SOME THOUGHTS ON THE ELUSIVE MECHANISM OF FLUIDELASTIC INSTABILITY IN HEAT EXCHANGER TUBE ARRAYS

David S. Weaver
McMaster University, Hamilton, Ontario, Canada

ABSTRACT

Flow-induced vibrations are a major source of tube failures in heat exchangers and the excitation mechanism of greatest concern is called fluidelastic instability. Despite more than 40 years of research, this mechanism is not fully understood. This paper critically examines the various attempts at modelling fluidelastic instability, emphasizing the contributions made to our understanding of the phenomenon and the model deficiencies. Some of the remaining mysteries are outlined and suggestions are made regarding future research.

1. INTRODUCTION

Flow-induced vibrations in shell and tube heat exchangers have become recognized as a major concern in the design of hardware because of premature tube failures. Such failures occur for tubes in cross flow and are typically due to fatigue or fretting wear at tube supports or at mid-span due to tube-to-tube clashing. There failures cannot be tolerated because of the enormous costs of repairs and production losses as well as potential safety issues.

One of the earliest publications reporting noise and vibration in heat exchangers was Baird (1954) and he attributed the problem to pressure pulsations in the tube wakes. For some years, heat exchanger vibrations were attributed to vortex shedding (see, for example, Livesey and Dye, 1962), and while this possibility was disputed by Owen (1965), Chen (1968) continued to consider the vibrations as due to "von Karman Streets". Roberts (1966) proposed that certain tube failures had been caused by jet-switching in the downstream tube wakes and, a few years later, Connors' (1970) designated the excitation mechanism as "fluidelastic instability". Most importantly, Connors' proposed a very simple relationship between dimensionless velocity and a mass-damping parameter which could be used in design and this equation with empirical coefficients became the industry standard for design.

Despite an increasing research effort during the 1970's, heat exchanger failures were becoming more problematic as well illustrated by Païdoussis (1980) and Pettigrew (1980). By this time, "fluidelastic instability" was recognized as the most serious excitation mechanism because it could cause serious damage to tube arrays in a matter of hours of service. A particularly graphic description of the destruction of a water-to-water U-tube heat exchanger is given by Yu (1986).

"Fluidelastic instability" is a self-excited vibration mechanism, so-called because of the interaction between the fluid forces acting on a structure and the structure's elastic response. The essence of the phenomenon is that the periodic fluid forces acting on the structure do not exist in the absence of structural motion. When the flow velocity exceeds a critical value, energy is transferred from the fluid to the structure and the vibration amplitude grows at a steep rate. Thus, this instability must be avoided and a substantial research effort has been undertaken, mostly driven by the nuclear industry, to establish the fluidelastic stability threshold criteria for design. The literature is now vast and a number of excellent reviews providing design guidelines have been published (Païdoussis, 1983; Chen, 1984; Weaver and Fitzpatrick, 1988; Pettigrew and Taylor, 1991; Schröder and Gelbe, 1999). These stability criteria are empirically based and in some cases continue to use Connors' equation. Schröder and Gelbe (1999) analyzed world data for fluidelastic instability and provided distinct lower bound design curves for each of the standard tube array geometries. The results are much more complex than Connors' equation. Price (2001) argued convincingly that there is little scientific justification for "using Connors' equation, or variations thereof, to predict the velocity at which instability will occur". At the same time, while a number of papers have been published on a variety of theoretical models, simplifying assumptions have been made and a full understanding of the underlying physics is still wanting. A thorough review of these theoretical models has been provided by Price (1995). Thus, at the present time, heat exchanger designers are dependent on empirical data to avoid fluidelastic instability in their hardware and no complete understanding of this excitation phenomenon exists.

It is perhaps curious that, after so much effort, a reliable theoretical model for fluidelastic instability in the tube arrays is still wanting. This paper is a 'thought-piece' which examines critically our state-of-knowledge of the mechanism of fluidelastic instability in tube arrays. While Price (1995) provided a full and rigorous account of all of the theoretical work to date, this brief article covers only the highlights and the emphasis is different. An attempt will be made to explain the principal contributions and deficiencies of the various approaches to the problem which have been taken, to provide possible reasons why our understanding is still incomplete, and to suggest potential directions for resolution of the remaining mysteries.

