POROUS MEDIA MODELLING FOR A ROW OF FUEL ASSEMBLIES UNDER SEISMIC LOADING

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ABSTRACT

The aim of this study was to develop a tractable model of a nuclear reactor core taking the complexity of the structure (including its nonlinear behaviour) and the fluid flow coupling into account. The mechanical behaviour modelling includes the dynamics of both the fuel assemblies and the fluid. Each rod bundle is modelled in the form of a deformable porous medium, so the velocity field of the fluid and the displacement field of the structure are defined over the whole domain. The fluid and structure are first modelled separately, before being linked together. The contact between fuel assemblies are accounted for by impact springs. The motion equations for the structure are obtained using a Lagrangian approach, and to be able to link up the fluid and the structure, the motion equations for the fluid are obtained using an Arbitrary Lagragian Eulerian approach. The finite elements method is applied to spatially discretize the equations. Simulations were performed to reproduce the dynamics of a six fuel assemblies row immersed in stagnant water on a shaking table which can reproduce a seismic loading, the maximum value of the impact forces are compared, results obtained show good agreement with the experimental data.

1. INTRODUCTION

This study is about the safety of nuclear reactor core when it is subjected to seismic loading. A reactor core is made up of several fuel assemblies. A fuel assembly is a complex structure composed of rods regularly spaced. A rod has a diameter of about 1cm, and a length of 4 m (Fig. 1). There are two types of rods: the guide tubes and the fuel rods. The guide tubes are welded to 10 grids regularly spaced, and the fuel rods contain enriched uranium fuel. Fuel rods are maintained by the grids, the grid/fuel rod contact is ensured by springs, so that the fuel rods can slip into the grids. Pressurized water crosses the core and the fluid flow is parallel to the rods' axis. A direct simulation taking into account all the contact phenomena and fluid-structure interaction would lead to too many degrees of freedom; engineers need simplified and accurate models to estimate impact forces between fuel assemblies under seismic excitation. A fuel assembly is frequently modelled by a single beam (Viallet et al, 2003; Pomîrleanu, 2005) which undergoes the fluid effects through added mass and damping. Special attention is paid to the impact model, these simple models allow engineers to simulate a row of fuel assemblies and make it possible to have the large number of simulations. Some researchers have proposed multi-beam models to represent a fuel assembly (Fontaine and Politopoulos, 2000); friction models are used to model rod-grid liaisons. These models have yielded good results in accordance with "in air" experiments (without the presence of water) but friction problems are numerically difficult to solve. These models don't represent the fluid flow through the core. Our purpose is to reduce the uncertainty margin of present day codes and since the fluid flow through the fuel assemblies induces non-negligible fuel assembly coupling, in this paper we propose a model that will take into account the nonlinear behaviour of both fluid and structure with few degrees of freedom.

2. MODEL FORMULATION

In this section we consider a core with N_{FA} fuel assemblies arranged in line submerged by stagnant water. The whole domain of the core is noted Ω_c. It is a two-phase parallelepiped containing the fluid domain Ω_f and the structure domain Ω_s ($\Omega_f \cap \Omega_s = \varnothing$ and $\Omega_f \cup \Omega_s = \Omega_c$), L_{cx}, L_{cy} and L_{cz} are the dimensions of Ω_c in the $\mathbf{e_x}$, $\mathbf{e_y}$ and $\mathbf{e_y}$ directions, note that L_{cx} is also the length of a fuel assembly. Fuel assemblies are embedded at their ends to the frame of which the displacement is imposed to simulate earthquake.

