Analysis of vortical and thermal flows of thermo-acoustic oscillation

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Summary

The thermo-acoustic oscillation (Taconis oscillation) in a closed long tube with large thermal gradient is studied by numerical simulation of the 2D compressible Navier-Stokes equations. Both ends of tube are hot ($T = T_H$), and the center of side wall is cold ($T = T_C$). We analyze the thermo-acoustic fields in the closed tube with different temperature ratios $\theta = T_H/T_C$. When the oscillation is observed, the time averaged pressure p_m and the pressure amplitude p_{amp} are almost constant in any tube cross section. On the other hand, the time averaged temperature T_m is not homogenous in the central cold region. The thermal boundary layer in the cold region may have a important role. We also observe two steady states between the temperature ratio $6.2 < \theta < 6.5$.

Introduction

Spontaneous oscillations of a gas column are frequently observed in a long tube when there is a nonuniform temperature distribution along the axis of the tube. It has been reported that the oscillation occurred in the pumping line from a liquid helium reservoir to a room temperature system. Taconis et al. [1] studied this oscillation in a gas-filled tube connected with a liquid helium reservoir. Thus these oscillations are called “Taconis oscillation”.

Theoretical studies of Taconis oscillations have been developed by Kramers [2] and Rott [3]. Kramers assumed that the thickness of viscous boundary layer on the tube wall was sufficiently small compared with the tube radius. He obtained the critical temperature ratio θ_{cri} over which a steady standing wave could exist. However, this temperature ratio θ_{cri} was extremely higher than that of the experimental results of helium gas. Rott analyzed Taconis oscillations in a 2D rectangular tube or in an axisymmetric tube with a closed hot end and an open cold end. He took into account a finite viscous boundary layer thickness and the material property of helium gas. He successfully obtained the critical temperature ratio θ_{cri} in a circular tube compared with the experiment. It is also shown in his paper that the value of the critical ratio of a 2D rectangular tube is not largely different from that of an axisymmetric tube.

Yazaki et al. investigated the pressure development of Taconis oscillations in the closed circular tubes experimentally [4]. In this experiment, both ends were closed and hot, and the central region of side walls was cold. They observed standing waves in the different geometrical and physical situations. They reported that the critical temperature ratio θ_{cri} in these experiments agreed with that by the theoretical analysis of Rott though the the experimental situation was different from theoretical one. It is difficult to observe the fluid

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flow in the tube experimentally. The research of Taconis oscillations in such a long tube has not been studied by numerical simulation. The purpose of this research is the numerical investigation of the dynamics of Taconis oscillation which occurs in the long tube similar to the experiment of Yazaki et al.

Formulation of problem

The fluid flow in a closed long tube is considered. For simplicity, we assumed that it is the two-dimensional rectangular tube in Fig.1(a). The fluid in the tube is gaseous helium. The basic equations are the 2D compressible Navier-Stokes equations of a perfect gas. The conservative form of the equations is

$$\frac{\partial}{\partial t}\boldsymbol{q}+\frac{\partial}{\partial x}\boldsymbol{E}+\frac{\partial}{\partial y}\boldsymbol{F}=\frac{1}{Re}\left(\frac{\partial}{\partial x}\boldsymbol{R}+\frac{\partial}{\partial y}\boldsymbol{S}\right) \tag{1}$$

where $\boldsymbol{q}=(\rho,\rho u,\rho v,e)$, ρ is the density, u and v are the velocity components in the x and y directions, and e is the total energy density, respectively. The specific heat ratio γ is 5/3 and the Prandtl number is 0.68. The viscosity and the thermal conductivity are determined by using the Sutherland law.

The wall temperatures of both ends are the room temperature $T_{\mathrm{H}}=300$K and the wall temperature of the central region is T_{C} ($<T_{\mathrm{H}}$). The wall temperature distribution is shown in Fig.1(b). We calculated the several temperature ratios $\theta=T_{\mathrm{H}}/T_{\mathrm{C}}$ between $5<\theta<12$. Since the tube is very narrow, we assume that flows are symmetric to the central axis of the tube. The tube length l is 28cm, the half of the tube width w is 0.7mm, and the length of temperature gradient Δl is 7.5mm.

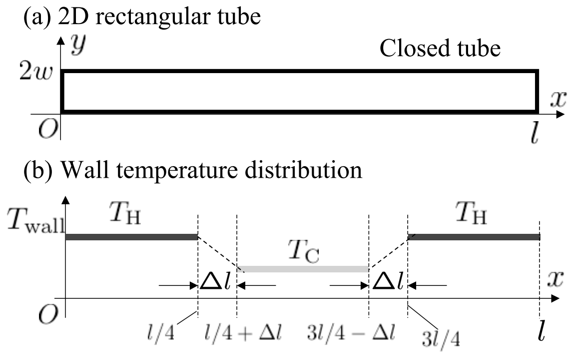

Figure 1: (a) Schematic of the 2D closed tube. (b) Wall temperature distribution.

