DYNAMICS OF CANTILEVERS SUBJECTED TO INTERNAL AND/OR EXTERNAL AXIAL FLOW: NEW DEVELOPMENTS AND INSIGHTS

Michael P. Païdoussis
Department of Mechanical Engineering, McGill University
817 Sherbrooke Street W., Montréal, QC, Canada H3A 2K6
mary.fiorilli@mcgill.ca

ABSTRACT

This paper presents some new work on the dynamics of cantilevers containing and/or immersed in external axial flow. The state of understanding of the dynamics, prior to this, is first reviewed. Then, some new work on: (i) the nature of the compressive force associated with the jet issuing from the free end of the pipe; (ii) the see-saw progress in understanding the dynamics of aspirating cantilevers — from the initial assessment in 1985 that flutter occurs at infinitesimally or quite small flow velocities, to the view that it does not occur at all, then that it may occur provided that dissipation is not too high, and finally that it does appear to occur, via experiments and CFD simulations; (iii) the nonlinear dynamics of pipes conveying fluid; (iv) the dynamics of systems subjected to internal flow which, after discharge, is forced to flow axially over the exterior of the pipe, predicting flutter at very low flow velocities.

1. INTRODUCTION

It has now been firmly established that the pipe conveying fluid has become an effective paradigm for dynamics of flexible systems conveying or immersed in axial flow, and indeed for all mass- or momentum-conveying systems (Païdoussis 1987, 1998, 2004; Païdoussis and Li 1993). In a recent paper, the radiation of knowledge gained to other dynamics problems across the wider realm of Applied Mechanics is charted (Païdoussis 2008): specifically, for cylinders, plates and panels in axial flow, and cylindrical shells immersed or containing flow.

On the other side of the coin, it has been shown how some of this curiosity-driven research, conducted with no engineering application in mind, has become directly applicable and useful in engineering and physiological applications 10, 20 or 30 years later (Païdoussis 1993). Examples are pipe- and shell-type Coriolis mass-flow meters, towed flexible barges and towed seismic arrays for oil/gas exploration, ichthyoid (fish-like) propulsion systems, deep-water risers, heat exchangers and nuclear reactor internals, and for pulmonary and haemodynamics research.

In this light, it is of interest to record and assess new developments concerning the dynamics of pipes conveying fluid, and by extension cylinders immersed in axial flow, both from the fundamentals viewpoint and for potential applications. Accordingly, in this paper, some new developments and new findings in this area are reviewed. Inevitably, along with new insights, some new questions arise, requiring further research for their resolution.

As the dynamics of *cantilevered* pipes and cylinders in axial flow is of particular interest from the dynamics point of view, and also in the interests of narrowing the scope and therefore probing deeper, this paper does not deal with cylinders or pipes with both ends supported, even though some interesting new research has been conducted in this area also; see, e.g., Karagiozis et al. (2005, 2007a,b, 2008), Modarres-Sadeghi et al. (2005, 2007a, 2008a).

2. BASIC DYNAMICS

Before embarking on the discussion of new developments, it is instructive to review the basic dynamics of pipes conveying fluid. This is a non-conservative, gyroscopic system, a *circulatory system* in Ziegler's (1968) classification. It is therefore not surprising to find, both theoretically and experimentally, that at sufficiently high flow velocity, the system loses stability by flutter, specifically by single-mode flutter. In greater detail, the dynamical behaviour is as follows: (i) as the flow velocity U is increased, the cantilever is subjected to increasing flow-induced damping; (ii) at higher U this damping begins to fade, eventually becoming negative; (iii) at sufficiently high U, flow-induced negative damping exceeds the positive structural one, and thus amplified oscillations (flutter) ensue.

From the nonlinear perspective, the loss of stability is via a Hopf bifurcation, subcritical or supercritical (Bajaj et al. 1980), and the flutter can be two-dimensional (planar) or three-dimensional (orbital) (Bajaj and Sethna 1984; Modarres-Sadeghi et al. 2008b). For the plain system, unadorned by additional springs or masses, this is a solitary bifurcation; with increasing flow, the amplitude of the flutter increases and the modal content in terms of cantilever-beam modes is continually enriched. Additional bifurcations do occur for the system embellished by springs and masses (just as the proverbial one, embellished by bells and whistles), and the dynamics becomes more complex and more interesting, as we shall briefly discuss in the following.

The mechanism underlying the generation of flutter was first elucidated by Benjamin (1961a) and elaborated upon by Gregory and Païdoussis (1966a); see Païdoussis (1998). In a putative cycle of periodic oscillation, the work done by the fluid on the pipe is

$$\Delta W = -M\,U\int_0^T [\dot{w}_L^2 + U\,\dot{w}_L\,w_L']\,dt, \qquad (1)$$

where M is the mass per unit length of the conveyed fluid, T is the period, $\dot{w}_L$ is the velocity of the free end (at $x = L$), and w_L' is the slope of the free end relative to the stretched-straight, undeformed equilibrium configuration. Clearly, when U is sufficiently small, the first term in the bracketed expression is dominant, and $\Delta W < 0$; i.e. any initial oscillation is damped out, assuming the flow is the only energy source. If U is sufficiently high, however, and the product $\dot{w}_L\,w_L'$ negative on the average, then ΔW may become positive, leading to flutter.

For reference, the simplest form of the equation of motion is

$$EI\,\frac{\partial^4 w}{\partial x^4} + M\,U^2\,\frac{\partial^2 w}{\partial x^2} + 2\,M\,U\,\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0, \qquad (2)$$

where EI is the flexural rigidity and m the mass of the pipe per unit length. Physically, the small-U damping regime is associated with the dominance of the Coriolis force, $2\,M\,U\,(\partial^2 w/\partial x \partial t)$ in the equation of motion, which can be viewed as $2\,M\,U\dot{\phi}$, with $\phi = \partial w/\partial x$; this same term gives rise to the first term in the brackets of (1). The second term is associated with the centrifugal force $M\,U^2(\partial^2 w/\partial x^2)$ in the equation of motion, which can be viewed $M\,U^2/R$, with R the local radius of curvature; an alternative interpretation is that this is a compressive force $-T(\partial^2 w/\partial x^2)$, where T would be a tension and $-T$ a compression.