2.1. Assumptions and method

The proposed formulation is subject to the following assumption :
H1 The fluid is viscous, incompressible and Newtonian.
H2 Gravity effects are neglected.
H3 The rod section does not deform.
H4 Distance between two rods (of the same fuel assembly) remains constant.
H5 Turbulent kinetic energy is negligible in comparison with the turbulent diffusion.
The procedure used to draw up the equations of motion governing the complex fluid/structure entity under investigation is summarized in Fig. 1. This procedure is based on a porous medium approach which gives access to an equivalent fluid model and an equivalent structure model both defined on the whole domain Ω_c. Motion equations for the equivalent fluid and the equivalent structure are first established separately. For the fluid part, global fluid flow equations through the rod bundle are obtained by spatially averaging the Navier Stokes equations written with an Arbitrary Lagragian Eulerian approach. The resulting equivalent fluid is characterized by the unknowns $\mathbf{V_{eq}}$ and P_{eq} which are defined in the whole domain Ω_c. Structure related effects on fluid are accounted for by a body force which is also defined in the whole domain. The structure equations are also space averaged but fuel assembly by fuel assembly, each fuel assembly is modelled as an equivalent structure satisfying a Timoshenko beam model with a nonlinear visco-elastic behaviour. The equivalent structure unknowns are reduced to the displacement $\mathbf{u_{eq}}$ and the rotation $\boldsymbol{\theta_{eq}}$ of each beam, with $\mathbf{U_{eq}}(x,y,z) = \mathbf{u_{eq}}(x) + \theta_{\mathbf{eq}}(x) \wedge (y\mathbf{e_y} + z\mathbf{e_z})$. Finally, fluid related effects on structure, $\mathbf{F_{fluid \to structure}}$, and structure related effects on fluid, $\mathbf{F_{structure \to fluid}}$, are of opposite sign and are accounted for by a body force which is defined in the whole domain.

2.2. Coupled model formulation

In this section we present the model for the fluid structure problem proposed by Ricciardi et al (2007) for a 2D problem.
Ω_c is subdivided (Fig. 2):

$$\Omega_c = \bigcup_{i=1}^{N_{FA}} \Omega_{FA_i}, \forall(i,j),\ \Omega_{FA_i} \cap \Omega_{FA_j} = \varnothing, \quad (1)$$

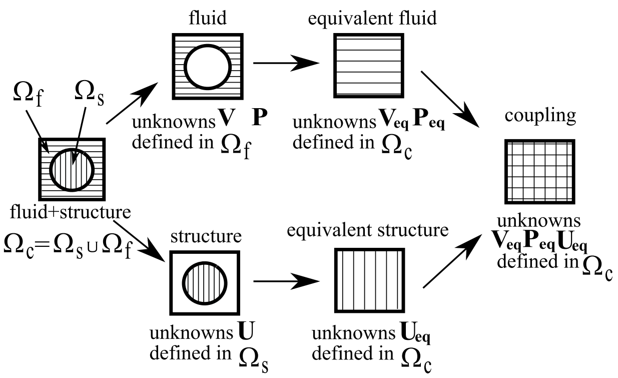

Figure 1: Porous modelling and method

where each fluid structure domain Ω_{FA_i} includes one fuel assembly.

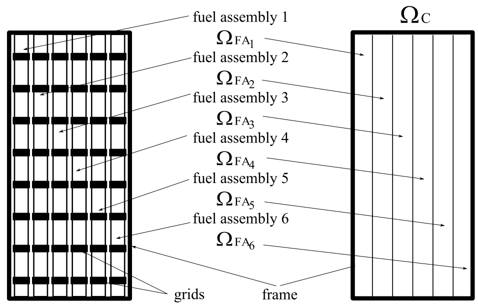

Figure 2: Modelling of six fuel assemblies

We have to solve the coupled equations in Ω_c,

$$\begin{aligned}
\rho_{eq}\frac{\partial \mathbf{V_{eq}}}{\partial t} &+ \rho_{eq} div \mathbf{V_{eq}} \otimes \mathbf{V_{eq}} = -\nabla P_{eq} \\
&+ \mu_{Teq}\Delta \mathbf{V_{eq}} + \rho_{eq}\frac{\partial \mathbf{u_{eq}}}{\partial t} \cdot \nabla \mathbf{V_{eq}} \\
&- \rho_{eq}\mathbf{V_{eq}} \cdot \nabla \frac{\partial \mathbf{u_{eq}}}{\partial t} \\
&+ \mathbf{F_{structure \to fluid}}, && (2) \\
div \mathbf{V_{eq}} &= 0, && (3) \\
m_{fa}\frac{\partial^2 \mathbf{u_{eq}}}{\partial t^2} &= \frac{\partial \mathbf{T}}{\partial x} + \mathbf{F_{fluid \to FA}} + \mathbf{F_{impact}}, && (4) \\
I_{fa}\frac{\partial^2 \boldsymbol{\theta_{eq}}}{\partial t^2} &= \frac{\partial \mathbf{M}}{\partial x} + \mathbf{e_x} \wedge \mathbf{T} + \mathbf{M_{fluid \to FA}}, && (5)
\end{aligned}$$

