AEROELASTIC ANALYSIS OF A SWEPT WING CONTAINING CUBIC AND FREEPLAY NONLINEARITIES IN A SUBSONIC COMPRESSIBLE FLOW

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ABSTRACT

Aeroelastic analysis of a swept wing containing cubic and freeplay nonlinearities is investigated. Previously developed aerodynamic model is modified for unsteady subsonic compressible flow using a well known compressibility correction factor for some of its terms. Applying this model in Lagrange equations and using strip theory and assumed modes, governing equations of aforementioned aeroelastic system are derived. For the sake of verification, the numerical solution results of derived equations are compared with experimental data for flutter speeds of some test cases. This comparison indicates a good agreement. Finally, dynamical responses of a wing containing either cubic or freeplay nonlinearity are obtained from the numerical results of equations developed from unmodified and modified aerodynamic models in order to investigate the different outcomes.

1. INTRODUCTION

Aeroelasticity is the science concerned with the interaction between the deformation of an elastic structure and acting aerodynamic loads. Calculation of instability boundary is one of the main aeroelastic problems and many methods have been employed to determine this aeroelastic instability boundary. Since combination of CFD and FE methods as cumbersome and time consuming procedures are needed to achieve the full and almost exact solution of the governing equations of an aeroelastic system in all air flow regimes, application of analytical aerodynamic model for the certain conditions of flows such as unsteady incompressible and compressible ones to simplify the aeroelastic equations is very favorable.

Governing aeroelastic equations for a 2-dof airfoil in an unsteady incompressible flow are derived by Theodorsen (1935) in the frequency domain ignoring nonlinearities. Also, governing aeroelastic equations of a 2-dof airfoil in an incompressible flow are presented by Fung (1969) in the time domain. These equations are solved via a numerical solution by Lee and Le Blanc (1986). Lee et al. (1998) used a standard fourth-order Runge-Kutta scheme to integrate the system of equations of a two dof airfoil containing cubic nonlinearity for given initial conditions in order to solve equations of motion in the time domain. In the issue of aeroelastic analysis of a wing in an incompressible flow, an experimental and linear analytical study of the flutter of sweptback cantilever wing in the frequency domain were presented by Barmby et al. (1950). Afterwards, many researchers have been studied the aeroelasticity of the wing. Recently, Ghadiri and Razi (2007) investigated the limit cycle oscillations of unswept rectangular cantilever wings containing cubic nonlinearity in an incompressible flow. They verified their formulation with the experimental data. This study is followed by the investigation of linear and nonlinear aeroelastic analyses of a swept wing in an incompressible flow and in the time domain (Razi and Ghadiri, 2008). Equations derived in this study were developed via applying fourier synthesis and Duhamel superposition to the equations previously derived by Barmby et al. (1950) in the frequency domain. LCO amplitude and frequency of the aeroelastic system were obtained using harmonic balance method and forth order Runge-Kutta method.

In the present work, the governing aeroelastic equations of a two dof swept cantilever wing with cubic and freeplay nonlinearity are derived through applying the strip theory and unsteady aerodynamics, and they are studied in the time domain in a compressible flow. In order to apply strip theory, mode shapes of the cantilever beam are used.

2. AEROELASTIC SYSTEM

Consider a two-dof swept rectangular wing rigidly connected to the fuselage and oscillating in pitch and plunge. The nondimensional plunge deflection is denoted by ξ, positive downward direction, and the pitch angle α, positive nose up, respectively. The sweep angle of the wing is Λ. Flow around the wing is assumed to be potential compressible with the Mach number below 0.7. In this study camber and thickness of the wing section are ignored. Elastic axis is uniform and rectilinear. For the assumption of potential flow, plunge and pitch displacements are small. The sketch of a swept wing and its section is plotted in figures 1 and 2. $\bar{y}$ is a coordinate directed along elastic axis and $\bar{x}$ is positive downward and normal to the $\bar{y}$ direction, while y is a coordinate parallel to free stream velocity direction and x is its Cartesian counterpart. As shown in figure 2, c, b, a_h and x_α are chord length, semi-chord length, dimensionless distance between elastic axis and mid-chord and nondimensional distance between elastic axis and centre of mass, respectively.

