USEFULNESS OF SENSITIVITY EQUATION METHOD IN FINDING INTER-CYLINDER COUPLINGS AND OVERVIEW OF ITS OTHER INTERESTS

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ABSTRACT

Flow-induced vibration of tube bundles remains an important problem in the nuclear industry. For several decades now, scientists have worked on developing a good theory which models well the behaviour of cylinders subjected to cross-flow. In all cases, understanding the influence of each tube on its neighbours is of primary importance. Given the difficulty and cost of measuring the coupling effects, we investigate numerical simulations as an alternative.

Sensitivity analysis is a method describing the influence of a parameter on a flow. It entails solving partial differential equations for the flow sensitivities which are obtained by differentiation of the Navier-Stokes equations. This paper investigates the potential of the SEM as a tool for determining the intercylinder coupling effects. The latter are quantified by the derivatives of lift and drag coefficients of all tubes with respect to the coordinates of the reference tube's center. The paper also presents SEM as a tool to obtain quick estimates of the flow on nearby geometry and to determine key parameters for design.

INTRODUCTION

Sensitivity analysis (Turgeon, 2001) describes the influence of a parameter on the flow. One approach to sensitivity analysis called *continuous SEM* involves solving partial differential equations for the flow sensitivities. The sensitivity equations are obtained by differentiation of the Navier-Stokes equations and solved by a Finite Element Method.

In this work, derivatives of the force coefficients have been computed for a rotated-triangle tube bundle subjected to a single-phase low Reynolds number flow. The numerical results show the expected symmetrical and anti-symmetrical behaviors of the flow that occur in this configuration.

Two other advantages of the Sensitivity Equation Method are shown here: the evaluation of flows on nearby geometries which gives, with only one numerical simulation, the flow for any nearby geometrical cylinder configurations and thus all force coefficients of the new configurations; and the determination of the key parameters for design.

The paper begins with a background in theory and the description of the problem studied. The steady-state governing equations for laminar incompressible fluids are then presented and the sensitivity equations derived. Calculations of force coefficients and their derivatives for a shape parameter follow. The numerical results are then presented. The other uses of SEM are then presented.

1. THEORETICAL BACKGROUND

Several theories for the prediction of fluidelastic instability in tube bundles have been proposed in the last twenty five years. Some are almost purely analytical requiring few experimental data. Others, less analytical, require measurement of a large number of coefficients. The quasi-static/quasi-steady model developed by (Price and Paidoussis, 1983) presents a good compromise by introducing a simplified analytical model for the velocity effects. For this reason only displacement dependent forces need to be determined experimentally.

The mathematical model is briefly explained as follows. The governing equations of motion, for a triangular tube bundle (Fig 1.) subjected to a cross-flow, is :

$$[\boldsymbol{M}]\ddot{\boldsymbol{X}} + [\boldsymbol{C}]\dot{\boldsymbol{X}} + [\boldsymbol{K}]\boldsymbol{X} = \boldsymbol{F} \quad (1)$$

where $\boldsymbol{X} = (x_C, y_C, ...y_6)$ is the cylinder displacement vector;$[\boldsymbol{M}]$,$[\boldsymbol{C}]$ and $[\boldsymbol{K}]$ are respectively the mechanical mass, damping and stiffness matrices and $\boldsymbol{F} = (F_{xC}, F_{yC}, \ldots, F_{x6}, F_{y6})$ the fluid force vector.

The fluid forces on the cylinder C may be written as :

$$\begin{Bmatrix} F_{xC} \\ F_{yC} \end{Bmatrix} = \frac{1}{2}\rho U_\infty^2 lD \left[\begin{Bmatrix} C_{DC} \\ C_{LC} \end{Bmatrix} (1 - 2\ \dot{x_C}\ D/U_G) + \begin{Bmatrix} C_{LC} \\ C_{DC} \end{Bmatrix} \dot{y_C}\ D/U_G \right] \quad (2)$$

where ρ is the fluid density, U_∞ the upstream velocity, D the tube diameter, l the tube length and U_G the gap velocity.