Numerical method

To solve the basic equations, we employ the block pentadiagonal matrix scheme [5]. Time advancement is by the second-order accurate three-point backward implicit scheme.

The convective terms are evaluated by using fourth-order central differencing and the viscous terms are evaluated by using second-order central differencing. The boundary conditions on the wall are non-slip, isothermal, and no pressure gradient in the normal direction of the wall. The initial state of the fluid is quiescent, $T = T_{\rm H}$, and $p = p_0$, where p_0 is the atmosphere pressure. The physical quantities are normalized by the tube length l, the density ρ_0, the acoustic velocity a_0 at $T = T_{\rm H}$ and $p = p_0$ respectively.

Results

When the temperature ratio θ is equal to 9.09, the steady oscillation is observed in the tube. The pressure fields at $t_a = 978.6$ and $t_b = 981.8$ are shown in Fig.2 when $\theta = 9.09$. The time difference $t_b - t_a$ is almost half of the resonant period. The tube walls are on $x/l = 0.0, 1.0$ and $y/l = 0.0$. The central axis of the tube, that is, symmetrical boundary is on $y/l = 2.5 \times 10^{-3}$. The pressure is low at the left end and high at the right end in Fig.2 (a), while the pressure is high at the left end and low at the right end in Fig.2 (b). It is noted that the value of the time averaged pressure is homogenous in the tube and the pressure is constant along the vertical line (x =constant) . The antinode of pressure oscillation exists at the both ends of the tube and the node exists at the center. The mode of this oscillation is a primary mode in the tube.

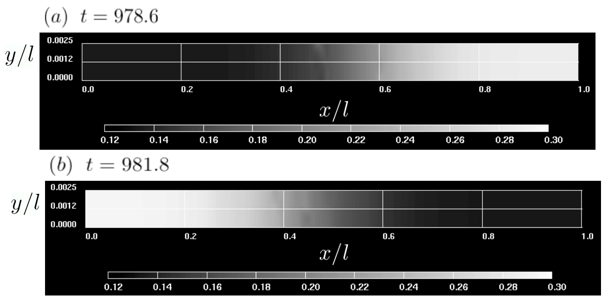

Figure 2: Pressure field in the tube. (a) time $t_a = 978.6$, (b) time $t_b = 981.8$ at $\theta = 9.09$. The time difference between (a) and (b) is almost half of the resonant period.

The time averaged temperature fields are shown in Fig.3. The temperature is time averaged over 10 periods of the primary oscillation. The time averaged temperature is not homogeneous along the vertical line in $0.1 < x/l < 0.9$. The thickness of the thermal boundary layer in the hot region $\delta_{\alpha\rm H}$ is $\delta_{\alpha\rm H} \approx 2 \times 10^{-3} = 0.8w/l$ at $\theta = 9.09$, and that in the central cold region $\delta_{\alpha\rm C}$ is $\delta_{\alpha\rm C} \approx 5 \times 10^{-4} = 0.2w/l$.

In order to study the dependency of the oscillation amplitude on the temperature ratio θ, θ is decreased from the oscillation state (A) at $\theta = 9.09$ ($T_{\rm H} = 300$H is fixed and $T_{\rm C}$ is increased). The steady oscillations are observed at $\theta \geq 5.88$ and the quiescent states in the tube are obtained at $\theta \leq 5.7$. On the other hand, when θ is increased from the quiescent

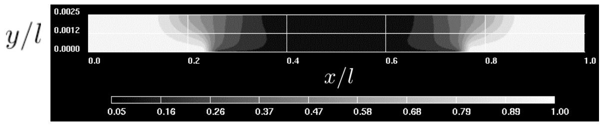

Figure 3: The time averaged temperature distribution at $\theta = 9.09$.

state (B) at $\theta = 5.7$, we obtain the steady oscillations at $\theta \geq 7.14$. The oscillation states in the closed circular tube were observed at $\theta > 5.5$ in the experiments of Yazaki et al. Their result is consistent with that by our numerical simulation in a 2D closed tube. The pressure amplitudes p_{amp} with the temperature ratios θ are shown in Fig.4. The pressure amplitudes from the oscillatory initial state (A) at $\theta = 9.09$ are shown by white circles, and those from the quiescent initial state (B) at $\theta = 5.7$ are shown by black squares. Two different steady states are obtained at $6.2 < \theta < 6.5$.