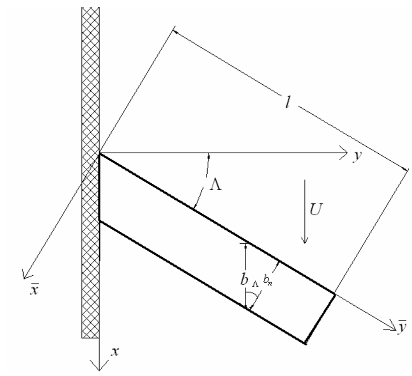

Figure 1: *Sketch of a swept rectangular wing*

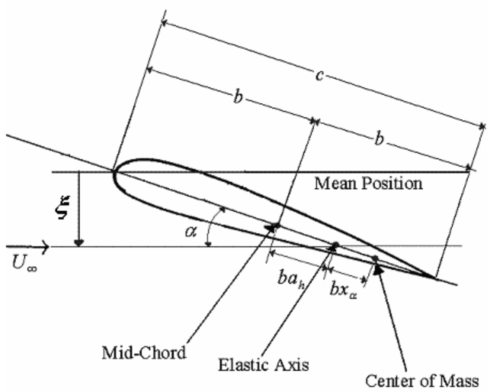

Figure 2: *Schematic figure of the wing section*

3. MODIFIED AERODYNAMIC MODEL

Unsteady aerodynamic model in the time domain for an element of a swept wing in an incompressible flow was derived by Razi and Ghadiri (2008) using a relation between Theodorsen and Wagner's functions obtained by Fourier synthesis and also Duhamel superposition formula as follows:

$$\begin{aligned}
dL = \pi\rho U_n^{\,2} b_n \Bigg[& 2\left[\begin{array}{l}\xi'(0)+\alpha(0)+\dfrac{1}{AR}\dfrac{\partial\xi(0)}{\partial\eta}\tan\Lambda+\left(\dfrac{1}{2}-a_h\right)\alpha'(0)\\ +\dfrac{1}{AR}\left(\dfrac{1}{2}-a_h\right)\dfrac{\partial\alpha(0)}{\partial\eta}\tan\Lambda\end{array}\right]\phi(\tau)\\
&+2\int_0^\tau \phi(\tau-\sigma)\left[\begin{array}{l}\xi''(\sigma)+\alpha'(\sigma)+\dfrac{1}{AR}\dfrac{\partial\xi'(\sigma)}{\partial\eta}\tan\Lambda+\left(\dfrac{1}{2}-a_h\right)\alpha''(\sigma)\\ +\dfrac{1}{AR}\left(\dfrac{1}{2}-a_h\right)\dfrac{\partial\alpha'(\sigma)}{\partial\eta}\tan\Lambda\end{array}\right]d\sigma\\
&+\left[\xi''+\alpha'+\frac{2}{AR}\frac{\partial\xi'}{\partial\eta}\tan\Lambda+\frac{1}{AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda+\left(\frac{1}{AR}\right)^2\frac{\partial^2\xi}{\partial\eta^2}\tan^2\Lambda\right]\\
&-a_h\left[\alpha''+\frac{2}{AR}\frac{\partial\alpha'}{\partial\eta}\tan\Lambda+\left(\frac{1}{AR}\right)^2\frac{\partial^2\alpha}{\partial\eta^2}\tan^2\Lambda\right]\Bigg],
\end{aligned} \tag{1}$$

