ROTOR INSTABILITY INDUCED BY OIL FILM AND POSSIBILITIES OF ITS SUPPRESSION THROUGH BEARING EXTERNAL EXCITATION

Šimek, J., Svoboda, R.
TECHLAB Ltd., Prague, Czech Republic

Tůma, J.
VŠB TU Ostrava, Czech Republic

ABSTRACT

The paper deals with rotor instability, which is encountered with rotors supported in sliding bearings. Possibilities to suppress instability by external excitation will be studied both theoretically and experimentally. The test stand enabling bearing bushing excitation by piezoactuators was designed and manufactured. Theoretical study of kinematic excitation influence on rotor behavior was carried out and some calculated results are shown. Strategy of further proceedings is outlined.

1. INTRODUCTION

With increasing speed of rotating machines problems with instability of rotor became more and more serious. The instability termed as "oil whirl" is caused by tangential component of hydrodynamic force occurring in narrow gaps of sliding bearings or labyrinth seals. Rotor instability is a very dangerous phenomenon, because the rotor vibrates with very high amplitudes reaching practically the whole of bearing clearance. Means for suppression of instability consist in changing conditions in the hydrodynamic film (specific load, oil inlet temperature), changing geometry of bearing gap or using additional damping (two oil films in series). However, in some cases the above-mentioned methods are not effective and it is necessary to look for more sophisticated methods of rotor stabilization.

2. EXPERIMENTAL TEST STAND

Some articles were published on the subject of quenching of self-excited vibrations by means of kinematic excitation, eg. Tondl (2005). This method was up to now used only for simple mechanical systems. External excitation of rolling bearings was used for reducing vibration amplitudes of aero engine rotors. However, no information is available about using external excitation for suppression of the rotor self-exciting vibrations. To investigate this possibility experimental stand was designed and manufactured (see Fig. 1 and 2).

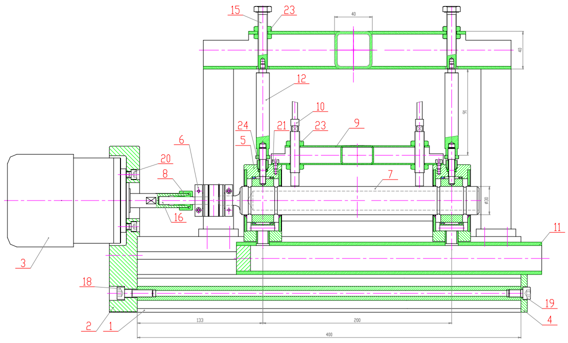


Figure 1: Longitudinal section of test stand

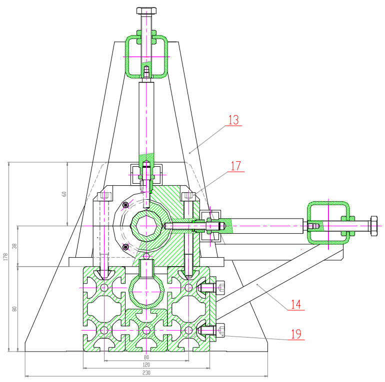


Figure 2: Crossl section of test stand

Test stand should enable running of the rotor up to its stability limit so that the effect of active control could be investigated. Bearing diameter of 30 mm permits to design the rotor as rigid. The base of the stand is constituted by the frame 1 composed of aluminous profiles. Driving motor clamping plane 2 is fastened to the frame by screws 18. High frequency motor 3, fixed in clamping plane 2, is connected to the test shaft 7 by elastic coupling 6, which is connected with the motor by short pin 15 and collet 8. Using the collet was necessary because the only available high speed motor for speeds up to 20.000 rpm is manufactured for drilling spindles. The motor is supplied by high-frequency current converter, enabling speed control by computer. The elastic coupling is of the multi-plate type. It constitutes two joints, thus separating the shaft from motor drive. Bearing pedestals 5 contain bearing bushings, inserted into pedestals with clearance, so that they can move. The bearing bushings are connected by means of screw bars to piezoactuators 12. Two vertically and two horizontally arranged piezoactuators enable excitation of both bearing bushings by practically arbitrary force. The piezoactuators are secured in frames 13 and 14 respectively, which are fastened to the stand base. Piezoactuators have maximum deviation of 60 μm, maximum force in tension/pressure 800/300 N. Four relative vibration sensors 10 are mounted in carriers 9, fastened to bearing pedestals to follow the rotor deviations.