$$\begin{aligned}
with,\ \mathbf{u_{eq}} &= \sum_i^{N_{fa}} u_{eqi}\mathbb{I}_{\Omega_{FAi}}, && (6) \\
\boldsymbol{\theta_{eq}} &= \sum_i^{N_{fa}} \boldsymbol{\theta_{eqi}}\mathbb{I}_{\Omega_{FAi}}, && (7)
\end{aligned}$$

where ρ_{eq} is the equivalent fluid density, μ_{Teq} is the equivalent fluid viscosity, m_{fa} is the mass per unit length of a fuel assembly, I_{fa} is the inertial moment per unit length of a fuel assembly, $\mathbf{M}$ is the bending moment, $\mathbf{T}$ is the shear force, $\mathbf{V_{eq}}$, P_{eq}, u_{eqi} and $\boldsymbol{\theta_{eqi}}$ are the unknowns, and $\mathbb{I}_{\Omega_{FAi}}$ denote the indicator function of Ω_{FAi}.
(2) and (3) are the equations governing the equivalent fluid motion, and (4) and (5) are the equations governing the equivalent structure motion.

2.3. Behaviour modelling

In this section we develop some terms introduced in the equations of the problem. (2), (3), (4) and (5) introduce the fluid/structure coupling forces $\mathbf{F_{structure \to fluid}}$, $\mathbf{F_{fluid \to FA}}$ and $\mathbf{M_{fluid \to FA}}$ which are expressed as functions of the unknowns. The bending moment $\mathbf{M}$ and the shear force $\mathbf{T}$ are linked to the equivalent structures unknowns by a nonlinear visco-elastic behaviour law. The impact between fuel assemblies accounted for in (4) by $\mathbf{F_{impact}}$ is modelled by unilateral linear springs.

2.3.1. Coupling forces

Since the inertial effects are not negligible in the present work, we decompose structure force in an inertial term and a drag term (Morison et al, 1950).

$$\mathbf{F_{structure \to fluid}} = \mathbf{F_I} + \mathbf{F_D}, \quad (8)$$

with :

$$\mathbf{F_I} = -\frac{m_f}{a^2}\frac{\partial}{\partial t}\left(\frac{\partial u_{eq_z}}{\partial t} - V_{eq_z}\right)\mathbf{e_z}, \quad (9)$$

$$\mathbf{F_D} = -\frac{C_D}{a^2}\left|\frac{\partial u_{eq_z}}{\partial t} - V_{eq_z}\right|\left(\frac{\partial u_{eq_z}}{\partial t} - V_{eq_z}\right)\mathbf{e_z}, \quad (10)$$

where a is the distance between two rod centers. The model proposed here is different from the one proposed by Ricciardi et al. (2007) where the core was subjected to axial flow whereas in the present work the water is stagnant. The model is completely defined given the numerical values of the constants m_f and C_D which depend on the geometry, the roughness and the casing.
Fluid forces acting on a fuel assembly are the opposite of the coupling forces at work in the fluid integrated over the fuel assembly cross-section :

$$\mathbf{F_{fluid \to FA}} = \int_{S_{fa}} \mathbf{F_{fluid \to structure}} dS, \quad (11)$$

$$\mathbf{M_{fluid \to FA}} = \int_{S_{fa}} z\mathbf{e_z} \wedge \mathbf{F_{fluid \to structure}} dS, \quad (12)$$

where S_{fa} is the area of the cross-section of a fuel assembly.