$$\begin{aligned}
dM_{E.A} = \pi\rho U_n^{\,2} b_n^{\,2} \Bigg[& 2\left(\frac{1}{2}+a_h\right)\left[\begin{array}{l}\xi'(0)+\alpha(0)+\dfrac{1}{AR}\dfrac{\partial\xi(0)}{\partial\eta}\tan\Lambda\\ +\left(\dfrac{1}{2}-a_h\right)\alpha'(0)+\dfrac{1}{AR}\left(\dfrac{1}{2}-a_h\right)\dfrac{\partial\alpha(0)}{\partial\eta}\tan\Lambda\end{array}\right]\phi(\tau)\\
&+2\left(\frac{1}{2}+a_h\right)\int_0^\tau \phi(\tau-\sigma)\left[\begin{array}{l}\xi''(\sigma)+\alpha'(\sigma)+\dfrac{1}{AR}\dfrac{\partial\xi'(\sigma)}{\partial\eta}\tan\Lambda\\ +\left(\dfrac{1}{2}-a_h\right)\alpha''(\sigma)+\dfrac{1}{AR}\left(\dfrac{1}{2}-a_h\right)\dfrac{\partial\alpha'(\sigma)}{\partial\eta}\tan\Lambda\end{array}\right]d\sigma\\
&-\left[\left(\frac{1}{2}-a_h\right)\alpha'+\frac{1}{2AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda\right]+a_h\left[\begin{array}{l}\xi''+\dfrac{2}{AR}\dfrac{\partial\xi'}{\partial\eta}\tan\Lambda+\dfrac{1}{AR}\dfrac{\partial\alpha}{\partial\eta}\tan\Lambda\\ +\left(\dfrac{1}{AR}\right)^2\dfrac{\partial^2\xi}{\partial\eta^2}\tan^2\Lambda\end{array}\right]\\
&-\left(\frac{1}{8}+a_h^{\,2}\right)\left[\alpha''+\frac{2}{AR}\frac{\partial\alpha'}{\partial\eta}\tan\Lambda+\left(\frac{1}{AR}\right)^2\frac{\partial^2\alpha}{\partial\eta^2}\tan^2\Lambda\right]\Bigg].
\end{aligned} \tag{2}$$

where AR and η are aspect ratio of the wing and nondimensional coordinate along the wing span, respectively. Applying the Kelvin's circulation theorem for this aeroelastic system, it can be deduced that the vortices developed on the wing surface shed vortices of equal strength and opposite rotation in the surrounding flow in order to produce no change in the overall circulation. These counter-rotating vortices would produce an induced flow that would effectively change the flow field around the wing. As the wing oscillates, a succession of these vortices would be continuously formed leading to unsteady flow around the wing dependent on the strength and distance of these vortices. Ignoring these vortices, the reduced frequency, k, can be assumed zero. Thus $C(k)$ or Theodorsen's function will be equal to 1 and the flow regime correspond to this assumption called quasi steady flow. In this case, equations for lift and moment acting on an element of a swept rectangular wing can be written as below:

$$
\begin{aligned}
dL = \pi\rho U_n{}^2 b_n \Bigg[& 2\left[\xi' + \alpha + \frac{1}{AR}\frac{\partial\xi}{\partial\eta}\tan\Lambda + \left(\frac{1}{2} - a_h\right)\left(\alpha' + \frac{1}{AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda\right)\right] \\
& + \left[\xi'' + \alpha' + \frac{2}{AR}\frac{\partial\xi'}{\partial\eta}\tan\Lambda + \frac{1}{AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda + \frac{1}{AR^2}\frac{\partial^2\xi}{\partial\eta^2}\tan^2\Lambda\right] \\
& - a_h\left[\alpha'' + \frac{2}{AR}\frac{\partial\alpha'}{\partial\eta}\tan\Lambda + \frac{1}{AR^2}\frac{\partial^2\alpha}{\partial\eta^2}\tan^2\Lambda\right]\Bigg]
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
dM_{E.A} = \pi\rho U_n{}^2 b_n{}^2 \Bigg[& (1+2a_h)\left[\xi' + \alpha + \frac{1}{AR}\frac{\partial\xi}{\partial\eta}\tan\Lambda + \left(\frac{1}{2} - a_h\right)\left(\dot{\alpha} + \frac{1}{AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda\right)\right] \\
& - \left[\left(\frac{1}{2} - a_h\right)\dot{\alpha} + \frac{1}{2AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda\right] \\
& + a_h\left[\xi'' + \frac{2}{AR}\frac{\partial\xi'}{\partial\eta}\tan\Lambda + \frac{1}{AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda + \frac{1}{AR^2}\frac{\partial^2\xi}{\partial\eta^2}\tan^2\Lambda\right] \\
& - \left(\frac{1}{8} + a_h{}^2\right)\left[\alpha'' + \frac{2}{AR}\frac{\partial\alpha'}{\partial\eta}\tan\Lambda + \frac{1}{AR^2}\frac{\partial^2\alpha}{\partial\eta^2}\tan^2\Lambda\right]\Bigg]
\end{aligned} \tag{4}
$$