The bearings are of circular cross section. Selected bearing clearance results in calculated stability limit of the rotor at about 11.000 rpm, which is about one halve of available speed range. The rigid shaft can be eventually replaced by an elastic one, enabling to test running through bending critical speeds while controlling the shaft relative vibration.

3. THEORETICAL ANALYSIS

3.1 Bearing forces

Outgoing from the pressure distribution in the bearing oil film based on the Reynold's equation of hydrodynamic lubrication, the hydrodynamic bearing forces acting on the shaft represent in general a nonlinear vector function of the shaft position $\boldsymbol{x}_b$ in the bearing and a linear function of its velocity:

$$\boldsymbol{F}_H(\boldsymbol{x}_b, \dot{\boldsymbol{x}}_b, \omega) \equiv \boldsymbol{K}_{\boldsymbol{H}}(\boldsymbol{x}_b, \omega) + \boldsymbol{B}_{\boldsymbol{H}}(\boldsymbol{x}_b, \omega)\dot{\boldsymbol{x}}_b \quad (1)$$

In case of zero shaft velocity $\dot{\boldsymbol{x}}_{\boldsymbol{b}} = \boldsymbol{0}$, when shaft rotates around its longitudinal axis only, for each angular velocity ω there is corresponding static equilibrium position $\boldsymbol{x}_{b0}(\omega)$ in which the bearing force is in balance with outer bearing load

$$-\boldsymbol{Q}_b = \boldsymbol{F}_H(\boldsymbol{x}_{b0}, \boldsymbol{0}, \omega) \equiv \boldsymbol{K}_{\boldsymbol{H}}(\boldsymbol{x}_{b0}, \omega) \quad (2)$$

Supposing small vibrations around this equilibrium position $\boldsymbol{x}_{b0}$ the hydrodynamic bearing force can be linearized, i.e. replaced by first two terms of Taylor series

$$\boldsymbol{F}_H(\boldsymbol{x}_b, \dot{\boldsymbol{x}}_b, \omega) \approx \boldsymbol{K}_{\boldsymbol{H}}(\boldsymbol{x}_{b0}, \omega) + \frac{\partial \boldsymbol{K}_{\boldsymbol{H}}}{\partial \boldsymbol{x}}(\boldsymbol{x}_b - \boldsymbol{x}_{b0}) + \boldsymbol{B}_{\boldsymbol{H}}(\boldsymbol{x}_{b0}, \omega)\dot{\boldsymbol{x}}_b \quad (3)$$

Introducing a relative shaft displacement with respect to this equilibrium position $\boldsymbol{y}_b = \boldsymbol{x}_b - \boldsymbol{x}_{b0}$ and relative velocity $\dot{\boldsymbol{y}}_b = \dot{\boldsymbol{x}}_b - \dot{\boldsymbol{x}}_{b0} \equiv \dot{\boldsymbol{x}}_b$, the hydrodynamic bearing force can be approximately replaced by relation

$$\boldsymbol{F}_H(\boldsymbol{x}_b, \dot{\boldsymbol{x}}_b, \omega) \approx -\boldsymbol{Q}_b - \boldsymbol{K}_b(\omega)\boldsymbol{y}_b - \boldsymbol{B}_b(\omega)\dot{\boldsymbol{y}}_b,$$

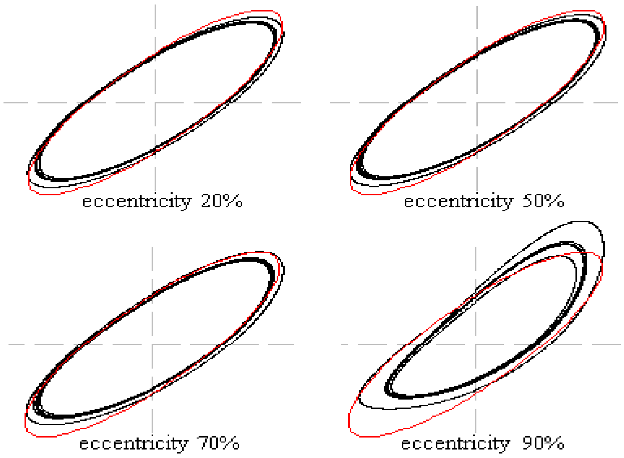
where $\boldsymbol{K}_b(\omega) = -\dfrac{\partial \boldsymbol{K}_{\boldsymbol{H}}}{\partial \boldsymbol{x}}(\boldsymbol{x}_{b0}(\omega), \omega) \quad (4)$

and $\boldsymbol{B}_b(\omega) = -\boldsymbol{B}_{\boldsymbol{H}}(\boldsymbol{x}_{b0}(\omega), \omega)$

are so called stiffness and damping matrices of the oil film in equilibrium position $\boldsymbol{x}_{b0}(\omega)$.