3. THE COMPRESSIVE FORCE $M\,U^2\,(\partial^2 w/\partial\,x^2)$

Physically, it is clear that, once U is high enough for the centrifugal/compressive force to overcome the flexural restoring forces (a situation that, if both ends of the pipe were supported, would give rise to divergence (buckling)), local bending would be increased; in the absence of a downstream support, this leads to a lateral movement of the deformed pipe, resisted by the Coriolis force as the angular velocity increases. The balance of these two tendencies (to exaggerate bending and hence lateral motion, and to resist its further increase) is at the core of the underlying mechanism for flutter.

However, although the Coriolis force is conceptually easy to accept, it is more difficult to conceive that fluid flow through the pipe can generate a compressive force; (indeed, it would be more intuitive to think that the flow generates a tensile force). To demonstrate its existence, a simple experiment was mounted (Rinaldi and Païdoussis 2007), in which a vertical cantilevered pipe conveying fluid was fitted with a special end-piece. This end-piece could either (i) allow the flow to go straight through or (ii) block it and force it out of several radial holes (Fig. 1). In arrangement (i), the system fluttered at a value of U slightly different from that without the end-piece. In arrangement (ii),

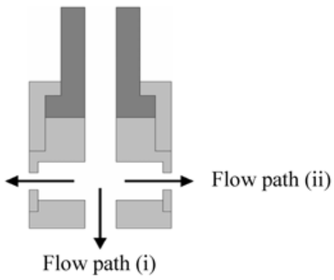

Figure 1: *The two possible flow paths at the end of the cantilever; when one is open, the other is blocked.*

however, the system remained stable, in its original stretched-straight equilibrium. Clearly, by momentum considerations, the fluid jet emerging from the free end and impacting on the blocked axial exit of the end-piece, generates a tensile force equal to MU^2. Hence, in the equation of motion we now have two terms, the compressive/centrifugal force $MU^2(\partial^2 w/\partial x^2)$ and the new tensile force $-MU^2(\partial^2 w/\partial x^2)$, cancelling each other out.

However, beyond this being a neat and quaint little experiment, it throws light into the mechanism of flutter that will be useful in what follows: the compressive force under discussion is the motive force behind the generation of flutter.

A parenthesis here on follower-force flutter of a cantilever (Beck's problem) is useful. It is recalled that a cantilever subjected to a compressive force remaining tangential to the free end (a "follower force") loses stability by flutter. To demonstrate experimentally its existence, the author once said that it would require "a rocket engine mounted on the free end of a beam column, or something similar!" (Païdoussis 1986), implying that this would be all but practically impossible. Yet, shortly after, Sugiyama et al. (1990) did just that! In the case of Beck's column, involving no Coriolis forces, the flutter is a two-degree-of-freedom coupled mode flutter, but flutter nonetheless, induced by the tangential compressive force. The action of the compressive force of the discharging jet on the cantilevered pipe is quite similar.

4. ASPIRATING PIPES

4.1. Background and status till 2005

In some areas of the Pacific Ocean, the sea floor is strewn with valuable minerals, such as manganese nodules. Inevitably, someone had the bright idea that it would be simpler to mine these minerals via a gigantic vacuum cleaner at sea than to dig them up on land (Fig. 2); refer, e.g. to Chung (1996), Deepak et al. (2001) and Xia et al. (2004).

It occurred to the author that if the "Miner" in Fig. 2 hit a ridge, the system would become temporarily a cantilevered pipe with an end-mass; the question that sprung to mind was: would it then flutter, just as a cantilever discharging fluid would? The first analysis on the aspirating cantilever was conducted by Païdoussis and Luu (1985), simply replacing U by $-U$ in the equation of motion and taking gravity and the end-mass into account. The intriguing result was obtained that the dynamics of the aspirating cantilever (with $-U$) was the mirror-image of that of the discharging one (with $+U$). Thus, in the absence of dissipation, the system would flutter at infinitesimal $|U|$, and regain stability at exactly the same value of $|U|$ at which the discharging cantilever would develop flutter!

This prediction was viewed with considerable skepticism by the authors themselves, if not others, and an experiment was mounted in 1986 in an apparatus shown diagrammatically in Fig. 3, involving an elastomer pipe in a reservoir (tank) such that water supplied to the sealed tank would enter the pipe at the free end and exit at the support — effectively being aspirated. As the flow velocity was increased to the maximum possible, the cantilever remained disconcertingly inert. It was reasoned that either the theory was incorrect or the damping associated with oscillation of the pipe in the surrounding water was strong enough to raise

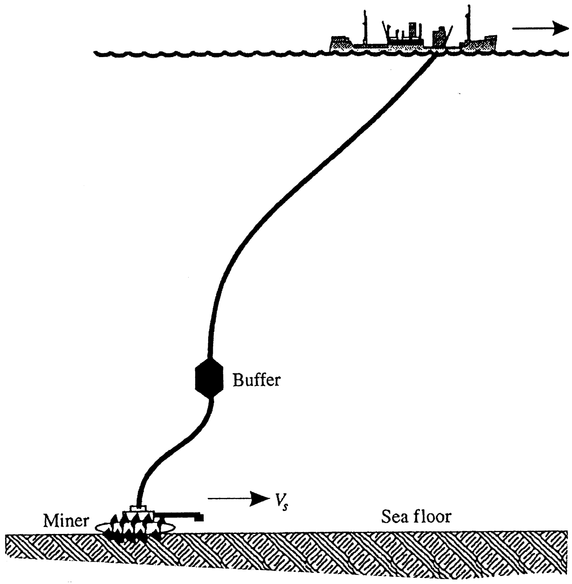

Figure 2: *The ocean-mining system, involving an aspirating pipe.*

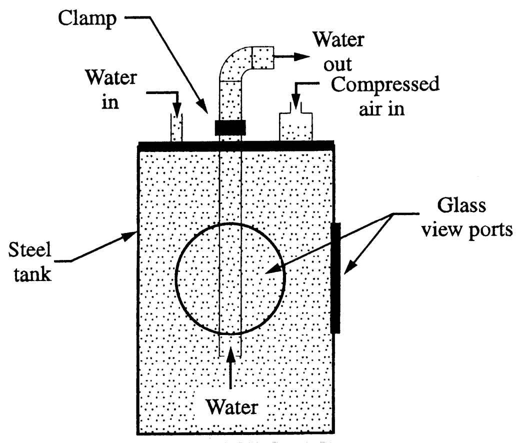

Figure 3: *The apparatus used in 1986 for experiments with an "aspirating pipe".*

the threshold of instability to unattainable levels. Part of the problem was that the pipe collapsed as a shell (in the 2nd circumferential mode) near the upstream end when the flow was high, because of the large inwards intramural pressure at that point. More experiments were planned with the upper part of the pipe in compressed air, but then "The Accident" intervened (Païdoussis 1999): the system was left under pressure overnight and a hose-clamp-secured joint slipped in the small hours: the water shot up to the ceiling and then showered all over the assembled instrumentation surrounding the experiment; some of it was irretrievably ruined, and the experiment was abandoned in disgust. The apparatus used and the accident are remarkably similar to those related to Richard Feynman's "sprinkler problem" (Gleick 1992).†

