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Recent Improvements to Meshfree Solutions of Partial Differential Equations

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Summary

The most popular meshfree radial basis function is the generalized multiquadrics (MQ), $\phi_j(||x-x_j||) = [1+\{(\mathbf{x}-\mathbf{x}_j)/c_j\}^2]^\beta$, where $\beta \geq -1/2$, c_j is the shape parameter, and $\mathbf{x}$ and $\mathbf{x}_j \in \Re^d$. ϕ_j depends only upon the radial distance between pairs of points in any dimensional space, $\Re$, and is a non-orthonomalized wavelet with translational, dilatational, and rotational invariance. The spatial partial derivatives are obtained by analytically differentiating ϕ_j; hence it is used in the solution of all forms of partial differential equations (PDEs), without the need to construct a mesh, especially over irregular, complex domains. $\phi_j$also possesses exponential convergence of O(λ^μ) where $0 < \lambda < 1$ and $\mu = c_j/h$, where h is the maximal point separation distance, $h = ||x-x_j||$). It is this exponential convergence rate that has interested many persons worldwide. Finite elements converge at the rate $O(h^p)$, where p is the polynomial order.

One can increase μ by refining the discretization similar to finite difference or element methods, or increase c_j. However, increasing μ increase ill-conditioning that is treated with preconditioners and domain decomposition. A recent paper shows a quad-tree or oct-tree adaptive refinement scheme is computationally more efficient because the PDE solution is only refined near steep gradient regions, Another paper demonstrated that by choosing large c_j near steep gradients, very accurate solutions can be obtained with a coarse discretization. Increasing c_j is vastly more efficient. One can employ an improved truncates singular value decomposition method that casts an ill-condition system of equations into a well-condition one, or one can use extended precision arithmetic. With extended precision arithmetic, μ is pushed to very large values, yet the CPU time is very modest since a very coarse discretization is required to obtain the same accuracy as a finely meshed finite element solution. Numerical results will be provided to demonstrate the efficiency of the meshless MQ method. The fast solvers for meshless MQ basis functions is still under development, and will most likely be a combination of the above-mentioned techniques.

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