2. EVOLUTION OF THINKING

2.1 Jet-Switch Mechanism

The first published work which indicates that heat exchanger tube failures were due to a self-excited vibration is Roberts (1966). Motivated by tube failures in a high performance boiler and the observation that the failures seemed to be confined to the rear tube rows, he considered a single row of closely spaced cylinders constrained to move in the streamwise direction. It had been observed experimentally that the wakes downstream of a tube row were bi-stable with pairing of these wakes producing alternately wide and narrow wakes. Measurements indicated that the drag on a cylinder with a narrow wake was higher than that of a cylinder with a wider wake. Thus, a cylinder slightly upstream of its two neighbours has a narrow wake produced by the converging wake flow and a cylinder slightly downstream of its two neighbours has a wider wake. With a suitable phase lag between cylinder motion and jet-switching, the resulting hysteresis in fluid force on a cylinder produces a mechanism for self-excited vibrations. If alternative cylinders in a row moved in the streamwise direction in-phase with one another and 180˚ out-of-phase with their adjacent cylinders, jet-switching would occur and, according to Roberts "are possibly applicable to some recent tube failures in a high performance boiler installation".

The main focus of Roberts' work was for the case when all of the cylinders in a row were aligned (no stagger). Examining this carefully in terms of Roberts' observations, it is seen that the cylinders restrained to move only in the streamwise direction out-of-phase with their neighbours are inherently stable to small disturbances. Whether moving upstream or downstream, the change in drag force opposes the motion. This is why Roberts found that the stability of the one degree-of-freedom system was a 'hard type', ie., the self-excitation mechanism is inherently nonlinear and requires a substantial disturbance to produce limit cycle oscillations.

Seitanis et al (2005) revisited this problem and carefully designed an apparatus to replicate Roberts' model using a row of 4 fixed tubes alternating with 3 flexible tubes, the latter being coupled together and restrained to move in the streamwise direction only. However, in their experiments, Seitanis et al could vary the mean streamwise location of the flexible tubes. In this way, they showed that the excitation mechanism was not due to jet-switching but rather negative fluid stiffness such as found in hydraulic valves (D'Netto and Weaver, 1987).

It is clear that Roberts' work broke new ground. He suggested for the first time the possibility that boiler tube failures could be the result of a self-excited vibration phenomenon, and was the first to introduce the dimensionless parameters now recognized as key to scaling fluidelastic instability. He included unsteady fluid terms in his model and recognized that a finite time was required for the jets to switch in response to the cylinder displacement. Nevertheless, his attribution of boiler tube failures to a jet-switch mechanism was incorrect.

There are a number of reasons why Roberts' research on the jet-switch mechanism failed to advance our understanding of fluidelastic instability in practical heat exchanger tube arrays.

1. It is applicable only to the downstream row of tubes in a tube bank which has sufficient open space downstream to permit coherent jet switching and pairing of wakes. Thus, it fails to explain the vast majority of heat exchanger tube failures.
2. It considers only a single row of tubes which are restrained to move in the streamwise direction and at the same frequency, 180˚ out-of-phase with the neighbouring tubes. This is not a practical

arrangement of tubes for a heat exchanger and does not represent the unstable tube model patterns observed for tube arrays.

3. Price (1995) has shown that Roberts' theory does not agree well with experiments other than Roberts' own data. Additionally Seitanis et al (2005) have demonstrated that the instability is due to negative fluid dynamic stiffness, implying that any jet-switching in the wakes may be the result of tube motion rather than the cause of it.

2.2 Connors' Displacement Mechanism

In 1970, the Heat Transfer Division of ASME sponsored a symposium on Flow-Induced Vibration in Heat Exchangers (Reiff ed, 1970) specifically to address the problems of the significant increase in the number of tube failures due to flow-induced vibration. Interestingly, many of the papers as well as the editors' Forward consider the problem to one of vortex-shedding resonance. However, the paper by Connors' (1970) stands out in identifying the excitation mechanism as "a fluid-elastic, time-independent, displacement mechanism". Like Roberts (1966), Connors' considered a single row of closely spaced tubes across the flow. However, he allowed all the tubes to be flexible and capable of whirling (2 degrees of freedom for each tube). Using a "quasi-steady" approach, he displaced the tubes relative to one another and measured the steady force versus tube position over an assumed vibration cycle. He found that if 3 tubes moved in synchronous elliptical orbits such that the gap between the central tube and its neighbours was smaller when the central tube was moving downstream than when moving upstream, net work could be done on the tube to produce a self-excited instability. The area of the force-displacement curve (work) was measured and used to determine an equivalent sinusoidal force in phase with vibration velocity which would produce the same amount of work. Equating this to the energy dissipated per cycle through viscous damping generated a stability criterion now known as Connors' equation;

$$\frac{V_p}{fd} = 9.9\left(\frac{m\delta}{\rho d^2}\right)^{0.5} \qquad (1)$$

where V_p is the gap velocity, f is the tube natural frequency, d is the tube diameter, m is the tube mass (including fluid added mass), δ is the logarithmic decrement of damping and ρ is the fluid density. In practice, the constant 9.9 is replaced by a so-called Connors' constant, K, which is, in fact, a variable. Connors' concluded that jet-switching was of secondary importance in his investigation and suggested that stability diagrams based on equation 1 provided a rational guide for designing tube arrays to avoid fluidelastic instability.