2.3.2. Fuel assembly behaviour

The coupling between grids and rods gives rise to complex contact and friction processes which make it difficult to establish the analytically overall behaviour from the averaged structure equation. It is therefore proposed to model the global nonlinear visco-elastic behaviour of the fuel assembly empirically. Since Pisapia et al (2003) have observed that the results obtained using methods based on the quadratic stiffness and damping give good agreement with the experimental data, a quadratic law was used to account for the stiffness, whereas the damping was assumed to be linear because the structural damping is very small in comparison with the fluid damping, so that any error in the structural damping will have negligible consequences :

$$\begin{aligned}
\mathbf{T} &= G_1 S_{fa}\left(\frac{\partial u_{eq_z}}{\partial x} + \theta_{eq_y}\right)\mathbf{e_z} \\
&+ G_2 S_{fa}\left|\frac{\partial u_{eq_z}}{\partial x} + \theta_{eq_y}\right|\left(\frac{\partial u_{eq_z}}{\partial x} + \theta_{eq_y}\right)\mathbf{e_z} \\
&+ \mu_G S_{fa}\frac{\partial}{\partial t}\left(\frac{\partial u_{eq_z}}{\partial x} + \theta_{eq_y}\right)\mathbf{e_z}, \quad (13) \\
\mathbf{M} &= \left(E_1 I\frac{\partial \theta_{eq_y}}{\partial x} + E_2 I\left|\frac{\partial \theta_{eq_y}}{\partial x}\right|\left(\frac{\partial \theta_{eq_y}}{\partial x}\right)\right)\mathbf{e_y} \\
&+ \mu_E I\frac{\partial^2 \theta_{eq_y}}{\partial t \partial x}\mathbf{e_y}, \quad (14)
\end{aligned}$$

where G_1, G_2, E_1, E_2 are stiffness coefficients, μ_G, μ_E are structural damping coefficients, and I is the quadratic moment.

2.3.3. Impact model

When two fuel assemblies strike, the impact surface is located at the grids, and the buckling of the grids will occur before two rods impact each other. So we model the contact between two fuel assemblies $\mathbf{F_{impact}}$ or between a fuel assembly and a wall by linear springs located at the grids positions. If the relative displacement of the fuel assemblies is lower than the gap, the contact is accounted for, else not. The impact between a fuel assembly and the frame is modelled in the same way expect that the frame is assumed to be rigid enough to not deform.

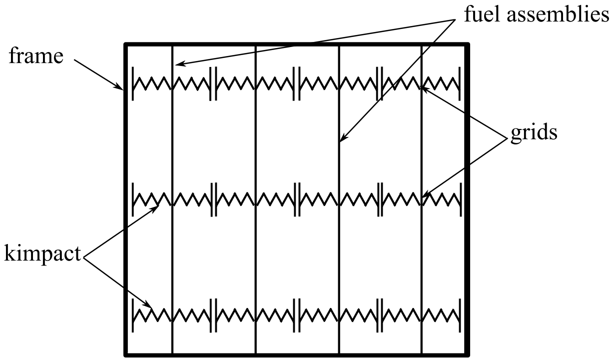

Figure 3: Impact model for four fuel assemblies with three grids

2.4. Discussion

We have proposed a global model of the behaviour of a PWR to avoid the large number of degree of freedom necessary to make a direct numerical simulation of the fluid and structure dynamics. Thus we transformed a fluid structure problem with a complex geometry (large number of rods linked by numerous contact friction) into a problem with a more simple geometry (the equivalent fluid and structure are parallelepipedic). It becomes possible to simulate both fluid and structure dynamics of a whole core. Some local informations are lost compared to a direct numerical simulation, as vibrations of rods into grids, but interactions between fuel assemblies, via fluid or contact, are conserved. Thus the effects of an external excitation (as earthquake or plan crash) on the impact forces between fuel assemblies can be simulated.
The model's equations need the identification of several coefficients : some of them are given by the geometry and physical characteristics of materials involved in the problem (m_{fa}, I_{fa}, S_{fa}, I, ρ, ρ_{eq}, μ_{Teq}, a), the other are introduced by empirical modelling and have to be identified by experiments (G_1, G_2, E_1, E_2, μ_G, μ_E, m_f and C_D).
Both equivalent fluid and structure have unknowns defined in the whole domain Ω_c, which means that, in the proposed model, there is no pure fluid zone accounted for. We assumed that distance between two rods remains constant (H4), but this is no more true at the interfaces between fuel assemblies, so pure fluid zones appear at those interfaces and between fuel assemblies and walls. The size of the pure fluid zones changes in time as the fuel assemblies move, and it could induce additional fluid forces on the fuel assemblies at the interfaces.