The time averaged temperature distributions in the cold region are observed for the initial state (A) to investigate these phenomena in detail. Figure 5 shows the temperature distribution at $\theta = 9.09, 7.14, 5.88$ in the cold region where the range of the temperature is between 0.05 and 0.17. The temperature of the fluid is almost equal to the cold temperature T_C of the tube wall at $\theta = 9.09$. However, at $\theta = 7.14$ and 5.88, the temperature of the fluid in the tube is lower than T_C.

Figure 6 shows the time and cross section averaged energy flux $< (e+p)u >$ in the length direction at $\theta = 9.09, 7.14, 5.88$ with the initial state (A). They are averaged over 10 periods of the primary oscillation and the half width of the tube. The energy flows toward the center of the tube when $\theta = 9.09$. On the other hand, in the cold region, the energy flux is almost zero when θ is 7.14, but the energy flows toward the hot end of the tube when $\theta = 5.88$. This flux is corresponding to the cold temperature of the fluid in the central region.

Figure 7 shows the acoustic intensity vector in the quarter region of the tube at $\theta = 9.09$ with the initial state (A). The wall temperature changes between $x/l = 0.25$ and 0.28. At $x/l = 0.25$ the acoustic energy flows to the cold region inside the boundary layer ($0 < y/l < 1.5 \times 10^{-3}$) and to the hot region outside the boundary layer ($1.5 \times 10^{-3} < y/l < 2.5 \times 10^{-3}$). At the point of the cold region ($x/l = 0.375$), the values of the acoustic energy inside the boundary layer ($0 < y/l < 5 \times 10^{-4}$) are much larger than those outside the boundary layer. This shows the role of the thermal boundary layer is important in the mechanism of the thermo-acoustic oscillation. Therefore, it is necessary to study the acoustic energy flow and thermal energy flow in detail in the cold region.

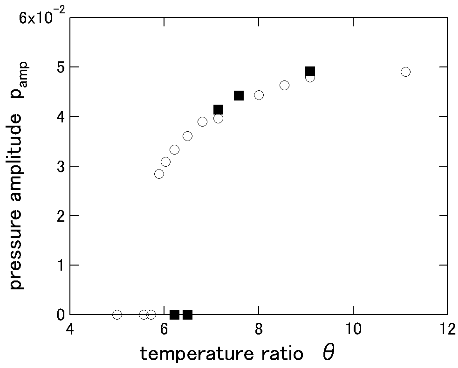

Figure 4: Relationship between the pressure amplitude and the temperature ratio. ◯: initial state (A) and ■: initial state (B).

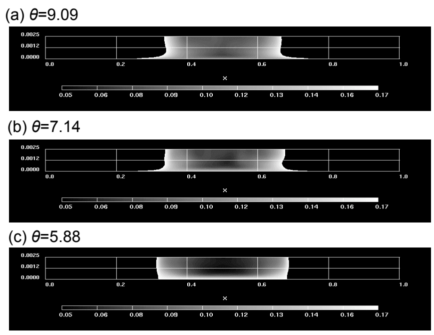

Figure 5: Time averaged temperature distribution in the cold region. (a) $\theta = 9.09$, (b) $\theta = 7.14$, and (c) $\theta = 5.88$ for the initial state (A).

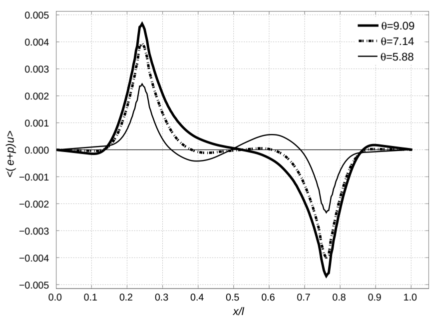

Figure 6: Time & cross section averaged energy flux in the length direction

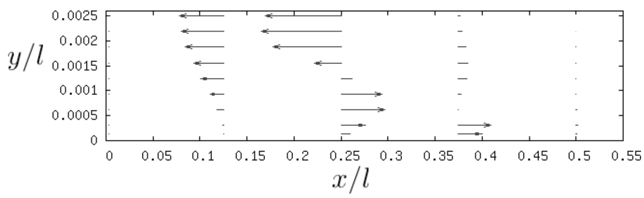

Figure 7: The acoustic intensity in the half region of the tube at $\theta = 9.09$. The initial state is (A).

Concluding remarks

We have successfully obtained Taconis oscillations in the closed tube. We observe two different steady states at the same temperature ratio. In the cold region, the time averaged temperature and the acoustic intensity inside the boundary layer are much different from those outside the boundary layer. To study the function of the boundary layer in the energy transportation is important to analyze the thermo-acoustic oscillation.

Acknowledgement

This work was supported by a Grant-in-Aid for 21st Century COE "Frontiers of Computational Science".

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