The general nonlinear description of bearing forces is necessary to study rotor systems with big excursions of the shaft in the bearings. Unlike the linearized case, where sophisticated methods of finding stiffness and damping bearing matrices are known, to find the full nonlinear description of the field of hydrodynamic forces in the frame of the whole bearing clearance range remains still a problem. Nonlinear forces have to be determined numerically on the sufficiently fine net, which requires huge amount of calculations and data. An analytical formulation of nonlinear bearing forces can be derived only for special simplified cases of so-called "short" or "long" circular bearings.

Calculations of shaft trajectories around different equilibrium positions show that linear description of bearing forces is sufficiently accurate for greater part of bearing area. Only in case of great eccentricities of equilibrium positions approaching bearing clearance the original elliptical trajectory is significantly deformed and shifted.



. *Figure 3: Examples of shaft trajectories*

With respect to intended purposes of decreasing and shifting resonance peaks in critical speeds as well as for improvement of rotor stability (both mentioned phenomena occur in the linear range of small shaft displacements in the bearing) the linear description of hydrodynamic bearing forces is fully adequate. Dynamic properties of the bearings are then described by a sequence of stiffness and damping matrices defined in a net of equilibrium positions, which correspond (in our case) to the net of shaft speeds $\{\omega_j\}$ covering the speed operating range.

3.2 Equations of motion of the system

The rotor shaft itself represents a linear dynamic system which can be modeled with satisfactory accuracy using standard finite element discretization procedures leading to the description of the system by mass and stiffness matrices of the shaft $\boldsymbol{M}_s$, $\boldsymbol{K}_s$ and displacement vector $\boldsymbol{x}$; the sub-vectors $\boldsymbol{x}_k$ (of length 4) represent deflection and tilting in cross-sections of the shaft, i.e. in connections of two adjacent finite elements. In case of linearized description of hydrodynamic bearing forces (on condition of small shaft deflections at bearing locations), the static part of bearing forces is in balance with weight vectors of shaft and bearing and dynamic part is described with help of stiffness and damping matrices. Introducing relative shaft displacement vector $\boldsymbol{x}$**,** where displacements are considered relatively to the joint line of (static) equilibrium position in both bearings, the complete rotor, oil bearing and bearing bushing system is described by two matrix equations, the first one for shaft motion, the second one for the bushings.

$$\begin{aligned} &\boldsymbol{M}\,\ddot{\boldsymbol{x}} + (\omega\boldsymbol{G} + \tilde{\boldsymbol{B}}_b(\omega))\dot{\boldsymbol{x}} + (\boldsymbol{K}_s + \tilde{\boldsymbol{K}}_b(\omega))\boldsymbol{x} = \\ &= \boldsymbol{n}(\omega,t) - (\boldsymbol{MP}\,\ddot{\boldsymbol{x}}_p + \omega\boldsymbol{GP}\dot{\boldsymbol{x}}_p) \end{aligned} \tag{5}$$

$$\begin{aligned} &\boldsymbol{M}_p\,\ddot{\boldsymbol{x}}_p + \boldsymbol{B}_p\dot{\boldsymbol{x}}_p + \boldsymbol{K}_p\boldsymbol{x}_p = \\ &= \boldsymbol{F}_a(t,\boldsymbol{\alpha},\boldsymbol{x},\boldsymbol{x}_p,...) + \boldsymbol{B}_b(\omega)\,\dot{\boldsymbol{x}}_b + \boldsymbol{K}_b(\omega)\boldsymbol{x}_b \end{aligned} \tag{6}$$

Matrices $\tilde{\boldsymbol{B}}_b(\omega), \tilde{\boldsymbol{K}}_b(\omega)$ denote the shaft-system related matrices with bearing matrices $\boldsymbol{B}_p(\omega)$, $\boldsymbol{K}_p(\omega)$ at appropriate positions and matrix $\boldsymbol{P}$ ensures a linear distribution of bushing displacements along the shaft.