3. NUMERICAL MODEL

The variational formulation of (2), (3), (4) and (5) is spatially discretized using a 2D Finite Element Method. We use a $Q9/4$ mixed "velocity-pressure" element, the velocity of the fluid is discretized with 9 nodes, and the continuous pressure with 4 nodes. In the case of the structure field, the Timoshenko beams are discretized using 3 nodes beam elements. The spatial mesh of a single fuel assembly is shown in Fig. 4. In the case of the fluid, the finite element mesh covers the whole domain of the fuel assembly. In the case of the structure, the mesh is one dimensional. The degrees of freedom are the displacement and the rotation of the nodes on the mean line of the beam. Therefore one structure element (for instance $S1$ in the Fig. 4) is superimposed on several fluid elements (for instance $F1$, $F2$, $F3$, $F4$ and $F5$ in the Fig. 4). Thus the number of degree of freedom related to the fluid is larger than those related to the structure.

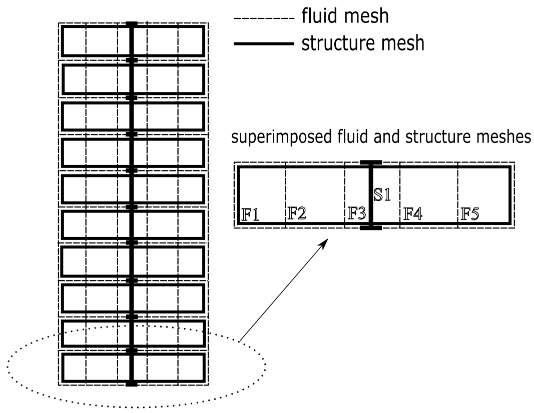

Figure 4: Fluid and structure mesh used to discretize one fuel assembly

For the temporal discretization procedure, two different classical schemes were chosen to discretize the fluid and structure equations, the fluid equations are temporally discretized with an Uzawa scheme, and the structure equation are discretized with a Newmark scheme. At each time step, a nonlinear system is solved with the Newton's method.

4. VALIDATION ON A ROW OF FUEL ASSEMBLIES

In this part, the 2D numerical model is validated on a row of six fuel assemblies immersed in stagnant water excited with seismic loading. Simulations were performed with 6 fluid and structure elements in the axial direction, and 6 fluid elements in the radial direction (1 element for each fuel assembly) and a time step $\Delta t = 12.5\ ms$.

4.1. Experimental device

The experimental device presents six scale one fuel assemblies immersed in stagnant water (Fig. 5). The six fuel assemblies are on a shaking table that can simulate seismic loading. The fuel assemblies are confined. Force sensors are located at the ends of the row. Displacements sensors are located on each grid of the first and the last fuel assemblies. For the others fuel assemblies, one displacement sensor is located on the fourth grid. Since the displacement sensors are fixed to the frame, the displacement measured is a relative displacement. An acceleration sensor is fixed to the shaking table. Tests were performed for several seismic loading at several level between $0.05g$ and $0.4g$ with a 2 mm width gap.