Since this aerodynamic model is true in steady state aerodynamics, it is possible to apply Prandtl-Glaurt compressibility correction factor β, in order to modify it for obtaining aerodynamic loads acting on wing in a compressible flow with Mach numbers less than 0.7. This limitation is mainly due to highly nonlinear behavior of flow in transonic regime.

$$
\beta = \frac{1}{\sqrt{1 - M_\infty{}^2}} \tag{5}
$$

where M_∞ is freestream Mach number. Comparing equations (1) and (2) with equations (3) and (4), it is obvious that all of the terms of the quasi steady equations are in common with the unsteady ones. Since these two aerodynamic models are both developed under linear aerodynamic theory, the modified quasi steady and unsteady equations can be superimposed. This will lead to modify only noncirculatory terms of the unsteady equations which are not concerned with the counter rotating vortices in the surrounding flow. Therefore, the proposed aerodynamic model is presented as follows for an element of a swept wing oscillating in an unsteady compressible flow:

$$
\begin{aligned}
dL = \pi\rho U_n{}^2 b_n \Bigg[& 2\begin{bmatrix} \xi'(0) + \alpha(0) + \frac{1}{AR}\frac{\partial\xi(0)}{\partial\eta}\tan\Lambda + \left(\frac{1}{2} - a_h\right)\alpha'(0) \\ + \frac{1}{AR}\left(\frac{1}{2} - a_h\right)\frac{\partial\alpha(0)}{\partial\eta}\tan\Lambda \end{bmatrix}\phi(\tau) \\
& + 2\int_0^\tau \phi(\tau - \sigma)\begin{bmatrix} \xi''(\sigma) + \alpha'(\sigma) + \frac{1}{AR}\frac{\partial\xi'(\sigma)}{\partial\eta}\tan\Lambda + \left(\frac{1}{2} - a_h\right)\alpha''(\sigma) \\ + \frac{1}{AR}\left(\frac{1}{2} - a_h\right)\frac{\partial\alpha'(\sigma)}{\partial\eta}\tan\Lambda \end{bmatrix} d\sigma \\
& + \beta \times \left[\xi'' + \alpha' + \frac{2}{AR}\frac{\partial\xi'}{\partial\eta}\tan\Lambda + \frac{1}{AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda + \left(\frac{1}{AR}\right)^2\frac{\partial^2\xi}{\partial\eta^2}\tan^2\Lambda\right] \\
& - \beta \times a_h\left[\alpha'' + \frac{2}{AR}\frac{\partial\alpha'}{\partial\eta}\tan\Lambda + \left(\frac{1}{AR}\right)^2\frac{\partial^2\alpha}{\partial\eta^2}\tan^2\Lambda\right]\Bigg],
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
dM_{E.A} = \pi\rho U_n{}^2 b_n{}^2 \Bigg[& 2\left(\frac{1}{2} + a_h\right)\begin{bmatrix} \xi'(0) + \alpha(0) + \frac{1}{AR}\frac{\partial\xi(0)}{\partial\eta}\tan\Lambda \\ + \left(\frac{1}{2} - a_h\right)\alpha'(0) + \frac{1}{AR}\left(\frac{1}{2} - a_h\right)\frac{\partial\alpha(0)}{\partial\eta}\tan\Lambda \end{bmatrix}\phi(\tau) \\
& + 2\left(\frac{1}{2} + a_h\right)\int_0^\tau \phi(\tau - \sigma)\begin{bmatrix} \xi''(\sigma) + \alpha'(\sigma) + \frac{1}{AR}\frac{\partial\xi'(\sigma)}{\partial\eta}\tan\Lambda \\ + \left(\frac{1}{2} - a_h\right)\alpha''(\sigma) + \frac{1}{AR}\left(\frac{1}{2} - a_h\right)\frac{\partial\alpha'(\sigma)}{\partial\eta}\tan\Lambda \end{bmatrix} d\sigma \\
& - \beta \times \left[\left(\frac{1}{2} - a_h\right)\alpha' + \frac{1}{2AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda\right] + \beta \times a_h\begin{bmatrix} \xi'' + \frac{2}{AR}\frac{\partial\xi'}{\partial\eta}\tan\Lambda + \frac{1}{AR}\frac{\partial\alpha}{\partial\eta}\tan\Lambda \\ + \left(\frac{1}{AR}\right)^2\frac{\partial^2\xi}{\partial\eta^2}\tan^2\Lambda \end{bmatrix} \\
& - \beta \times \left(\frac{1}{8} + a_h{}^2\right)\left[\alpha'' + \frac{2}{AR}\frac{\partial\alpha'}{\partial\eta}\tan\Lambda + \left(\frac{1}{AR}\right)^2\frac{\partial^2\alpha}{\partial\eta^2}\tan^2\Lambda\right]\Bigg].
\end{aligned} \tag{7}
$$