The excittaing forces generated by piezoactuators acting on bearing bushings are formally declared by a force vector $\boldsymbol{F}_a$ in dependence on parameters and shaft and bearing displacements $\boldsymbol{x}$, $\boldsymbol{x}_p$ respectively. The resulting effect of these additional bearing excitations on rotor dynamics will strongly depend on their form and functional relations. Based on the kind and character of generated excitation such rotor systems can be divided into three basic groups:

a) Rotors with kinematic excitation of bushings,
b) Rotors with parametric excitation of bushings,
c) Rotors with active control of exciting forces.

In case of kinematical excitation the exciting force acting on the bushings is not dependent on shaft or bushing deflections and represents therefore another external excitation which can be unlike unbalance exciting forces of arbitrary non-synchronous frequency. If the source of exciting force is sufficiently robust the kinematic trajectory instead of kinematic force can be prescribed. As will be shown in next chapter, kinematic excitation enables to change the course of response, but does not influence the rotor stability.

In both remaining cases the exciting forces acting on bushings are generated so, that either in dependence on the bearing deflections simulate periodically variable stiffness of bearing bushings (parametrical excitation) or in the appropriate manner respond to the deflection at specified shaft location (active control). Through parametrical excitation as well as active control both rotor stability and rotor response can be affected.

3.3 Kinematic excitation of bearing bushings

Provided that the kinematic trajectories of bearing bushings are prescribed, the rotor system is defined by a single linear differential equation (5)

$$\begin{aligned} &\boldsymbol{M}\,\ddot{\boldsymbol{x}} + (\omega\boldsymbol{G} + \tilde{\boldsymbol{B}}_b(\omega))\dot{\boldsymbol{x}} + (\boldsymbol{K}_s + \tilde{\boldsymbol{K}}_b(\omega))\boldsymbol{x} = \\ &= \boldsymbol{n}(\omega,t) - (\boldsymbol{MP}\,\ddot{\boldsymbol{x}}_p + \omega\boldsymbol{GP}\dot{\boldsymbol{x}}_p) \end{aligned}.$$

As the bearing deflections $\boldsymbol{x}_p$ are known the second term on the right side of equation represents (analogous to unbalance force) another exciting force only. As a linear system with constant coefficient matrices these equation can be solved by standard methods. The frequency and modal properties of the system are determined by corresponding eigenvalue problem

$$\begin{aligned} &\boldsymbol{S}(\lambda,\omega)\,\hat{\boldsymbol{x}} \equiv \\ &(\lambda^2\boldsymbol{M} + \lambda(\omega\boldsymbol{G} + \tilde{\boldsymbol{B}}_b(\omega)) + (\boldsymbol{K}_s + \tilde{\boldsymbol{K}}_b(\omega)))\,\hat{\boldsymbol{x}} = \boldsymbol{0} \end{aligned}$$

while time-response amplitudes are given by relation

$$\begin{aligned} &\boldsymbol{x}(t) \equiv Re\left\{\boldsymbol{S}^{-1}(i\omega,\omega)\hat{\boldsymbol{n}}\omega^2 e^{i\omega t}\right\} - \\ &- Re\left\{\boldsymbol{S}^{-1}(i\Omega,\omega)(-\Omega^2\boldsymbol{MP} + i\Omega\omega\boldsymbol{GP})\hat{\boldsymbol{x}}_p e^{i\Omega t}\right\} \end{aligned} \tag{7}$$

where Ω represents angular frequency of kinematic excitation, in general different from shaft angular frequency ω. The stability properties given by solution of eigenvalue characteristics are therefore not depend on any kind of kinematic excitation $\boldsymbol{x}_p$. On the other hand, the course of rotor response, as an envelope of time-response amplitude vectors, can be substantially changed and modeled by appropriate choice of bushing trajectory parameters.

Provided, that no bearing trajectory is prescribed but external periodic forces act on bearing bushings, the entire system is described by equations (5),(6) with exciting force $\boldsymbol{F}_a(\Omega,t)$ independent of both rotor and bushing deflection. But this is still a linear system of differential equations, whose stability cannot be dependent neither on unbalance exciting forces $\boldsymbol{n}(\omega,t)$, nor on kinematic exciting forces $\boldsymbol{F}_a(\Omega,t)$ and the solution of which is of the same type as in (7).