The drag and lift coefficients, respectively C_{DC} and C_{LC}, are based on the pitch velocity and have the following form :

$$\begin{Bmatrix} C_{LC} \\ C_{DC} \end{Bmatrix} = \begin{Bmatrix} C_{L0} \\ C_{D0} \end{Bmatrix} + \sum_{i=C,1}^{6} \left[g_i(x_i, y_i, y_C) \begin{Bmatrix} \frac{\partial C_{LC}}{\partial x_i} \\ \frac{\partial C_{DC}}{\partial x_i} \end{Bmatrix} + h_i(y_i, y_C) \begin{Bmatrix} \frac{\partial C_{LC}}{\partial y_i} \\ \frac{\partial C_{DC}}{\partial y_i} \end{Bmatrix} \right] \quad (3)$$

where C_{L0} and C_{D0} are respectively the lift and drag coefficients for the steady state, and the functions g_i and h_i represent the apparent inter-cylinder displacements including time delay effects.

For all quasi-static/steady models, the force coefficients and their derivatives are the experimental inputs. The purpose of this work is to investigate whether the sensitivity equations method could provide an approximation of these derivatives and therefore reduce the dependence on experiments. It is important to understand that sensitivities are not restricted to seven moving tubes and can be applied to more complex theories (n-tubes model).

2. GEOMETRY AND SIMULATION STRATEGY

A section consisting of 7 cylinders and 6 half-cylinders was modeled, matching the experimental geometry in the laboratory (Fig. 1).

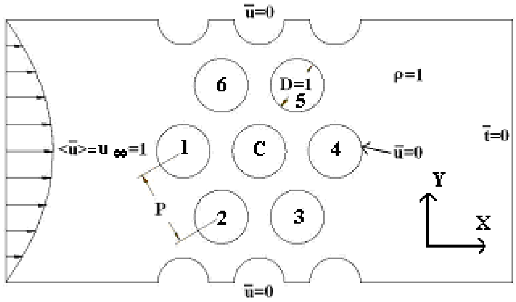

Figure 1: *Geometry of the simulation*

Experiments show that cross-flow transverse to an isolated cylinder becomes unsteady for a Reynolds number Re between 30 and 40. Vortex shedding phenomenon appears then. Similar unsteadiness occurs in tube bundles. Knowing this and in order to have a full laminar flow, the simulation is done on a rotated-triangle tube bundle with $P/D = 1.5$ and subjected to a single-phase low Reynolds number flow ($Re = 20$). For the calculation of the Reynolds number, we use the upstream velocity U_∞ and the diameter of the cylinders D.

3. GOVERNING EQUATIONS

3.1. Fluid flow equations

The steady flow of an incompressible fluid is described by the continuity and momentum Navier-Stokes equations in an Eulerian frame of reference :

$$\nabla \cdot u = 0 \quad (4)$$
$$\rho u \cdot \nabla u = \nabla \cdot \boldsymbol{\sigma} \quad (5)$$

where ρ is the density (constant here), u the fluid velocity, and $\boldsymbol{\sigma}$ the total Newtonian fluid stress tensor (pressure and viscous forces) given by :

$$\boldsymbol{\sigma} = \mu(\nabla u + (\nabla u)^T) - pI \quad (6)$$

where μ is the dynamic viscosity (constant here), and p is the fluid pressure. Dirichlet and Neumann boundary conditions are imposed on boundary segments Γ_D and Γ_N respectively :

$$u \cdot \hat{n} = \bar{u} \quad \text{on} \quad \Gamma_D \quad (7)$$
$$\boldsymbol{\sigma} \cdot \hat{n} = \bar{t} \quad \text{on} \quad \Gamma_N \quad (8)$$

3.2. Sensitivity equations

The sensitivity of the flow characteristics is obtained by differentiation of the governing equations with respect to an arbitrary parameter $\boldsymbol{a}$.