4. AEROELASTIC EQUATIONS

Like any other aeroelastic system, the well known Lagrange equations can be used to obtain the governing aeroelastic equations of aforementioned aeroelastic system. Assuming the rectangular wing as a uniform cantilever beam, its first mode shapes obtained from Barmby et al. can be used for plunge and pitch degree of freedom. In this manner, Terms concerned with kinetic and potential energies are derived easily. In order to obtain generalized forces corresponding to plunge and pitch displacements, strip theory is used for analytical aerodynamic model of unsteady compressible flow, equations (6) and (7).

Inserting derived equations for kinetic and potential energies and also generalized forces in the Lagrange formulas, the governing aeroelastic equations of a swept rectangular wing in subsonic compressible flow are derived in the time domain as follows:

$$
\begin{aligned}
& c_0\xi_1'' + c_1\alpha_1'' + c_2\xi_1' + c_3\alpha_1' + c_4\xi_1 + c_5\alpha_1 + c_6 w_1 + c_7 w_2 \\
& \quad + c_8 w_3 + c_9 w_4 + c_{10} G(\xi_1) = f(\tau) \ ,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
& d_0\xi_1'' + d_1\alpha_1'' + d_2\alpha_1' + d_3\alpha_1 + d_4\xi_1' + d_5\xi_1 + d_6 w_1 + d_7 w_2 \\
& \quad + d_8 w_3 + d_9 w_4 + d_{10} M(\alpha_1) = g(\tau) \ .
\end{aligned} \tag{9}
$$

where ξ_1 and α_1 are time dependent dimensionless plunge and pitch displacement, the prime sign denotes differentiation with respect to the dimensionless time τ, coefficients c_0, c_1,…, c_{10} and d_0, d_1,…, d_{10} are concerned with nondimensional parameters of the wing, constants of the Wagner's function, freestream mach number, sweep angle and nondimensional parameters of the wing and also assumed mode shapes. They are different from the coefficients of previously derived equations in previous works. Expressions $f(\tau)$ and $g(\tau)$ are dependent on initial conditions, nondimensional time and constants of the Wagner's function. Also, w_1,…,w_4 are the well known integral variable introduced by Lee and Leblanc (1986) and terms $G(\xi_1)$ and $M(\alpha_1)$ are the functions representing concentrated structural nonlinearities.