All computer simulation carried out with different kinds and parameters of kinematic bushing excitation prove, that stability threshold of the system does not change, but significantly is changed steady response in which both synchronous and non-synchronous components, corresponding to unbalance forces and exciting bushing forces are observed. Examples of such responses in stable and unstable region are presented in Fig. 4 and 5. Stable trajectory in Fig. 4 corresponds to the shaft speed of 5000 rpm and bushing frequency of 3000 rpm. Unstable trajectory of the same rotor at 8250 rpm and exciting bushing frequency 4125 rpm is shown in Fig. 5. Shaft speed 8250 rpm is close to stability threshold of the system without kinematic excitation at ≈8150 rpm.

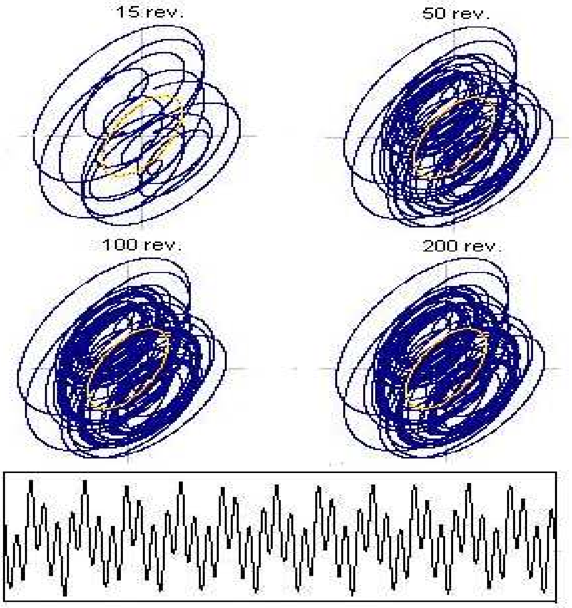


Figure 4: Stable trajectory of the rotor with kinematicaly excited bushings

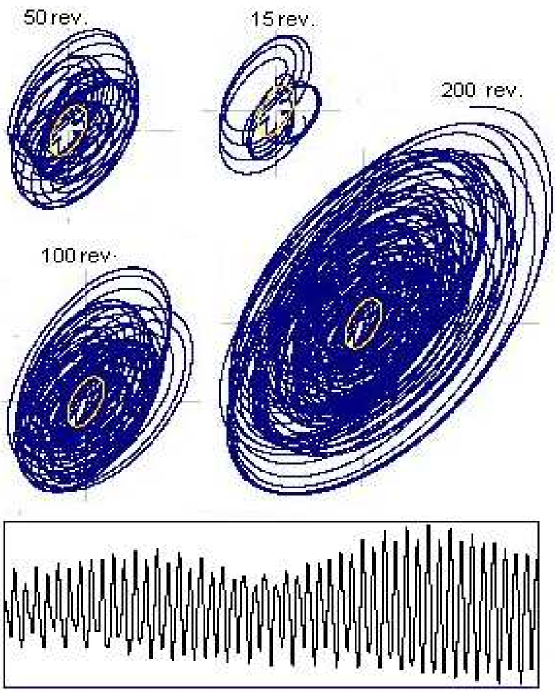


Figure 5: Unstable trajectory of the rotor with kinematicaly excited bushings

3.4 Parametric excitation of bearing bushings

In case of parametric excitation the external force $\boldsymbol{F}_a$ acting on bearing bushing has to be modeled in dependence on bushing deflection $\boldsymbol{x}_p$, so that it simulates variable stiffness of bushing mounting. We shall assume harmonic course of stiffness variation only with generally non-synchronous angular frequency Ω. The resulting stiffness of bushing mounting will be therefore of the form

$$\boldsymbol{K}_p = \boldsymbol{K}_{p0} + \boldsymbol{K}_{pc}\cos(\Omega t) + \boldsymbol{K}_{ps}\sin(\Omega t) \qquad (8)$$

where $\boldsymbol{K}_{pc}$, $\boldsymbol{K}_{ps}$ can be arbitrary, in general non-symmetrical matrices. Applying this variable bushing stiffness mounting to equations of motion (5), (6) we get the resulting differential equations formally expressed in the form

$$\begin{aligned} &\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{B}(\omega)\dot{\boldsymbol{x}} + (\boldsymbol{K}_0(\omega) + \boldsymbol{K}_c\cos(\Omega t) + \\ &\qquad + \boldsymbol{K}_s\sin(\Omega t))\boldsymbol{x} = \boldsymbol{f}(\omega t) \end{aligned} \qquad (9)$$

with constant mass and damping matrices $\boldsymbol{M}, \boldsymbol{B}(\omega)$ respectively and time-dependent periodic stiffness matrix $\boldsymbol{K}(\omega,\Omega t)$ of the analyzed dynamic system. The aim is to find stiffness matrices $\boldsymbol{K}_{pc}$, $\boldsymbol{K}_{ps}$ (containing exciting parameters) together with exciting frequency Ω so that all solutions of homogenous part of equation (9) are stable.