Therefore, we define the flow sensitivities as the partial derivatives of the velocity $u = (u_x, u_y)$ and the pressure p with respect to the parameter $\boldsymbol{a}$:

$$s_u^a = \frac{\partial u}{\partial a} \quad ; \quad s_p^a = \frac{\partial p}{\partial a} \tag{9}$$

The derivatives of the other variables, with respect to $\boldsymbol{a}$, are denoted by a prime. We obtain the following fluid sensitivity equations by differentiation of Eqns. (4) and (5) :

$$\nabla \cdot s_u^a = 0 \tag{10}$$
$$\rho(s_u^a \cdot \nabla u + u \cdot \nabla s_u^a) = \nabla \cdot \boldsymbol{\sigma'} \tag{11}$$

where $\boldsymbol{\sigma'}$ is the sensitivity of the Newtonian stress tensor, given as :

$$\boldsymbol{\sigma'} = \mu \left(\nabla s_u^a + (\nabla s_u^a)^T \right) - s_p^a I \tag{12}$$

The Dirichlet and Neumann boundary conditions are differentiated in the same way.

In the present work, the shape parameter a is either X_C for an inline displacement or Y_C for a transverse displacement.

3.3. Induced forces and derivatives

The drag and lift forces are obtained by projection and integration of the reaction term along the surface of the cylinder, with $\hat{n} = \{\hat{n}_x, \hat{n}_y\}^T$ as the outward normal to the cylinder and $\boldsymbol{F}$ the force vector. :

$$\boldsymbol{F} = \begin{Bmatrix} F_D \\ F_L \end{Bmatrix} = \int_{\Gamma_a} \boldsymbol{\sigma} \cdot \begin{Bmatrix} \hat{n}_x \\ \hat{n}_y \end{Bmatrix} d\Gamma \tag{13}$$

To compute stability derivatives, we proceed as follows. The parameter a being the centroïd of the circle, the material derivative has to be considered. It includes two separate contributions : the Eulerian term $\boldsymbol{\sigma'}$ and an additional so-called *transpiration term* which represents the change of frame of reference, from Eulerian to Lagrangian.

$$\frac{D}{Da} \int_{\Gamma_a} \boldsymbol{\sigma} \cdot \hat{n} d\Gamma = \int_{\Gamma_a} \Big((\boldsymbol{\sigma'} + \nabla \boldsymbol{\sigma} \cdot \Pi_a) \cdot \hat{n} \cdot J + \boldsymbol{\sigma} \cdot \frac{D\hat{n}}{Da} \cdot J + \boldsymbol{\sigma} \cdot \hat{n} \cdot \frac{DJ}{Da} \Big) d\Gamma_0 \tag{14}$$

where J is the Jacobian of $\boldsymbol{X}$. The Jacobian J and derivative $D\hat{n}/Da$ characterize the deformation of the surface. In our case, the cylinder is non-shrinkable and in translation so, respectively, $J = 1$ and $D\hat{n}/Da = 0$, so that only the first term is non zero. The *transpiration term* reduces to $\nabla \boldsymbol{\sigma} \cdot \Pi_a$, which yields the following expression for the stability derivatives (for exemple with F_L and $a = Y_C$) :

$$\frac{DF_L}{DY_C} = \int_{\Gamma_a} \left(\boldsymbol{\sigma'} + \nabla \boldsymbol{\sigma} \cdot \Pi_{Y_C} \right) \hat{n}_y d\Gamma \tag{15}$$

3.4. Normalized calculations

For the calculation of the equations we take normalized terms:

$$U = 1 \quad ; \quad \rho = 1 \quad ; \quad D = 1 \quad ; \quad \delta a = 1 \tag{16}$$

Taking for the viscosity:

$$\mu = \frac{1}{R_e} \rho U D = 0.05 \tag{17}$$

Thus the obtained sensitivities with respect to X and Y are normalized.

