

A Novel Reduced-Basis Method for Certified Real-Time Solution to Solid Mechanics with Exact Upper and Lower Bounds

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Summary

It has been recently discovered by the authors that an upper bound solution to a solid mechanics problem can be produced by the newly developed Linearly Conforming Point Interpolation Method (LC-PIM) using a Smoothed Galerkin Projection. This paper presents a novel SGP_RBM method for real-time computation of solid mechanics problems, using the Smoothed Galerkin Projection together with the dimension-reducing technique used in the reduced-basis method (RBM). The SGP_RBM is a systematical, rapid and reliable computation technique providing an upper bound of “exact” solution. Making use of the fact that the standard RBM based on the Galerkin projection produces a lower bound of the exact solution, we establish RBM models that provide a certify the solution to solid mechanics problems with both exact upper bound (by SGP_RBM) and lower bound (by GP_RBM). To efficiently build the RBM models for both SGP_RBM and GP_RBM, an asymptotic error estimation with a greedy adaptive procedure is used. The present method is applied to analyze a cantilever beam with an oblique crack to verify proposed RBM technique in terms of accuracy, convergence, bound properties and computational savings. Both theoretical study and numerical results have demonstrated that the present method is very efficient method for real-time solutions providing both upper and lower bounds to the “exact” solution for solid mechanics problems. The CPU time for the online computation can be reduced as much as 600 times.

keywords: Meshfree method; reduced-basis method; asymptotic error estimation; linear elasticity; point interpolation method; greedy adaptive procedure; real-time computation; error estimation; numerical method.

References

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