Using well known procedure, the system of linear differential equations of second order (9) can be converted into the system of differential equations of the first order and double dimension

$$\begin{aligned} &\dot{\boldsymbol{y}} = \boldsymbol{A}(t)\boldsymbol{y}, \\ &\boldsymbol{A}(t) = \boldsymbol{A}_0 + \boldsymbol{A}_C\cos(\Omega t) + \boldsymbol{A}_S\sin(\Omega t) \end{aligned} \qquad (10)$$

The system matrix $\boldsymbol{A}(t)$ with dimension n is therefore a periodic function with period $p=2\pi/\Omega$. As is well known from the theory of linear differential equations, there exists a system of n linear independent solutions $\boldsymbol{y}_k(t)$ of equation (10), which can be aranged to the fundamental matrix $\boldsymbol{Y}(t)$ satisfying original equations

$$\dot{\boldsymbol{Y}} = \boldsymbol{A}(t)\boldsymbol{Y}\ . \qquad (11)$$

Because any solution of linear differential equation can be expressed by a linear combination of functions from a fundamental system, the two different fundamental systems $\boldsymbol{Y}(t)$, $\boldsymbol{Z}(t)$ are mutually connected by relation

$$\boldsymbol{Z}(t) = \boldsymbol{Y}(t)\cdot\boldsymbol{C}, \quad \boldsymbol{C} = konst. \qquad (12)$$

As a consequence of periodicity, the matrix $\boldsymbol{Y}(t+p)$ is also a fundamental system and therefore

$$\boldsymbol{Y}(t+p) = \boldsymbol{Y}(t)\cdot\boldsymbol{W}_Y \qquad (13)$$

Matrix $\boldsymbol{W}_Y$ is so called 'monodromy matrix' and is determined unambiguously except for a conformity transformation $\boldsymbol{C}^{-1}.\boldsymbol{W}_Y.\boldsymbol{C}$. The spectrum matrix $\boldsymbol{S}$ from modal decomposition

$$\boldsymbol{W}_Y = \boldsymbol{V}_Y\cdot\boldsymbol{S}\cdot\boldsymbol{V}_Y^{-1} \qquad (14)$$

is therefore identical for all monodromy matrices and is determined by system matrix $\boldsymbol{A}(t)$ only. On the basis of periodicity of the system matrix and modal decomposition of monodromy matrix, the value of functions from arbitrary fundamental system for $t \to \infty$ can be estimated by inequality

$$\begin{aligned} lim_{t\to\infty}\|\boldsymbol{Y}(t)\| = lim_{k\to\infty}\|\boldsymbol{Y}(t_0 + kp)\| \le \\ \le \boldsymbol{K}_1 \cdot lim_{k\to\infty}\|\boldsymbol{S}^{\boldsymbol{k}}\| \cdot \boldsymbol{K}_2 \end{aligned} \tag{15}$$

If all eigenvalues s_i of spectral matrix $\boldsymbol{S}$ lie inside the unit circle, i.e. if $|s_i| < 1$, the right side of inequality (15) tends to zero and all solutions $\boldsymbol{y}_k(t)$ are stable.

This criterion enables to judge the stability of the system with concrete numerically expressed elements of system matrix $\boldsymbol{A}(t)$. Corresponding monodromy matrix can be then determined by numerical integration of the system through the first time period $0 \div p$. Because the spectral properties do not depend on actual instance of monodromy matrix, it is possible to start numerical integration with arbitrary initial conditions; stability properties of the system will be determined by eigenvalues of the calculated monodromy matrix. However, this procedure is not too favorable for finding stability conditions of the equation (9) in dependence on a set of exciting parameters contained in stiffness matrices in formula (8).