4. NUMERICAL RESULTS

4.1. Pressure and Velocity field for flow and sensitivities

Computations provide visualizations of the flow and its sensitivity with respect to the central cylinder coordinates. Fig. 2 shows an example of the sensitivity $\partial u/\partial Y_C$. We can see therefore the impact on the flow of the given cylinder motion through the sensitivity fields. The field of positive sensitivities being down the central cylinder and the negative one being up the cylinder, it shows that when the central cylinder goes up the velocity increases down the cylinder resulting in a negative lift force.

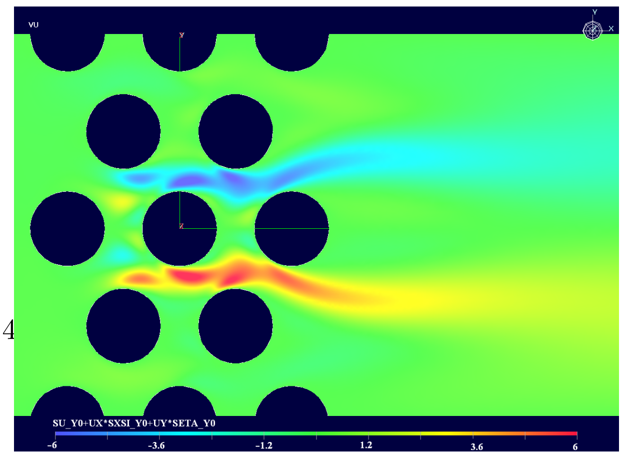

Figure 2: *Lagrangian sensitivity of the u-velocity for a Y-displacement of the central cylinder*

4.2. Force coefficients and stability derivatives

The force coefficients are defined as :

$$C_{L/D} = \frac{F_{L/D}}{\frac{1}{2}\rho D U_\infty^2} \tag{18}$$

The corresponding stability derivatives become :

$$\frac{\partial C_{L/D}}{\partial a} = \frac{1}{\frac{1}{2}\rho D U_\infty^2} \frac{\partial F_{L/D}}{\partial a} \tag{19}$$

where a may be either the inflow (X) or crossflow (Y) displacement of a cylinder. Table 1 presents force coefficients and their derivatives.

	C_L	C_D	$\frac{\partial C_L}{\partial Y_C}$	$\frac{\partial C_D}{\partial Y_C}$	$\frac{\partial C_L}{\partial X_C}$	$\frac{\partial C_D}{\partial X_C}$
C	0.0	23.2	-31.2	0.0	0.0	2.5
1	0.0	17.0	0.2	0.0	0.0	10.2
2	-0.5	22.9	2.0	6.0	10.8	-13.5
3	0.0	24.0	5.0	0.0	-2.2	9.7
4	0.0	12.2	5.7	0.0	0.0	-5.7
5	0.0	24.0	5.0	0.0	2.2	9.7
6	0.5	22.9	2.0	-6.0	-10.8	-13.5

Table 1: *force coefficients and stability derivatives of cylinders with respect to displacements Y_C and X_C of cylinder C*

The force coefficients and derivatives presented in Table 1 are the inputs needed for the quasi-static/steady theory outlined in section 1.

5. EVALUATION OF FLOWS ON NEARBY GEOMETRY

Another use of sensitivity information is to predict the flow behavior for a nearby geometry without having to perform a full flow recalculation. The procedure simply uses Taylor series in parameter space. Let $\Gamma(a_0)$ denote the reference baseline and $\Gamma(a_0 + \Delta a)$ the nearby geometry. Let Φ be any solution variable. Its value at $a_0 + \Delta a$ is obtained by linear Taylor series in a-space using the baseline values of the flow and its sensitivities :

$$\Phi(\hat{X}, a_0 + \Delta a) \approx \Phi(\hat{X}, a_0) + \Delta a \frac{D\Phi}{Da}(\hat{X}, a_0) \tag{20}$$

