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Newton-Cotes Quadrature Formulae and their Superconvergence for Some Hypersingular Integrals in Boundary Element Methods

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Summary

Many scientific and engineering problems, such as the problems in acoustics, electromagnetic scattering and fracture mechanics, can be reduced to integral equations with hypersingular kernels on the boundary. We use boundary element methods for solving these equations. The very important topic is how to evaluate the hypersingular integrals efficiently, which should be understand in the Hadamard finite-part sense.

In this paper we discuss Newton-Cotes method and its development for hypersingular integrals on an interval and on a circle for 2d problems. The method on an interval was first studied by Linz in 1985, then a modified trapezoidal rule applicable for the case that the singular point coincides with a mesh node, and a quadrature scheme using geometric meshes are suggested by Yu in 1992. After that, the composite Simpson rules and corresponding indirect methods have been developed by Wu, Du and Yu. In recent yeas the superconvergence results for composite trapezoidal and Simpson rules have been obtained, and the corresponding indirect methods based on them are also presented to improve the accuracy and avoid the problem of mesh selection. As for the hypersingular integral on a circle, the similar trapezoidal rules with uniform mesh and geometric meshes are discussed, and related superconvergence results for the composite trapezoidal rule are also presented here. Lastly, some numerical experiments are given to verify the theoretical analysis and to show the validity for solving some hypersingular equations.

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