Feynman used to amuse his physicist colleagues at the Institute for Advanced Study at Princeton by asking whether a rotary sprinkler as shown in Fig. 4(a), would rotate in the same sense when it is *aspirating* fluid as in Fig 4(b), or in the opposite sense. Feynman could apparently argue convincingly either way. Eventually, Feynman decided to do an experiment, which was remarkably similar to the author's. He immersed the lawn sprinkler into a glass jar filled with water, with an outlet connected to the sprinkler and a compressed air supply into the jar, to force the water into the sprinkler and out. With increasing pressure and flow, the sprinkler refused to budge, up to the point where the glass jar exploded, spraying water all over. The result was that Feynman was banished from the laboratory henceforth!

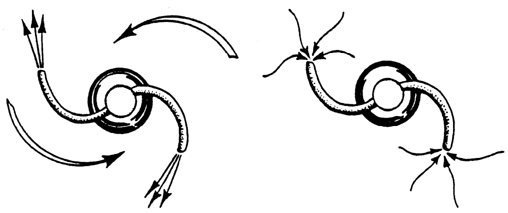

Figure 4: (a) *The discharging rotary sprinkler;* (b) *the aspirating sprinkler.*

Returning the aspirating pipe, clearly, we have a paradox. Theory predicted that the aspirating pipe loses stability for infinitesimal (or very small) flow velocity, but experiments showed the system to remain stable, at least to the maximum attainable flow prior to pipe collapse. Hence, reversing the flow direction in the experiments did not invert the stability behaviour of the pipe. Similarly, in Feynman's sprinkler, reversing the flow direction did not reverse (nor replicate) the direction of rotation.

That remained the status of this paradox till 1999, when a paper entitled "Aspirating pipes do not flutter at infinitesimally small flow" was published (Païdoussis 1999). The main thesis behind this assertion was that in the aspirating case there is a depressurization at inlet, because the intake flow is not a reversed jet-flow, but something like a sink. If the fluid in the pipe is uniformly pressurized to a pressure $\bar{p}$, the equation of motion becomes

$$EI\frac{\partial^4 w}{\partial x^4}+(\bar{p}A+MU^2)\frac{\partial^2 w}{\partial x^2} -2MU\frac{\partial^2 w}{\partial x\partial t}+(M+m)\frac{\partial^2 w}{\partial t^2}=0, \quad (3)$$

where A is the internal cross-sectional area of the pipe. If there is a depressurization at inlet $(\bar{p}<0)$, this would apply throughout the pipe (over and above the friction-induced distributed pressure loss which is totally counterbalanced by the induced traction; see (Païdoussis (1998)). It was estimated that for pure sink flow, $\bar{p}A=-\rho AU^2=-MU^2$, and hence the compressive/centrifugal term in (3) disappears. Perhaps influenced by the fact that this is the motive force for flutter (Section 3), it was concluded, without the benefit of any calculations, that flutter cannot occur.

In the same way, for the sprinkler, it was argued that the centrifugal force MU^2R on the curved pipe would be exactly cancelled by the depressurization force $\bar{p}A/R$, with $\bar{p}A=-MU^2$.

As pointed out by Kuiper and Metrikine (2005), however, the conclusion that the aspirating pipe is immune to flutter could be erroneous for two reasons: the Coriolis force was "conveniently" left out of the picture, and the depressurization may have been overestimated. Using stream-tube/ Bernoulli-equation arguments, they found $\bar{p}A\simeq -\frac{1}{2}MU^2$, even though conceding that $-MU^2<\bar{p}A<-\frac{1}{2}MU^2$.

This forced a reappraisal of the dynamics of the aspirating system by Païdoussis et al. (2005), to be discussed in the next section.

Suffice it to say here that, indeed,

$$\varDelta W=\int_0^T[MU\dot{w}_L^2-(MU^2+\bar{p}A)\dot{w}_L w_L']dt, \quad (4)$$

where U has been replaced by $-U$ *vis-à-vis*

† The author is grateful to Dr David J. Maull, who made him aware of this analogous problem, while on a visit to Cambridge in 1995.

equation (1); hence, in principle, flutter would be possible even if $MU^2 + \bar{p}A = 0,$ as a result of $\bar{p} < 0.$ But $\Delta W > 0$ could still arise via the first term. This would suggest that the nonobservance of flutter in the experiment was likely due to the high damping associated with the surrounding water.

4.2. Reappraisal of the analytical model

With the benefit of the doubts raised by Kuiper and Metrikine and some useful insights by Pramila (1992), the analytical model was re-evaluated (Païdoussis et al. 2005) by making three key assumptions, as follows.

(i) The mean flow velocity of the fluid just facing the inlet is $-v,$ where $0 < |v| < \left|\tfrac{1}{2}U\right|$ — with $|v| = 0$ as in Païdoussis (1999), and $|v| = \left|\tfrac{1}{2}U\right|$ according to Kuiper and Metrikine (2005); introducing $v/U = \alpha,$ we have $0 < \alpha < \tfrac{1}{2}.$

(ii) There is a sudden change in the fluid velocity as the fluid enters the pipe: in the direction tangential to the pipe centreline (the ξ-direction in Fig. 5) from $-v$ to $-U$ and in the transverse z-direction from $-v\sin\chi \equiv -v\,w_L'$ to $\dot{w}_L - Uw_L'$ if the fluid enters the pipe tangentially (Fig. 6(b)). The axial component gives rise to a depressurization $\bar{p}A = -(1-\alpha)MU^2,$ where $\alpha = v/U.$ The transverse component gives rise to a shear force at the free end $EI\,w_L''' - MU\,\dot{w}_L = 0.$ The unrealistic scenario where the flow vector $-v$ remains unchanged in the equilibrium direction was also considered (Fig. 6(a)), giving $EI\,w_L''' - MU(\dot{w}_L - \alpha U\,w_L') = 0.$ Hence, if we consider the intake to be not quite tangential but to lag