For the fluid variables, the material derivative becomes the Eulerian sensitivity since we are looking at a fixed point in space. The pressure, for example, is calculated as :

$$p(\hat{X}, a_0 + \Delta a) \approx p(\hat{X}, a_0) + \Delta a \cdot s_p^a \tag{21}$$

However for quantities like $C_{L/D}$ computed a parameter dependent boundary, one must use the material derivative :

$$\int_{\Gamma(a_0+\Delta a)} [\boldsymbol{\sigma} \cdot \hat{n}]_{a_0+\Delta a}\, d\Gamma = \int_{\Gamma(a_0+\Delta a)} \left[\sigma + \Delta a \frac{D\sigma}{Da}\right]_a \hat{n} d\Gamma$$

Since the cylinder is translated as a rigid body, the surface Γ keeps its original shape :

$$\Gamma(a_0 + \Delta a) = \Gamma(a_0)$$

The force coefficients for the displaced cylinder are therefore :

$$C_L(a_0 + \Delta a) = C_L(a_0) + \Delta a \cdot \frac{DC_L}{Da} \tag{22}$$

$$C_D(a_0 + \Delta a) = C_D(a_0) + \Delta a \cdot \frac{DC_D}{Da} \tag{23}$$

In order to verify the nearby solution, we computed solutions for a sequence of increasing central cylinder displacements in Y-and-X-directions. We compare the contours of the nearby solution and those of the recalculation. For a displacement of 2% of the diameter D, the Taylor approximation closely matches the solution obtained by direct calculation.(Fig. 3 and 4).

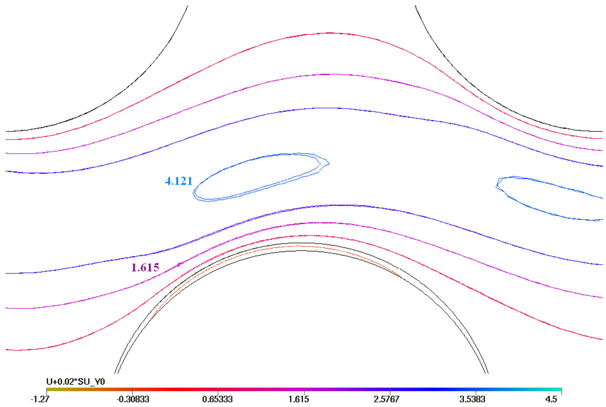

Figure 3: *Contours of u velocity for Taylor series and recalculation ($a = Y_C$)*

For a displacement of 5% of D, the superposition of contours is not as good but still provides acceptable values of the force coefficients.

Force coefficients are obtained by recalculation and by linear Taylor approximation for X- and Y-displacements ranging from 0 to 20 % of D. The

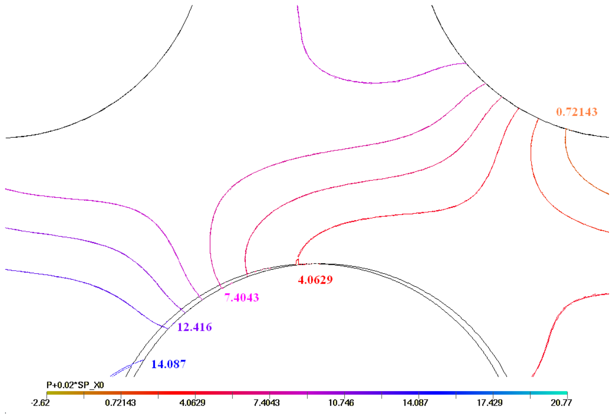

Figure 4: *Pressure Contours for Taylor series and recalculation ($a = X_C$)*

variation of C_D for a X-displacement and C_L for an Y- displacement being quasi equal to zero, the relative error is negligible. The other variations are shown on Fig. 5.