Blevins (1974) used similar assumptions to Connors' regarding relative displacements of tubes in a row and derived a stability equation which reduces to equation 1 when the appropriate constants are used. Blevins (1977) then extended his model to tube arrays. An interesting implication of the Connors-Blevins displacement mechanism is that if adjacent tubes in an array do not move in specific synchronous orbits at the same frequency, the self-excited motion is stabilized. This prospect was studied by Weaver and Lever (1977) who carefully detuned adjacent tubes in a tube array to determine the effect on the stability of the array. It was found that small detuning (up to about 3% difference in frequencies) resulted in up to a 40% increase in the critical velocity but a frequency difference of 10% or more had no effect. Indeed, a single flexible tube in a rigid array can become unstable. Connors' displacement mechanism is incapable of predicting this behaviour.

Connors' contribution to the practical prediction of self-excited vibrations in tube banks is undeniable. He introduced the name 'fluidelastic instability' in relation to vibrations in tube arrays and presented explicitly the form of the stability equation which became known as Connors' equation and the industry standard for designing heat exchangers against fluidelastic instability. Connors' contribution to the understanding of the basic mechanism of fluidelastic instability is not so clear, noting the following:

1. Connors considered a tube row which is incapable of representing the general behaviour of a typical heat exchanger tube array and excludes the effects of array geometry which is now known to have a significant effect on array stability (see, for example, Schröder and Gelbe, 1999).
2. Connors' displacement mechanism depends on an assumed tube modal pattern which is not generally observed and is incapable of predicting the fact that a single flexible tube can become unstable in a rigid array.

3. Connors' equation does not depend on the mechanics of the displacement mechanism but can be derived from a simple dimensional analysis as discussed by Price (2001). It could be argued that if such an approach had been used in the first place, the influence of array geometry on the stability threshold would not have been neglected and the square root dependence of reduced velocity on the mass-damping parameter would not have been so rigidly adhered to in many design guidelines.

2.3 Unsteady Theory

Tanaka and Takahara (1981) measured the unsteady fluid forces on a square array of tubes with a pitch ratio of 1.33 A central tube in the array was harmonically oscillated in the transverse and streamwise directions in flowing water and measurements were made of the magnitude and phase of the force on the four closest neighbouring tubes with respect to the forced tube. Both the forcing frequency and flow velocity were varied, producing a range of reduced velocity from 1.5 to 200. Thus, lift and drag force coefficients for a tube as a result of its own motion and that of its immediate neighbours could be plotted as a function of reduced velocity. Tanaka et al (1982) also studied a square array with a pitch ratio of 2 using the same approach. Importantly, the results showed that a single flexible tube constrained to move in the transverse direction could become unstable in a rigid array. Thus, this approach included a negative damping mechanism since this is the only mechanism that can produce dynamic instability in a linear one degree-of-freedom oscillator. Chen (1983a,b) developed a general unsteady theoretical model with force coefficients which represented all of the fluid forces on a cylinder due to its own motion as well as that of its surrounding neighbours. He used the measured data of Tanaka and Takahara and found good agreement with experimental results for the fluidelastic stability threshold. This is not surprising since the theory simply replicated the experiments used to generate the force coefficients. From his work, Chen argued that there were in fact two distinct mechanisms causing fluidelastic instability, negative damping and fluid stiffness. The former could cause a single degree of freedom system to become unstable while the latter required multiple degrees of freedom. Additionally, Chen argued that the damping mechanism was dominant at low values of the mass-damping parameter while the fluid stiffness mechanism was dominant at high values.

The works of Tanaka and Takahara (1981) and Chen (1983a,b) represent an important break-through in our understanding of fluidelastic instability in tube bundles. These papers treated a group of tubes which can be considered representative of a practical heat exchanger tube array and demonstrated that both negative damping and fluid stiffness (coupling between tubes) played roles in fluidelastic instability. The results also showed that fluidelastic instability could not be accurately modelled using Connors' equation.