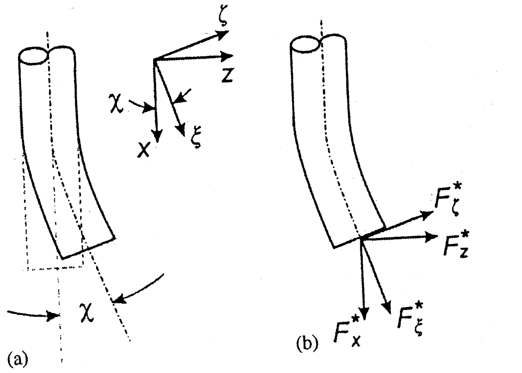

Figure 5: (a) *Free end of the pipe and definition of coordinates and the angle χ;* (b) *definition of the forces exerted by the fluid at the free end of the pipe*

slightly behind the motion, we may write the shear as

$$EI\,w_L''' - MU\,(\dot{w}_L - (1-\delta_s)\alpha U\,w_L') = 0, \qquad (5)$$

where $0 < \delta_s < 1;$ as we shall see, δ_s is closer to 1 than to 0.

(iii) There is an additional non-negligible tension induced by the flow near the free end of the pipe, on the pipe lips, such that $(T - pA)_L = (1-\alpha)(1+\bar{\gamma})MU^2,$ with $\bar{\gamma}$ being of $\mathcal{O}(1)$.

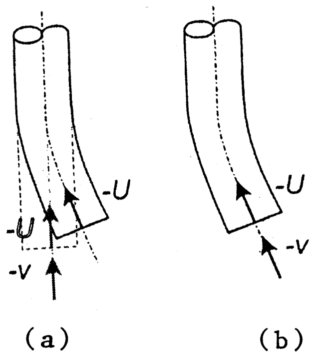

Figure 6: (a) *The unrealistic intake flow (vertical before entry);* (b) *tangential, follower entry of the flow into the pipe.*

Hence, taking into account the added mass of the surrounding fluid, M_a per unit length, introducing a viscous damping with coefficient c and viscoelastic damping in the pipe (with coefficient α^*), and incorporating the shear force via a Dirac delta function $\delta(x-L),$ the equation of motion becomes

$$\begin{aligned} EI\left(1+\alpha^*\frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4} &+ \left[1-(1-\alpha)(1+\bar{\gamma})\right]MU^2\frac{\partial^2 w}{\partial x^2} \\ &- 2MU\frac{\partial^2 w}{\partial x \partial t} + c\frac{\partial w}{\partial t} + (M+m+M_a)\frac{\partial^2 w}{\partial t^2} \qquad (6) \\ &+ MU\left[\frac{\partial w}{\partial t} - (1-\delta_s)\alpha U\frac{\partial w}{\partial x}\right]\delta(x-L) = 0. \end{aligned}$$

Some calculations were conducted with this model, with the following typical results:

(a) if $\delta_s = 0,$ the unrealistic "vertical" flow entry condition of Fig. 6(a), the system remains stable for flutter irrespective of α, though it could develop divergence, but at high flow velocities;

(b) for $\delta_s = 1$ (tangential entry) and $c = \alpha^* = 0,$ the system generally develops flutter;

(c) for $\delta_s = 1$ and reasonable values of c and α^*, substantial flow velocities are needed for flutter to develop.

Hence, the overall conclusion is that (i) flutter is possible, but (ii) whether or for what flow velocity it occurs depends on the details of the intake flow, specifically on the parameters α and δ_s, and also (iii) on the amount of dissipation in the pipe and from the surrounding fluid. It was concluded that the best way to assess these would be via CFD simulations, particularly in the vicinity of the intake, and work started on this at McGill in 2005, as discussed in Section 4.4.

4.3. Experiments

Recently, further applications of aspirating pipes have emerged, other than ocean mining of minerals, and hence the definitive answer as to whether aspirating pipes flutter or not at reasonably low flow rates has assumed a degree of urgency. Specifically, long pipes aspirating cold water are part of the design for liquifying natural gas on a ship in the vicinity of the undersea production well, rather than transporting it to land and liquifying it there. Also, some considerable interest has been shown in the exploitation of the extremely important gas hydrate deposits (e.g. methane crystal deposits) in the Arctic and near-Arctic waters.

Kuiper (2008) has conducted some experiments involving very long pipes aspirating water; the lower part of the pipe was immersed in the water, while the upper part was in air, in an effort to reduce dissipation and to obtain flutter, if it exists, within the range of flows attainable. The observations were rather curious. At a difficult-to-define precisely critical flow, orbital periodic oscillations of rather small amplitude were observed, which gradually declined, being succeeded by a quasi-stationary state displaying very small, chaotic-looking vibrations; and then back to periodic oscillations, and so on. The amplitude A of the periodic oscillations was of the order of 0.1 m, for a pipe with diameter D = 0.075 m and length L = 4.75 m; thus A/D ~ 1.3 and A/L ~ 0.02.

This, together with the alternating, quasi-bistable nature of the oscillation created doubts as to it being true flutter; e.g., supposing it could be related to a circulatory flow generated in the reservoir feeding the flow to the pipe. All possible "improvements", however, did not eliminate the oscillation, though some succeeded in reducing its amplitude further.

Using a modified form of the apparatus in which "The Accident" occurred in 1986, new experiments were conducted at McGill (starting in 2007) using entirely air-flow (Fig. 7), the advantage being that dissipation with the surrounding fluid is reduced very substantially. The lower pressure loss in the pipe also eliminated the collapse near the support. Experiments were conducted with elastomer pipes $(D_o \simeq 15.9\,\text{mm},\ L \simeq 424\,\text{mm})$, in one case fitted with a central blade which confined motions to a plane. The observed behaviour was remarkably similar to Kuiper's: beyond a critical flow velocity the pipe started oscillating in an orbital path, sometimes clockwise, sometimes counterclockwise, at small frequency (~ 1 Hz). The oscillation was interrupted by phases of smaller-amplitude shuddering quasi-chaotic motions. The amplitude was quite small $(A/D_o \simeq 0.012$ to 1.2, depending on the flow velocity; or, at most, $A/L = 0.044)$, much smaller than when the flow was discharging $(A/L > 0.25)$. When motions were 2-D, the amplitude was considerably smaller, but the intervals of "shuddering" less pronounced.