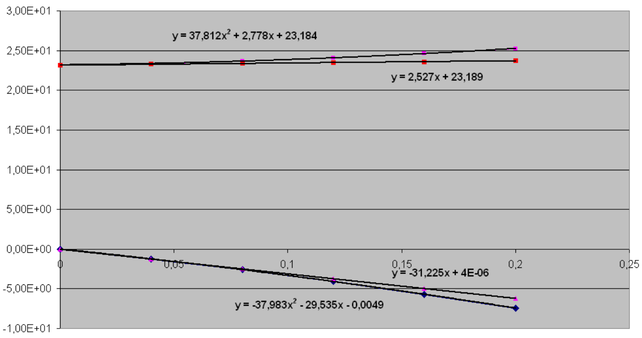

Figure 5: *Evolution of C_D for an Y displacement and C_L for a X displacement*

The relative error between the first order Taylor extrapolation and the full recalculation is given in Tables 2 and 3.

disp. Y/D	Recalc.	Taylor ser.	relative err.
4%	23.35	23.29	0.25%
8%	23.64	23.39	1%
12%	24.06	23.49	2%
16%	24.60	23.59	4%
20%	25.25	23.69	6%

Table 2: *Error in C_D for a displacement along X*

The nearby flow approximation is good enough to predict flows and force behavior for a range of perturbation of approximately 12% of D.The Taylor series yields good results for broad interval of displacements. For the Y-displacements,

disp. Y/D	Recalc.	Taylor s.	relative err.
4%	-1.26	-1.25	1%
8%	-2.60	-2.50	4%
12%	-4.08	-3.75	8%
16%	-5.72	-5.00	12%
20%	-7.43	-6.25	16%

Table 3: *Error in C_L for a displacement along Y*

the behavior is more non-linear. The Taylor series extrapolation becomes unreliable for perturbations larger than 12%, giving an error bigger than 10%. It would be then necessary to use a second order Taylor series extrapolation. Second order Taylor series extrapolation could also provide the evolution for the stability derivatives.

6. DETERMINATION OF KEY PARAMETERS FOR DESIGN AND UNCERTAINTY ANALYSIS

In any experiment it is important to determine the reliability of the measurements, which we can quantify with an uncertainty analysis. Sensitivity data provides information as to how large the uncertainty of the measurement will be. Furthermore it provides also a possibility to reduce this uncertainty during the experiment design. By analyzing values of the sensitivities, we can know how strongly a parameter perturbation will affect the flow and therefore its measurements. The uncertainty of a function Φ due to an uncertainty δa in a parameter a is:

$$\delta\Phi = \pm\delta(a) \cdot s^a_\Phi = \pm \left(\frac{\delta a}{a_0}\right)\left(\frac{\partial\Phi}{\partial(a/a_0)}\right) \quad (24)$$

a_0 being the nominal value of the parameter a. Thus, to compare the influence of the different parameters, it is important to study scaled sensitivities $s^{\tilde{a}}_\Phi$, which are calculated as follows:

$$s^{\tilde{a}}_\Phi = a_0 s^a_\Phi \quad (25)$$

In this study, we measure the induced forces on the central cylinder : $\Phi = F_D$ and $\Phi = F_L$. The different parameters that can influence the measurements are then:

- The position of the central cylinder $(s^{\tilde{Y}_F}_{F_D}, s^{\tilde{Y}_F}_{F_L}, s^{\tilde{X}_F}_{F_D}, s^{\tilde{X}_F}_{F_L})$
- The flow-rate $(s^{\tilde{U}_\infty}_{F_D}, s^{\tilde{U}_\infty}_{F_L})$

- The density ρ and viscosity μ $(s_{F_D}^{\tilde{\rho}}, s_{F_L}^{\tilde{\rho}}, s_{F_D}^{\tilde{\mu}}, s_{F_L}^{\tilde{\mu}})$
- The diameter of the cylinder $(s_{F_L}^{\tilde{D}}, s_{F_L}^{\tilde{D}})$