To derive a more acceptable criterion we introduce a new fundamental system $\boldsymbol{Z}(t)$

$$\boldsymbol{Z}(t) \equiv \boldsymbol{Y}(t) \cdot \boldsymbol{V_Y}, \qquad \boldsymbol{W}_Y = \boldsymbol{V_Y} \cdot \boldsymbol{S} \cdot \boldsymbol{V_Y^{-1}}. \tag{16}$$

With respect to (13) and (14) the system $\boldsymbol{Z}(t)$ satisfies

$$\boldsymbol{Z}(t+p) = \boldsymbol{Z}(t) \cdot \boldsymbol{S} \tag{17}$$

It holds therefore for single solutions $\boldsymbol{z}_j(t)$

$$\boldsymbol{z}_j(t+p) = s_j\, \boldsymbol{z}_j(t), \tag{18}$$

which a variant of well known Floquet's theorem. Strictly speaking this equation is valid in case of mutually different eigenvalues s_j. If spectral matrix $\boldsymbol{S}$ is of general Jordan structure with canonical Jordan cells on its diagonal, the equation (18) holds for solutions $\boldsymbol{z}_j(t)$, which correspond the first members in Jordan cells only. But this fact does not mean any restriction, because the stability depends purely on eigenvalues s_j and other solutions are not interesting for stability purposes. The number m of unequal eigenvalues s_j and linear independent solutions $\boldsymbol{z}_j(t)$ satisfying Floquet's formula (18) is less or equal to the number of Jordan cells in spectral decomposition of monodromy matrix. Defining characteristic exponents α_k by formula

$$s_k = exp(\alpha_k p), \tag{19}$$

the function

$$\boldsymbol{q_k}(t) = e^{-\alpha_k t}\boldsymbol{z_k}(t) \tag{20}$$

is a periodic function with period p and satisfies differential equation

$$\dot{\boldsymbol{q}}_k = (\boldsymbol{A}(t) - \alpha_k \boldsymbol{I})\boldsymbol{q_k}. \tag{21}$$

If we define -analogous to eigenvalue problem with constant matrix $\boldsymbol{A}$- a characteristic problem of periodic solutions for periodic matrices $\boldsymbol{A}(t)$

$$\dot{\boldsymbol{q}} = (\boldsymbol{A}(t) - \alpha)\boldsymbol{q}, \tag{22}$$

it follows from above derived relations, that for this problem exist exactly m characteristic values α_k (and corresponding periodic functions $\boldsymbol{q}_k(t)$). The numbers $s_k = exp(\alpha_k p)$ are at the same time eigenvalues of the system monodromy matrix. Therefore, the original system of differential equations (11) is stable if all its characteristics exponents α_k have negative real part:

$$Re(\alpha_k) < 0, \quad k=1,..,m. \tag{23}$$

There is a certain similarity between stability conditions of systems of linear differential equations with constant coefficients and systems with periodic coefficients: in case of constant matrix $\boldsymbol{A}$ for all eigenvalues λ_k conditions $Re(\lambda_k) < 0$ must be satisfied, in case of periodic matrix $\boldsymbol{A}(t)$ conditions $Re(\alpha_k) < 0$ for all characteristic exponent α_k must be satisfied.

To find periodic solution of characteristic problem (22), the Fourier series expansion can be used. This technique leads in general to the solution of eigenvalue problems with matrix of infinite dimension. But in our case of matrix $\boldsymbol{A}(t)$ containing members with $cos(\Omega t)$ and $sin(\Omega t)$ components only, we limit the solution for technical purposes on the first three members of Fourier series

$$\boldsymbol{q}(t) \approx \boldsymbol{q_0} + \boldsymbol{q_C}\, cos(\Omega t) + \boldsymbol{q}_S\, sin(\Omega t). \tag{24}$$

Substitution of this expression to the characteristic problem (22) leads to the eigenvalue problem for calculation of characteristic exponents α_k

$$(\tilde{\boldsymbol{A}} - \alpha \boldsymbol{I}) \cdot \tilde{\boldsymbol{q}} = \boldsymbol{0} \tag{25}$$

with matrix $\tilde{\boldsymbol{A}}$ given by formula

$$\begin{aligned} \tilde{\boldsymbol{A}} &= \boldsymbol{A_0} - \frac{1}{2}(\boldsymbol{A_C}\boldsymbol{A}_0\boldsymbol{Q^{-1}}\boldsymbol{A_C} + \boldsymbol{A}_S\boldsymbol{A}_0\boldsymbol{Q^{-1}}\boldsymbol{A}_S) \\ &\quad - \frac{\Omega}{2}(\boldsymbol{A_C}\boldsymbol{Q^{-1}}\boldsymbol{A}_S - \boldsymbol{A}_S\boldsymbol{Q^{-1}}\boldsymbol{A}_C), \\ \boldsymbol{Q} &= \boldsymbol{A_0^2} + \Omega^2\boldsymbol{I}. \end{aligned} \tag{26}$$