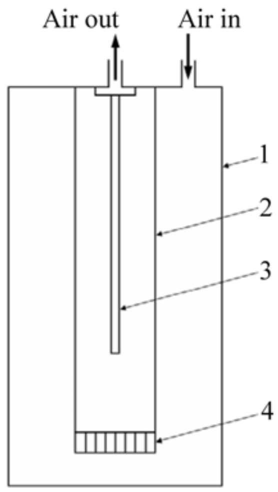

Figure 7: *Modified form of the apparatus of Fig. 3: 1. outer steel tank; 2. plexiglas "protector" to ensure no disturbance from the fluid flowing in; 3. elastomer cantilevered pipe; 4. honeycomb to regularize the flow to pipe intake.*

Taking these two experiments together, along with the analytical findings summarized in the last paragraph of Section 4.2, it would appear that the aspirating system can, indeed, develop flutter at sufficiently high flows, provided that dissipation is kept rather small. Nevertheless, in view of the several assumptions made in the analytical model and the peculiarity and smallness of the observed oscillations, corroboration by numerical simulation would obviously be highly desirable.

4.4. CFD study of the aspirating pipe

This study began in 2005 at McGill and, although not pursued full-time, began yielding useful results only in 2007. The pipe was modelled in 2-D in water via the ANSYS and FLUENT software. Later, the more powerful CFX was used instead. It should be said at the outset that the problem of flow-induced instability, whether by discharging or aspirating flow, proved to be much more resistant to CFD analysis, as compared, say, to cross-flow-related VIV (vortex-induced vibrations).

The initial numerical experiments aimed at studying the flow at the intake of an aspirating pipe, with the aim of leading to reasonable estimates of the parameters α, $\bar{\gamma}$ and δ_s in equation (6). For this purpose a lateral or pendular oscillation of varying amplitude and frequency was imposed on the pipe. It was found that δ_s was very nearly 1, so that the intake was nearly tangential, while the values of α and $\bar{\gamma}$ used to obtain the results in Section 4.2 are in the correct range.

A typical figure showing a flow-velocity vector plot for an oscillating, aspirating cantilever is shown in Fig. 8.

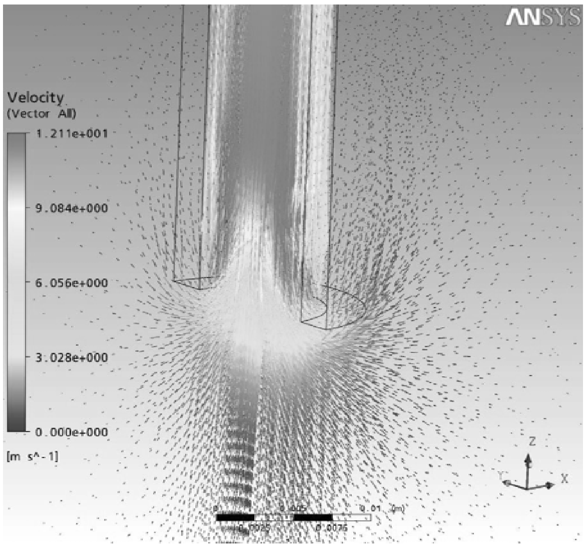

Figure 8: *Sample flow-velocity vector plot at intake of aspirating cantilever, obtained with ANSYS and CFX.*

The CFD analysis was pursued on the much more difficult question of whether self-excited oscillations are possible (Giacobbi 2007). The very fine grid necessary to model the flow near the intake, on the one hand, and the relatively large deformations of the fluid finite elements near the free end created problems; also, the slowness of the calculations (involving days of running time on a Pentium 4, dual-core computer with 2 GHz processor) militated against getting an easy answer. However, some preliminary results have recently been confirmed and the answer is that *self-excited oscillation, i.e. flutter, does occur* at flow velocities near those observed experimentally.

These results will not be elaborated here, as a more complete account will be given shortly elsewhere (Giacobbi et al., 2008).

5. NONLINEAR DYNAMICS OF DISCHARGING CANTILEVERS

A considerable amount of work has been conducted on this topic recently which is worth mentioning, though space limitations preclude a detailed description.

First, the nonlinear equations for 3-D motion were derived (Wadham-Gagnon et al. 2007), correct to $\mathcal{O}(\varepsilon^3)$. Then the dynamics of the system in the presence of an additional support, made up of an array of springs, typically at 0.6 or 0.75*L*, was considered (Païdoussis et al. 2007). Typical dynamical behaviour, in some cases supported by experiments, is as follows: the system loses stability by planar flutter, and thereafter performs two-dimensional (2-D) or 3-D periodic, quasiperiodic and chaotic oscillations; in other cases, the system loses stability by divergence, followed at higher flows by oscillations in the plane of divergence or perpendicular to it, again periodic, quasiperiodic or chaotic. See, e.g., Fig. 9.

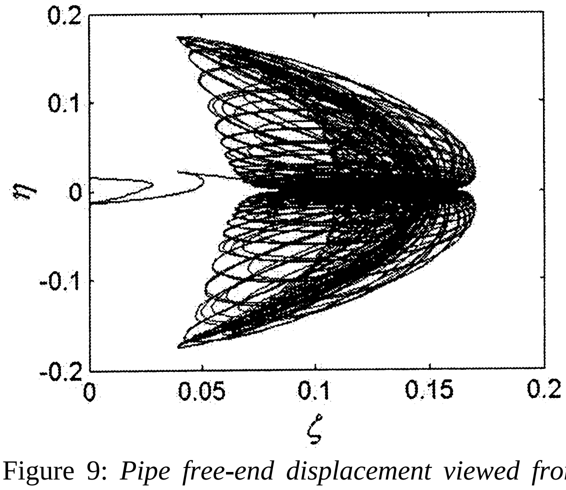

Figure 9: *Pipe free-end displacement viewed from above for quasiperiodic oscillations of a cantilevered pipe with additional spring support.*

The dynamics in the presence of an end-mass, rather than a spring support, was also revisited (Modarres-Sadeghi et al. 2007b). Typically, in one case, a sequence of periodic, quasiperiodic and chaotic oscillations, followed by 3-D quasiperiodic and chaotic motions, was observed. See, e.g., Fig. 10.