Despite the major contributions of this work, there are a couple of significant concerns:

1. The measured force coefficients as a function of reduced velocity are strongly dependent on tube array geometry and cannot be generalized. Thus, difficult and laborious experiments would have to be carried out for every array geometry of interest and this makes this approach impractical as a design tool.
2. Being completely dependent on empirical force coefficients measured over a range of reduced velocity, this approach does little to explain the physics of the phenomenon and several mysteries remain. The stability plot (reduced velocity vs mass-damping parameter) shows a discontinuity which Chen argued was due to a shift from one mechanism to another but which Price (1995) demonstrated was more likely due to a sudden shift in the phase angle between the fluid force and cylinder displacement. It also remains a mystery, if Chen's argument is correct that the damping mechanism is dominant at low mass-damping parameter values, why most of the experimental data and more recent theoretical analyses indicate that the critical reduced velocity becomes weakly dependent on the mass-damping parameter in this low range. It is similarly mysterious why damping has a significant effect at high mass-damping parameter values where Chen argues that the stiffness mechanism is dominant.

2.4 Semi-Analytical, Quasi-Steady and Quasi-Unsteady Models

Motivated by experiments which showed that a single flexible cylinder in a rigid array became unstable at essentially the same critical velocity as a fully flexible array (Weaver and Lever, 1977), Lever and Weaver (1982) developed a very simple first principles analytical model dubbed semi-analytical by Price (1995). The flow through a general array of tubes was modelled using the unsteady Bernoulli equation in flow channels shaped by the array geometry. The wake regions were neglected and the unsteady fluid forces were obtained by integrating the perturbation pressure on the tube surface, arising from the tube motion, over the region of flow attachment. A time delay was introduced to provide a force in phase with velocity, arguing that fluid inertia would prevent instantaneous changes in flow distribution in response to tube motion. The theory was later modified (Lever and Weaver, 1986a,b) and extended to include the effects of the motion of neighbouring tubes (Yetisir and Weaver, 1993a,b). Given all the simplifications, agreement with experimental results was surprisingly good, especially for in-line and parallel triangular arrays. The results suggest that the essence of fluidelastic instability is captured by this simple model. However, the results are also sensitive to the phase lag model which was more intuitive than rigorous. This is a serious deficiency in this model.

Price and Païdoussis (1984) developed a semi-empirical approach which represents a compromise between the extensive measurements required by the unsteady model of Chen (1983a) and the semi-analytical model of Lever and Weaver (1982). This and subsequent papers (Price and Païdoussis, 1986a,b; Price et al, 1990) were based on the quasi-steady assumption that the fluid forces acting on a fixed cylinder displaced from its static equilibrium position can be measured and used to represent the fluid forces on the cylinder during its motion. Assuming that the lift and drag coefficients on a cylinder can be expressed as a linear function of the displacements of the cylinder and its immediate surrounding neighbours, the measured fluid forces can be used in the model to examine the stability of the tube. These authors used various symmetry arguments to reduce the number of measurements required and constrained the relative tube oscillation modes to make the stability computations more tractable. Importantly, Price and Païdoussis introduced a time delay in the fluid coupling between various tubes and a phase shift based on a so-called flow retardation effect.

The Price and Païdoussis quasi-steady approach provided reasonable agreement with experimental observations and accounts for both the so-called fluid damping and fluid stiffness controlled instabilities. However, it shares the same significant deficiency as the Lever and Weaver semi-analytical approach, the lack of a rigorous treatment of the phase differences between fluid forces and cylinder displacement. This time lag is especially significant at low mass-damping parameter values and is essential for the negative damping mechanism.

It is worth noting that the unsteady model of Chen(1983a,b), the semi-analytical model of Lever and Weaver (1982) and the quasi-steady model of Price and Païdoussis (1984) all show multiple regions of instability and a weak dependence of the critical velocity on the mass-damping parameter at low values of this parameter. While Price (2001) discusses the evidence in support of the existence of some multiple stability boundaries, the various models predict that they go on indefinitely. Usually the multiple regions of instability have been restricted to two using physical arguments. Blevins (1977) argued that the quasi-steady theory was only valid for values of reduced velocity greater than about 10 but Price et al (1988) suggested that the limitations may be even greater for closely spaced bluff bodies. However, the unsteady and analytical models are not constrained by the quasi-steady assumption and, therefore, the unrealistic multiple instability regions cannot be blamed on the apparent limitations of the quasi-steady assumption at low reduced velocities. At low reduced velocities, O(1), the tube velocity is of the same order as the flow velocity so the relative velocity vector produces a significant component of drag force opposing a tube's transverse motion. This could explain the apparent stabilization at low reduced velocities. However, the predicted multiple regions of instability are associated with increasingly large phase angles (multiples of 2π), which cannot be physically realistic. Apparently, none of the existing models deal correctly with the dissipation of propagating disturbances. Additionally, it may be that the deficiencies of the assumed phase lag models show up more prominently at low reduced velocities.