The definition formula of matrix $\tilde{\boldsymbol{A}}$ is therefore a little complicated and for analysis as well as calculations some simplification is desirable. Outgoing from formulation of the system equations (10), dynamical properties of the original rotor without parametric excitation are determined by modal characteristics of the system

$$\dot{\boldsymbol{y}} = \boldsymbol{A}_0 \boldsymbol{y}. \qquad (27)$$

Denoting $\boldsymbol{\Lambda}_0$ diagonal matrix of complex eigenvalues and $\boldsymbol{V}_0$ modal matrix of corresponding left-hand eigenvectors of non-excited system, the eigenvalue problem (26) for characteristic exponents α_k can be transformed to the equivalent eigenvalue problem expressed in modal coordinates generated by matrix $\boldsymbol{V}_0$. In these coordinates the matrix $\tilde{\boldsymbol{A}}$ has a more transparent form

$$\tilde{\boldsymbol{A}} = \boldsymbol{\Lambda_0} - \frac{1}{2}(\tilde{\boldsymbol{A}}_{\boldsymbol{C}} \boldsymbol{D}_{\lambda} \tilde{\boldsymbol{A}}_{\boldsymbol{C}} + \tilde{\boldsymbol{A}}_S \boldsymbol{D}_{\lambda} \tilde{\boldsymbol{A}}_S + \tilde{\boldsymbol{A}}_{\boldsymbol{C}} \boldsymbol{D}_{\Omega} \tilde{\boldsymbol{A}}_S - \tilde{\boldsymbol{A}}_S \boldsymbol{D}_{\Omega} \tilde{\boldsymbol{A}}_C), \qquad (28)$$

$$\tilde{\boldsymbol{A}}_{\boldsymbol{C}} = \boldsymbol{V}_0^{-1} \boldsymbol{A}_{\boldsymbol{C}} \boldsymbol{V}_0 , \qquad \tilde{\boldsymbol{A}}_S = \boldsymbol{V}_0^{-1} \boldsymbol{A}_S \boldsymbol{V}$$

$$\boldsymbol{D}_{\lambda} = diag\left[\frac{\lambda_k}{\lambda_k^2 + \Omega^2} \right], \quad \boldsymbol{D}_{\Omega} = diag\left[\frac{\Omega}{\lambda_k^2 + \Omega^2} \right].$$

In this expression the terms of type $\tilde{\boldsymbol{A}}_{\boldsymbol{C}} \boldsymbol{D}_{\lambda} \tilde{\boldsymbol{A}}_{\boldsymbol{C}}$ represent contributions of exciting parameters on resulting characteristic exponents. Regarding the lay-out of original eigenvalues of non-excited system on the matrix diagonal, the form of this matrix is efficient for numerical processing. With respect to the higher number of freedom of the original system and therefore higher dimension of matrix $\tilde{\boldsymbol{A}}$, the eigenvalue and stability problems cannot be solved analytically. That is why finding and analyzing relations among parameters of external parametric excitation guaranteeing rotor stability will not be an easy problem.

4. CONCLUSIONS

Test stand was designed and manufactured to study possibilities to affect dynamics of rotors supported in hydrodynamic journal bearings by means of external excitation, The stand enables to achieve speeds more than 20000 rpm and to act on both bearings by arbitrary excitation force in two directions. Performed theoretical study provided some means for prediction of excitation mode on the rotor behavior. It was shown, that kinematic excitation can affect amplitudes of vibration but not the stability limit. To suppress rotor instability will apparently require use of more sophisticated excitation modes realized through piezoactutor control.

ACKNOLEDGEMENT

This work was supported by the Czech Scientific Foundation under project No. 101/07/1345 “Active control of journal bearings aimed at suppressing the rotor instability”.

5. REFERENCES

Tondl,, A., Nabergoj, R., Ecker, H.: Quenching of self-excited vibrations in a system with two unstable vibration modes. Proc. of 7th Int. Conference on Vibration Problems – ICOVP (2005)

Ehmann, C., Liebich, R.: Piezoactuators for vibration reduction of aero engine rotors – a case study. ISCORMA-3, Ohio 2005, paper 307

Šimek, J.: Test stand for active control of sliding bearings aimed at suppressing the rotor instability, Technical report TECHLAB No. 07-405, 2007