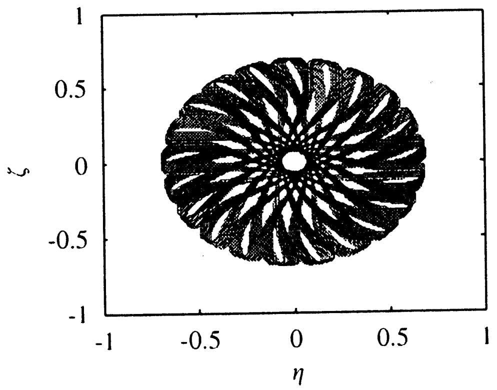

Figure 10: *Pipe free-end displacement as in Fig. 9 for quasiperiodic oscillations of a cantilevered pipe with an end-mass.*

For both variants of the system, it is clear that the post-critical dynamics of the system (the dynamics beyond the threshold of the first instability) is very rich: 2-D and 3-D oscillatory motions, periodic, quasiperiodic or chaotic, following loss of stability by static divergence or flutter. Moreover, the bifurcations beyond the first one are often associated with distinctive changes in the modal form, amplitude and/or frequency of the motion. It is also clear that further and more systematic exploration of the parameter space would yield rich rewards in terms of interesting dynamical behaviour.

6. CANTILEVERS WITH INTERNAL AND EXTERNAL FLOW

A considerable amount of work was conducted on this topic; see, e.g. Hannoyer and Païdoussis (1978), Païdoussis and Besançon (1983), Wang and Bloom (1999). Mostly, the internal and external flows were considered to be independent of each other. Recently, however, a case where the outer flow is actually the reversed inner flow has been studied. This problem (Fig. 11(a)), an idealized model for a drilling system with a floating drill-bit (Fig. 11(b)), was initially tackled by Luu (1983).

It was later re-studied as a fundamental problem involving *a reverse external axial flow*, which could also find applications in MEMS/nanotechnology as a system in which the damping could be controllably small by approaching the flutter threshold, advantageously lying at rather low flow velocities (Païdoussis et al. 2008).

6.1 Cantilever subjected to external axial flow

The equations of motion of a cantilever in axial flow, in its simplest form, may be written as

$$EI\frac{\partial^4 w}{\partial x^4} + MU^2\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + F_v + (M+m)\frac{\partial^2 w}{\partial t^2} = 0, \qquad (7)$$

where M is the virtual (or added) mass of the fluid per unit length associated with lateral movements of the cylinder, and F_v stands for terms associated with transverse and longitudinal *viscous* forces and a base drag acting on the cylinder per unit length — not given explicitly here for simplicity (Païdoussis 2004). Thus, the equation of motion is quite similar to (2) for internal flow, apart from the viscous terms, though the meaning of M is quite different.

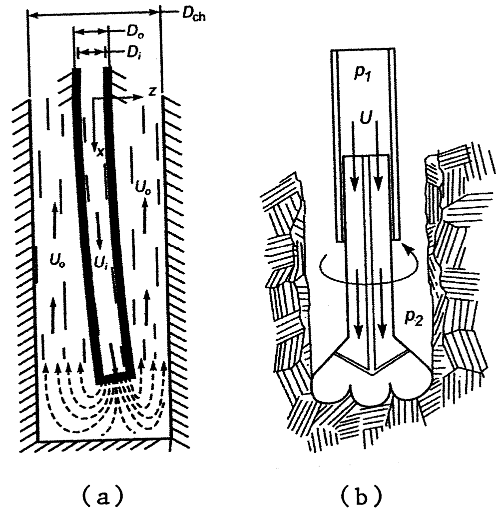

Figure 11: (a) *The cantilevered pipe conveying fluid which, after discharge, reverses direction and flows over the pipe on the outside in the annulus.* (b) *diagram of a drill-string with floating drill-bit (from Den Hartog 1969).*

The boundary conditions are the same — particularly zero shear and bending moment at the free end — if the cantilever is cut "square" at $x = L$, i.e. if the free end is very blunt. If, on the other hand, there is a tapered, streamlined downstream end, there is a shear force at $x = L$, which in its simplest form may be written as

$$-EI\frac{\partial^3 w}{\partial x^3} - fMU\left(\frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x}\right) + (fM+m)\,x_e\frac{\partial^2 w}{\partial t^2} = 0, \qquad (8)$$

where $x_e = (1/A)\int_{L-l}^{L} A(x)\,dx$, l being the length of the tapered end, and f is a slenderness/streamlining parameter for the tapered end; if it is well streamlined, $f \to 1$; if it is blunt, then $f \to 0$.

The nonlinear dynamics of this problem has recently been studied in a three-part paper (Païdoussis et al. 2002; Lopes et al. 2002; Semler et al. 2002), re-examining along the way the linear and physical dynamics which had been studied before, by Païdoussis (1966a,b, 1973).

The linear dynamics of this system is as follows: the system with a tapered free end generally loses stability by *divergence* (buckling) and at higher flows develops flutter. If the end is blunt, however, divergence does not occur, similarly to a cantilevered pipe conveying fluid, precisely because of the absence of the tapered end. Unlike the pipe, however, which is "naturally" blunt-ended, a blunt cantilever in external flow is immune to flutter also, but for different reasons; in this case, though the lift force is high, there is also a base drag associated with the blunt end, which results in suppressing flutter (Païdoussis et al. 2002).

More specifically, the divergence may be understood via the time-independent form of (8),

$$EI\,\frac{\partial^3 w}{\partial x^3} + fM\,U^2\,\frac{\partial w}{\partial x} = 0. \qquad (9)$$

There is a lift generated at the tapered end, akin to the lift on a delta wing (Triantafyllou 1998). If $f = 0$, however, this lift vanishes, and with it the divergence.

Similarly, the work done by the fluid on the cylinder in a putative cycle of oscillation is

$$\begin{aligned}\Delta W = &-(1-f)\,M\,U\int_0^T [\dot{w}^2 + U\,\dot{w}\,w']_L \\ &+ \tfrac{1}{2}\,M\,U^2\,c_b\int_0^T [\dot{w}\,w']_L \\ &+ [\text{work due to frictional terms}].\end{aligned} \qquad (10)$$

For a streamlined end, most of the energy input is due to the first, inviscid term. For a blunt end $(f = 0,\, c_b \geq 1)$, the second term is sufficiently large to cancel the large energy input due to the first term; note the difference in sign of the $M\,U^2[\dot{w}\,w']_L$ terms in the first and second terms of equation (10). [Interestingly, for a *highly* streamlined end $(f = 1,\, c_b = 0)$, all the energy input comes from the frictional terms!]