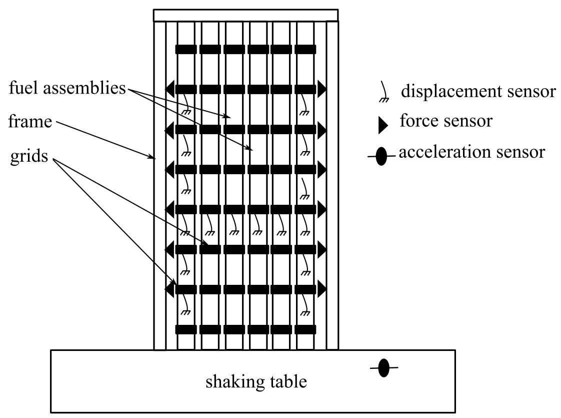

Figure 5: Experimental device

4.2. Results

We compare here experimental data and simulation for the in water configuration. Simulations were performed for excitation level. We can see on Fig. 6 that the results are in good agreement with experimental data. The maximal impact force between the casing and the fuel assemblies given by the simulations show good agreement with experiments (Fig. 7). As it can be observed with experimental data, the maximal impact force increases with the level excitation.
We can observe that the simulations reproduce displacements of the fuel assemblies with a better agreement for the centred fuel assemblies than for the fuel assemblies located at the ends, and this only with in water simulations. This is due to the limitation of the model that doesn't take into account the presence of pure fluid zones between the walls and the fuel assemblies located at the ends. That pure fluid zones could induce fluid forces on fuel assemblies (see 2.4).
We have seen that the model reasonably reproduces the fuel assemblies dynamics, but it seems to be useless as we have to perform experiments to obtain the coefficients used in the model. However the aim of the model is not to simulate six fuel assemblies but a whole core. So coefficients can be obtained with low cost experiments on few fuel assemblies with appropriate casing, and used afterward to simulate more fuel assemblies with various operating conditions. For instance, various configurations mixing end of life and beginning of life fuel assemblies can be analysed under various excitations conditions.

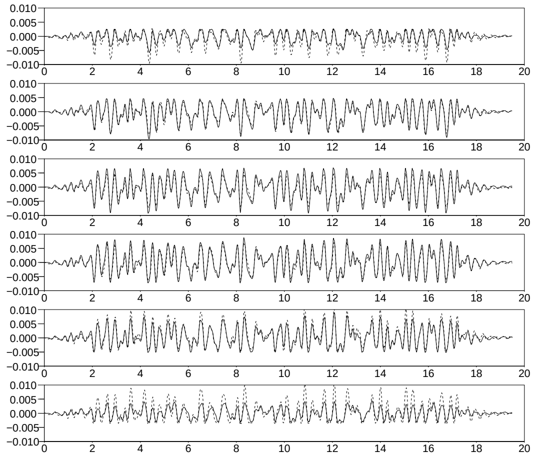

Figure 6: Relative displacements of the fourth grids of the fuel assemblies (m) vs time (s), comparison simulation (continuous line) experimental data (dash line), in water with 0.4 g excitation level

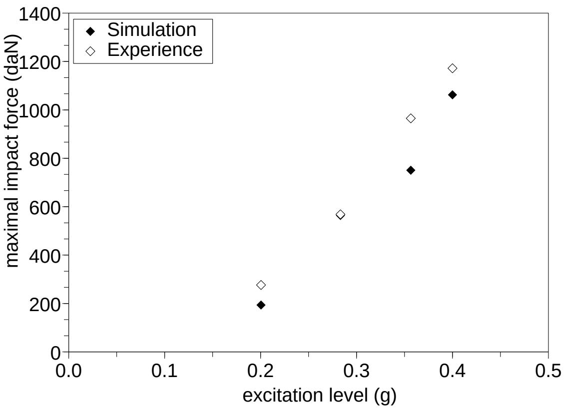

Figure 7: Comparison of the maximal impact force for different excitation level, in water

5. CONCLUSION

In this study, an overall model for a nuclear reactor core was developed using a porous medium method. One of the main features of this model is the fact that the dynamics of both fluid and structure, the nonlinear behaviour, and fluid-structure coupling are taken into account. Fuel assemblies are assimilated to porous media with nonlinear visco-elastic behaviour. The fluid equations, written with an ALE approach, are space averaged in order to determine the overall behaviour of the fluid. The fluid-structure coupling is provided by a body force based on fluid forces acting on a rod. Numerical 2D model was drawn up using a Finite Element Method for the spatial discretization, and the temporal discretization is performed using two classical schemes, one for the structure, and the other for the fluid. Numerical simulations were performed and compared with experimental data on a row of six scale one fuel assemblies immersed by water, with a 0.4 g seismic excitation. Since the displacements and the impact forces obtained with the numerical model are in good agreement with experimental data, in the future the model presented in this paper could be used for design reactor core to seismic loading.

6. REFERENCES

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