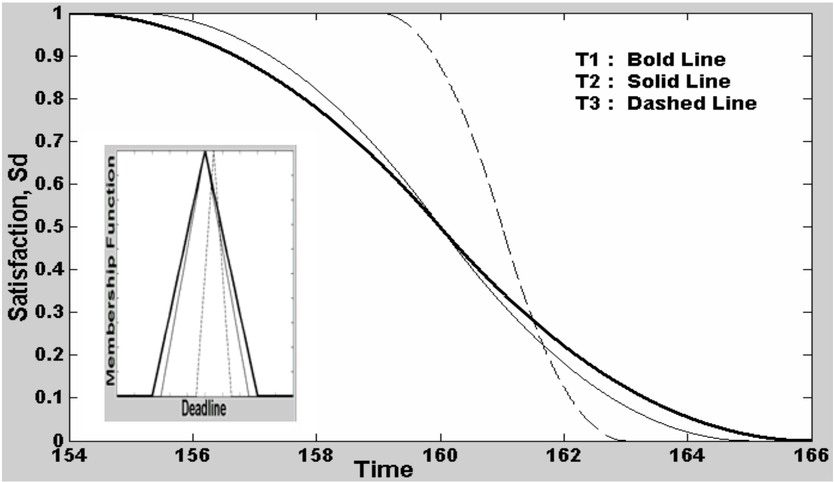
Table- 1 : Task Characteristics

Task Name	Worst Case Execution Time	Fuzzy Deadline			Period
		Triangular MF (I)	Triangular MF (II)	Trapezoidal MF	
T_1	50.8	[154 160 166]	[154 160 163]	[154 157 163 166]	166
T_2	75.7	[155 160 165]	[155 160 166]	[155 157.5 162.5 165]	166
T_3	35.6	[159 161 163]	[159 161 165]	[159 160 162 163]	166

Table-2: Satisfaction Intervals, Satisfaction of Schedulability and Optimal Schedule

Membership Function Type	Satisfaction Intervals	Corresponding Task Order	Satisfaction of Schedulability	Modified Deadlines	Optimal Schedule
Triangular MF (I)	[0.0000, 0.2222]	$[T_3, T_2, T_1]$	0.2113	T_1 : 162.0995 T_2 : 161.7496 T_3 : 161.6998	**$[T_3, T_2, T_1]$**
	[0.2222, 0.2813]	$[T_2, T_3, T_1]$			
	[0.2813, 0.5000]	$[T_2, T_1, T_3]$			
	[0.5000, 1.0000]	$[T_1, T_2, T_3]$			
Triangular MF (II)	[0.0000, 0.3088]	$[T_1, T_3, T_2]$	0.2103	T_1 : 160.6171 T_2 : 162.2744 T_3 : 162.2485	**$[T_1, T_3, T_2]$**
	[0.3088 , 1.0000]	$[T_1, T_2, T_3]$			
Trapezoidal MF	[0.0000, 0.2778]	$[T_3, T_2, T_1]$	0.2817	T_1 : 161.9647 T_2 : 161.6372 T_3 : 161.6549	**$[T_2, T_3, T_1]$**
	[0.2778, 0.3333]	$[T_2, T_3, T_1]$			
	[0.3333, 0.5000]	$[T_2, T_1, T_3]$			
	[0.5000, 1.0000]	$[T_1, T_2, T_3]$			

Table-2 summarizes the results obtained. Figures-3, 4 & 5 shows the plot of the Satisfaction Function, S_d for all the three different cases considered.



Figurere-3 : Plot of Satisfaction for Triangular MF (I)

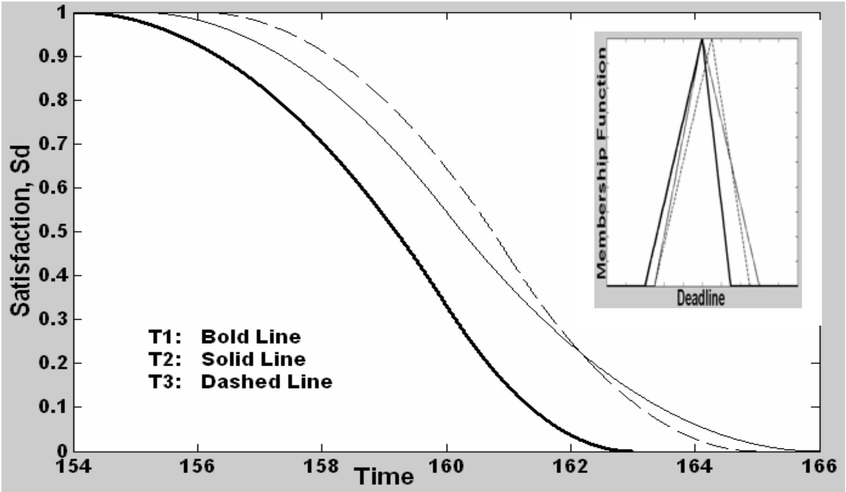


Figure-4 : Plot of Satisfaction for Triangular MF(II)

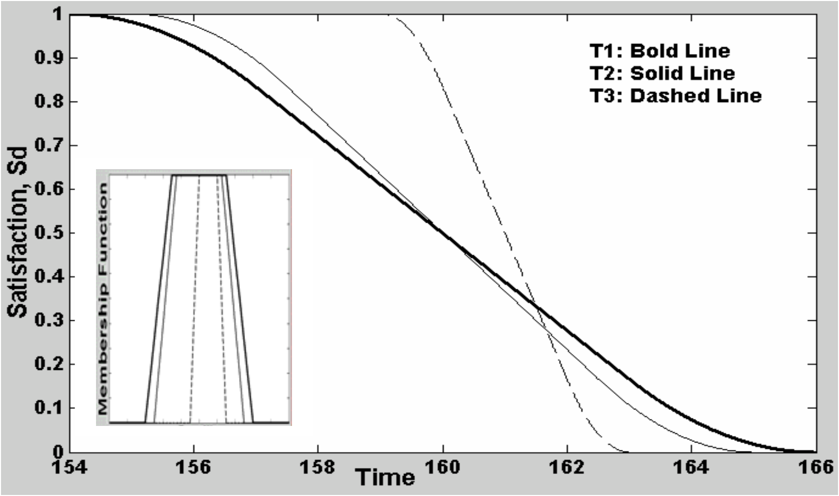


Figure-5: Plot of Satisfaction for Trapezoidal MF

V. CONCLUSION

While introducing different fuzzy membership functions to model the uncertainty in the deadline, contrary to the conclusive remarks of Litoiu et. al. [13] that small changes in the fuzzy numbers do not affect the task priority, we observed that if the completion time varies then the optimal tasks schedules changes at different points for various membership functions of the deadlines. This has been observed as we find different satisfaction crossover points of modified deadlines of the tasks for different membership functions of the tasks deadlines. The satisfaction of schedulability and task order for the task set is also different when we consider different membership function shapes for the task deadlines.

Thus, it is quite logical that the schedule designers face dilemma in choosing appropriate membership functions shapes when the processing times and deadlines have associated uncertainty. We have demonstrated the effect of various choices of membership functions on the real-time scheduling by taking simple numerical examples in this paper. This will allow the designers to make a proper choice depending on the application at hand and adjust the sensitivity of task priorities to change in the fuzzy parameters of the task scheduling model. Therefore, when we consider the uncertainty in the timing parameters, the task scheduling problem in real-time systems becomes very much interesting and in future we wish to do further research in this direction.

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