6.2 Dynamics of the reversing flow system

The discussion in Section 6.2 was for external axial flow, from the clamped end towards the free end of the cantilever. In the case of Fig. 11(a), however, the external axial flow is from the free end towards the clamped one.

The dynamics of the system in Fig. 11(a) proved to be quite interesting. The behaviour with increasing flow could be of two distinct types.

(a) For a wide enough annular channel, the external up-flow velocity, U_o, is much smaller than that of the internal one, U_i. Accordingly, the dynamics of the system is controlled by the internal flow. For relatively small flow velocities, therefore, the system is increasingly damped as U_i is increased; see (i) in the first paragraph of Section 2 (though eventually of course, the trend would be reversed, as described in the foregoing).

(b) For a narrow enough annular passage, $U_o > U_i$, and the dynamics is controlled by the external flow, the effect of which is destabilizing, in a similar way as for the aspirating pipe, and the system is subject to increasing *negative* damping as the flow velocity is increased.

In this case, for a small enough U_i,

$$\begin{aligned}\Delta W \approx &-M_i\,U_i\int_0^T (\dot{w}^2)_L \\ &+ M_o\,U_o\int_0^T [\dot{w}_L^2 - U_o\,\dot{w}_L\,w_L']\,dt,\end{aligned} \qquad (11)$$

which, clearly, can be positive. Indeed, for an elastomer tube with D_o = 15.7 mm, D_i = 9.525 mm, L = 443 mm in a cylindrical channel of diameter D_{ch} such that D_{ch}/D_o = 1.2, with water flow, the system loses stability at a dimensionless flow velocity

$u_i = (M_i/EI)^{1/2} U_i\,L = 0.25$

which corresponds to $U_i = 0.18$ m/s only.

Thus, theoretically at least, one can conceive of this as a system which, when attached to another (e.g. a MEMS-type microcantilever) can be used to generate a controllably low overall damping, thus a high Q factor, and do so at relatively low flow velocities, which makes it suitable for such operations (with small power requirements).

An experimental programme has been initiated at McGill to validate the theoretical findings and explore further the practicability of such devices for biomolecular detection and AFM (atomic force microscopy).

7. CONCLUSION

The dynamics of cantilevered pipes conveying fluid and cylinders in axial flow was reviewed and some new work described, putting the stress on physical behaviour, rather than on the underlying mathematical and numerical work. Specifically, the following were presented:

(i) some work on the nature of the compressive force in cantilevered pipes discharging fluid;

(ii) the crab-walk-type progress of work on the dynamics of aspirating pipes (now this way, now that), coming full circle from the original assertion in 1985 that the system loses stability at infinitesimal flow velocities (in the absence of dissipation) to much the same conclusion in 2008;

(iii) some exciting new work on the nonlinear dynamics of pipes conveying fluid when enhanced by additional spring supports or an end-mass;

(iv) the dynamics of pipes subjected to internal flow which then reverses direction and flows axially over the exterior of the pipe.

This work, and its natural extensions across kindred areas of Applied Mechanics (Païdoussis 2008), continues yielding new challenges and new insights, as well as becoming increasingly useful for engineering applications.

Acknowledgements

The support by the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

REFERENCES

Bajaj, A.K., et al., 1980, Hopf bifurcation phenomena in tubes carrying fluid. *SIAM Journal of Applied Mathematics*, **39**: 213-230.

Bajaj, A.K., Sethna, P.R., 1984, Flow induced bifurcations to three-dimensional oscillatory motions in continuous tubes. *SIAM Journal of Applied Mathematics*, **44**: 270-286.

Benjamin, T.B., 1961, Dynamics of a system of articulated pipes conveying fluid. I. Theory. *Proceedings of the Royal Society (London)* A, **261**: 457-486.

Chung, J.S., 1996. Deep ocean mining: technologies for manganese nodules and crusts. *International Journal of Offshore and Polar Engineering*, **6**: 244-254.

Deepak, C.R., et al., 2001, Development tests on the underwater mining system using flexible riser concept. *Proceedings 4th (2001) ISOPE Ocean Mining Symposium*, pp. 94-98.

Den Hartog, J.P., 1969, John Orr memorial lecture: recent cases of mechanical vibration. *The South African Mechanical Engineer*, **19**: 53-68.

Giacobbi, D.B., 2007, A numerical and experimental study of the dynamics of aspirating cantilever pipes. Undergraduate Honours Thesis, Department of Mechanical Engineering, McGill University.

Giacobbi, D.B., et al., 2008, CFD study of a cantilevered pipe discharging or aspirating fluid. *6th Int'l Conference on Engineering Computational Technology*, Athens, Greece.

Gleick, J. 1992 *Genius*. New York: Pantheon Books.

Gregory, R.W., Païdoussis, M.P., 1966, Unstable oscillation of tubular cantilevers conveying fluid. I. Theory. *Proceedings of the Royal Society (London)* A, **293**: 512-527.

Hannoyer, M.J., Païdoussis, M.P., 1978, Instabilities of tubular beams simultaneously subjected to internal and external axial flows. *ASME Journal of Mechanical Design*, **100**: 328-336.

Karagiozis, K.N., et al., 2005, An experimental study of the nonlinear dynamics of cylindrical shells with clamped ends subjected to axial flow. *Journal of Fluids and Structures*, **20**: 801-816. Addendum: 2006, **22**: 595-596.

Karagiozis, K.N., et al., 2007a, Effect of geometry on the stability of cylindrical clamped shells subjected to internal fluid flow. *Computers and Structures*, **85**: 645-659.

Karagiozis, K.N., et al., 2007b, Transmural pressure effects on the stability of clamped cylindrical shells subjected to internal fluid flow: Theory and experiments. *International Journal of Non-Linear Mechanics*, **42**: 13-23.

Karagiozis, K.N., et al., 2008, Nonlinear stability of cylindrical shells subjected to axial flow: Theory and Experiments. *Journal of Sound and Vibration*, **309**: 637-676.