Note however that U_∞ , ρ , D and μ are not linearly independent ($R_e = \rho U_\infty D/\mu$). Fig.6 shows values of the scaled sensitivities listed above. Thus, if we want to measure precisely the forces on the central cylinder at its central position, only the Y position of the cylinder has to be measure very precisely for the lift force and the diameter D is the most important parameter for the drag force. Comparison of scaled sensitivities is then a useful method during pre-design but the picture could be more complete if we know the uncertainty of the key parameters. Indeed, there are of course certain parameters that are much harder to control than others. The consequences are that while the diameter has a big sensitivity but is easy to measure precisely, the flow rate might be less sensitive but is harder to measure precisely, making its contribution to the uncertainty bigger than the viscosity one. Thus, if for example, we have an uncertainty of 5% on the flow rate and an uncertainty of 0.1% for the other parameters, the contribution of each parameter on the force coefficients is shown in fig.7.

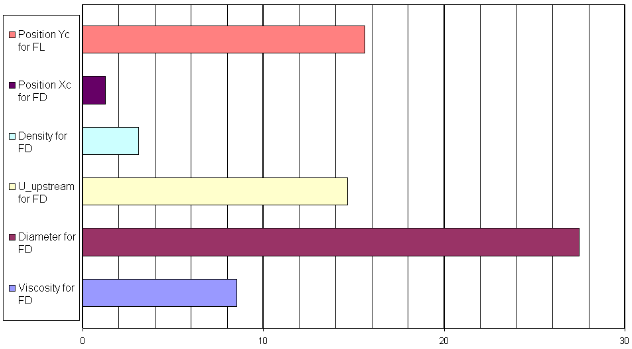

Figure 6: *Sensitivities of drag coefficients - Values relative to the nominal forces F_D or F_L*

Once the system is build and measurements are done, it is also important to do an uncertainty analysis. The total uncertainty is calculated as follows:

$$(\delta \Phi)_{tot} = \sum |\delta a_i||s^{\tilde{a_i}_\Phi}| \quad (26)$$

In this case, we would have for the drag force, an uncertainty of $(\delta \Phi)_{tot}(F_D) = \pm 1.13$ for a value of $F_D = 11.6$ and for the lift force, an uncertainty of $(\delta \Phi)_{tot}(F_L) = \pm 0.16$ for a value of $F_L = 0$.

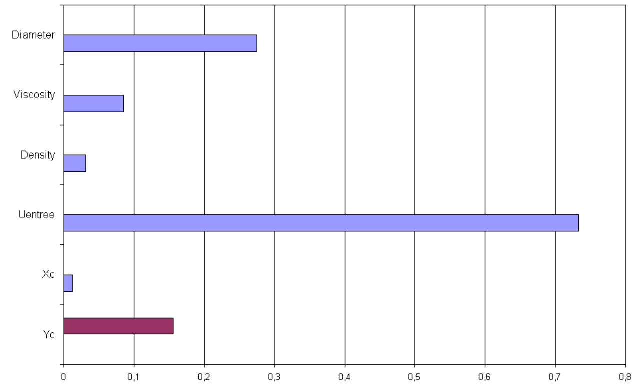

Figure 7: *Contributions of key parameters - Values relative to the nominal forces F_D or F_L*

CONCLUSION

In this paper, we have investigated the potential for sensitivities to determine stability derivatives for a tube bundles in cross-flow. The stationary Navier-Stokes equations are used to describe the flow. We have shown then that sensitivities are a potentially useful method for the calculation of the force derivatives and is powerful to compute nearby solutions and determine influential parameters for uncertainty analysis.
This method is very useful for this laminar case but to increase the Reynolds number is necessary. Unsteady flow simulations for $R_e = 100$ and $R_e = 200$ are being computed but certain fundamental difficulties due to the sensitivities behavior are faced.

7. REFERENCES

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