Recognizing the problems with the intuitive phase lag models of the earlier semi-analytical and quasi-steady models, Granger and Païdoussis (1996) developed what they called a 'quasi-unsteady' model. Considering a single flexible tube in a rigid array, they determined an analytical expression for the fluid forces on the impulsively displaced tube based on satisfying continuity and the Navier Stokes equations. The motion-induced forces are in the form of a fluid added mass and an unsteady component which arises from the vorticity, generated in the tube boundary layer, diffusing and convecting away in the mean flow. Granger and Païdoussis call this a “memory effect” because it represents the decay of disturbances generated by tube motion. This transient is modelled using a linear combination of decaying exponentials, the parameters of which need to be determined experimentally. Used as an extension of the Price and Païdoussis model, this quasi-unsteady theory shows a marked quantitative improvement over the quasi-steady theory for in-line square, normal triangular and parallel triangular arrays. This agreement is remarkable in that the theory considers only a single flexible cylinder in a rigid array while most of the experimental data was from fully flexible arrays. This raises interesting questions about the relative importance of fluid coupling between cylinders at high mass-damping parameter values where the fluid stiffness mechanism is supposed to dominate. This quasi-unsteady theory also raises interesting questions about its proper fit to experimental data at low mass-damping parameter values where the authors have speculated that vortex shedding may influence the fluidelastic stability threshold. Despite the many questions raised, the Granger and Païdoussis quasi-unsteady theory represents an important step forward in understanding the mechanics of fluidelastic instability. It provides a physical explanation for the time delay necessary to create the negative damping mechanism and is based on a fundamental fluid mechanics model.

While these various theoretical models have contributed substantially to our understanding of fluidelastic instability, none of them has proven capable of predicting or explaining all of the stability behaviour characteristics observed in tube arrays. Some of the significant remaining questions are:

1. What is the relative importance of the negative damping and fluid stiffness mechanisms over the range of the mass-damping parameter and how is this influenced by tube array geometry?
2. Why does the critical reduced velocity become weakly dependent on the mass-damping parameter in the low value range where the damping mechanism is supposed to be dominant?
3. At low values of the mass-damping parameter, do multiple regions of stability exist and what is the influence, if any, of vortex shedding?

3. Concluding Remarks

While substantial progress has been made in understanding fluidelastic instability in heat exchanger tube arrays, nearly 50 years of research has failed to provide an entirely satisfactory explanation or completely reliable theoretical model of the phenomenon. Some possible reasons for this are:

1. Tube failures were a serious industrial problem which needed to be addressed. Corporations seem to have been more interested in obtaining data for their particular hardware to predict safe stability limits than in understanding the phenomena. The appearance of a simple design formula in the form of Connors’ equation and empirical data to establish design guidelines was apparently sufficient for their needs.
2. The problem has been revealed to be much more complex than originally envisaged. Early models were incorrect or too simplistic, and some later research, in an effort to make the problem more tractable, introduced overly simplified models which failed to include factors essential to a full understanding of the phenomenon. There is a huge variety of tube pitch ratios and patterns in use and, while these are ignored in some design guidelines, subtle differences can sometimes have significant effects on the stability threshold.
3. The required force measurements are very difficult. The coherent fluid dynamic forces at or near the stability threshold are small and difficult to distinguish from background noise. This is because the tube displacements are typically less than 0.01d at instability and the turbulent pressure fluctuations generated by the tube array are relatively large.

At the present time, there exist empirical design guidelines which are adequate for predicting fluidelastic instability in standard array geometries under idealized operating conditions. However, these guidelines are typically lower bounds on the data and there is considerable scatter. The published curves do not account for tube array pitch ratio, indeed, some of them do not even distinguish between array geometries. Such curves may be seriously conservative, especially at low mass-damping parameters. These guidelines cannot predict the stability of non-standard array geometries or of tubes adjacent to open tube lanes or exposed to leakage flows around the tube shell. This is not to mention the case of two-phase flow where the effects of void fraction and flow regime add significant complexities (Pettigrew and Taylor, 1993). Thus, the continued search for a complete understanding of fluidelastic instability in heat exchanger tube arrays is a difficult but very worthy undertaking.

Progress seems most likely if:
1. The model used in the study is geometrically scaled in cross-section to the prototype and consists of sufficient tubes that the flow patterns and coupling of tubes with their neighbours is included.
2. The model has the capability of including both negative damping and fluid stiffness mechanisms.
3. The various model parameters can be precisely and independently controlled so that the results are not adversely affected or misinterpreted because of unconsidered parameter variations.

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