Kuiper, G.L., 2008, Stability of offshore risers conveying fluid. Ph.D. Thesis, Delft Technical University, The Netherlands.

Kuiper, G.L., Metrikine, A.V., 2005, Stability of submerged free-hanging riser. *Journal of Sound and Vibration*, **280**: 1051-1065.

Lopes, J.-L., et al., 2002, Linear and nonlinear dynamics of cantilevered cylinders in axial flow. Part 2: The equations of motion. *Journal of Fluids and Structures*, **16**: 715-737.

Luu, T.P., 1983, On the dynamics of three systems involving tubular beams conveying fluid. M.Eng. Thesis, Department of Mechanical Engineering, McGill University, Montreal, Québec, Canada.

Modarres-Sadeghi, Y., et al., 2005. A nonlinear model for an extensible slender flexible cylinder subjected to axial flow. *Journal of Fluids and Structures*, **21**: 609-627. Addendum: 2006, **22**: 597.

Modarres-Sadeghi, Y., et al., 2007a, The nonlinear behaviour of a slender flexible cylinder pinned or clamped at both ends and subjected to axial flow. *Computers and Structures*, **85**: 1121-1133.

Modarres-Sadeghi, Y., et al., 2007b, Dynamics of cantilevered pipes conveying fluid. Part 3: Three-dimensional dynamics in the presence of an end-mass. *Journal of Fluids and Structures*, **23**: 589-603.

Modarres-Sadeghi, Y., et al., 2008a, Experiments on vertical slender flexible cylinders clamped at both ends and subjected to axial flow. *Philosophical Transactions of the Royal Society* A **366**: 1275-1296.

Modarres-Sadeghi, Y., et al., 2008b, Three-dimensional oscillations of a cantilever pipe conveying fluid. *International Journal of Non-Linear Mechanics*, **43**: 18-25.

Païdoussis, M.P., 1966a, Dynamics of flexible slender cylinders in axial flow. Part 1: theory. *Journal of Fluid Mechanics*, **26**: 717-736.

Païdoussis, M.P., 1966b, Dynamics of flexible slender cylinders in axial flow. Part 2: experiments. *Journal of Fluid Mechanics*, **26**: 737-751.

Païdoussis, M.P., 1973, Dynamics of cylindrical structures subjected to axial flow. *Journal of Sound and Vibration*, **29**: 365-385.

Païdoussis, M.P., 1986, Flow-induced instabilities of cylindrical structures. In *Proceedings 10th U.S. National Congress of Applied Mechanics* (ed. J.P. Lamb), pp. 155-170.

Païdoussis, M.P., 1987, Flow-induced instabilities of cylindrical structures. *Applied Mechanics Reviews*, **40**: 163-175.

Païdoussis, M.P., 1993, *Calvin Rice Lecture*: Some curiosity-driven research in fluid-structure interactions and its current applications. *ASME Journal of Pressure Vessel Technology*, **115**: 2-14.

Païdoussis, M.P., 1998, *Fluid-Structure Interactions: Slender Structures and Axial Flow*. Volume 1. London: Academic Press.

Païdoussis, M.P., 1999, Aspirating pipes do not flutter at infinitesimally small flow. *Journal of Fluids and Structures*, **13**: 419-425.

Païdoussis, M.P., 2004, *Fluid-Structure Interactions: Slender Structures and Axial Flow*. Volume 2, London: Elsevier Academic Press.

Païdoussis, M.P., 2008, The canonical problem of the fluid-conveying pipe and radiation of the knowledge gained to other dynamics problems in Applied Mechanics. *Journal of Sound and Vibration*, **310**: 462-492.

Païdoussis, M.P., Besançon, P., 1981, Dynamics of arrays of cylinders with internal and external axial flow. *Journal of Sound and Vibration*, **76**: 361-380.

Païdoussis, M.P., Li, G.X., 1993, Pipes conveying fluid: a model dynamical problem. *Journal of Fluids and Structures*, **7**: 137-204.

Païdoussis, M.P., Luu, T.P., 1985, Dynamics of a pipe aspirating fluid, such as might be used in ocean mining. *ASME Journal of Energy Resources Technology*, **107**: 250-255.

Païdoussis, M.P., et al., 2002, Linear and nonlinear dynamics of cantilevered cylinders in axial Flow. Part 1: Physical dynamics. *Journal of Fluids and Structures*, **16**: 691-713.

Païdoussis, M.P., et al., 2007, Dynamics of cantilevered pipes conveying fluid. Part 2: Dynamics of the system with intermediate spring support. *Journal of Fluids and Structures*, **23**: 569-587.

Païdoussis, M.P., et al., 2008, Dynamics of a long tubular cantilever conveying fluid downwards, which then flows upwards around the cantilever as a confined annular flow. *Journal of Fluids and Structures*, **24**: 111-128.

Pramila, A., 1992, Undamped cantilever aspirating fluid may be stable. *Rakenteiden Mekaniikka*, **25**: 3-14.

Rinaldi, S., Païdoussis, M.P., 2007, Dynamics of a cantilevered pipe conveying fluid fitted with a stabilizing end-piece. *Proceedings CANCAM 2007*, Ryerson University, Toronto, pp. 507-508.

Semler, C., et al., 2002, Linear and nonlinear dynamics of cantilevered cylinders in axial flow. Part 3: Nonlinear dynamics. *Journal of Fluids and Structures*, **16**: 739-759.

Sugiyama, Y., et al., 1990, Experiment on flutter of cantilevered columns subjected to rocket thrust. In *Proceedings 31st AIAA Structures, Structural Dynamics & Materials Conference*, pp. 1893-1898. Paper AIAA-90-0948-CP.

Triantafyllou, M.S., 1998, Private communication.

Wadham-Gagnon, M., et al., 2007, Dynamics of cantilevered pipes conveying fluid. Part 1: Nonlinear equations of three-dimensional motion. *Journal of Fluids and Structures*, **23**: 545-567.

Wang, X., Bloom, F., 1999, Dynamics of a submerged and inclined concentric pipe system with internal and external flows. *Journal of Fluids and Structures*, **13**: 443-460.

Xia, et al., 2004, Hydraulic lifting of manganese nodules through a riser. *ASME Journal of Offshore Mechanics and Arctic Engineering*, **126**: 72-77.

Ziegler, H. 1968, *Principles of Structural Stability*. Waltham, Mass: Blaisdell.