5. STRUCTURAL NONLINEARITIES

For better understanding of the aeroelastic behavior of a swept wing in the speed upper than linear flutter boundary, it is essential to consider its aeroelastic nonlinearities. A comprehensive review of these nonlinearities includes aerodynamic and structural ones has been presented by Lee et al. (1999).

Distributed structural nonlinearities governed by elastodynamic deformations that affect the whole structure are beyond the scope of this paper. In the other hand, in this study concentrated structural nonlinearities such as cubic and freeplay ones commonly can be found in control mechanisms or connecting parts of the wing. They are usually modeled as a spring with a nonlinear stiffness coefficient. Cubic nonlinearities are classified as being either softening or hardening. They are mathematically represents by following formula for the pitch degree of freedom (Lee et al., 1999):

$$M(\alpha_1) = \beta_\alpha \alpha_1 + \beta_{\alpha^3} \alpha_1^{\ 3} \qquad (10)$$

However, when a system contains freeplay nonlinearity for small displacements, the spring shows different behavior in compare with the time it is exposed to larger displacements. In other words, this kind of nonlinearity for the pitch degree of freedom can be presented by equations written below [refer to figure 3 for definition of parameters of equation (11)] (Lee et al. 1999):

$$M(\alpha_1) = \begin{Bmatrix} M_0 + \alpha - \alpha_f & \alpha < \alpha_f \\ M_0 + M_f(\alpha - \alpha_f) & \alpha_f \le \alpha \le \alpha_f + \delta \\ M_0 + \alpha - \alpha_f + \delta(M_f - 1) & \alpha_f + \delta < \alpha \end{Bmatrix} \qquad (11)$$

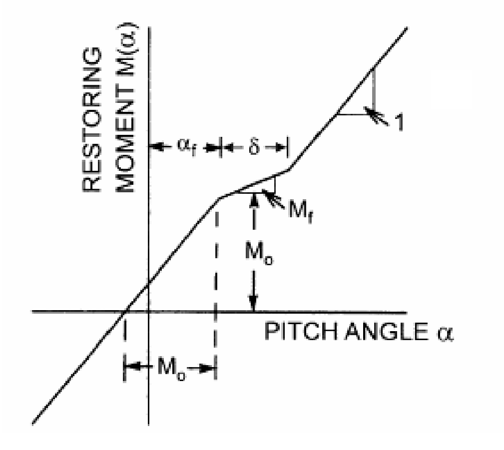

Figure 3: *Restoring moment versus pitch angle as the mathematical model of freeplay nonlinearity*

6. RESULTS AND DISCUSSION

Linear aeroelastic analysis of the swept wing is carried out in order to verify the derived formulations. For this reason, experimental data for the flutter speed of the uniform cantilever wing of Barmby et al. (1950) are used. The physical characteristics of the tested wings and their nondimensional parameters have been presented in their report. The standard 4th order Runge-Kutta method was applied to obtain numerical solutions of the resulting set of first-order differential equations and as a result the aeroelastic instability boundary of the system. Experimental and calculated flutter speeds for seven different cases are given in Table 1. The flow Mach numbers in all cases are between 0.3 and 0.7, in compressible flow regime. It can be seen that this formulation provides good agreement with the experimental data, and the error of our proposed equations with the modified aerodynamic model compared to the experimental data in all seven cases is below 10 percent. In the strip theory approximation, the chord wise pressure distribution at any spanwise station is assumed to depend only on the downwash at that station given by the two-dimensional aerodynamic theory and to be independent of the downwash at any other spanwise station and it is the main reason of the difference between experimental and calculated data for the flutter speed. However, by our comparison, it is shown that this error can be ignored. The assumption of two-dimensional flow, applying only the first mode shape of a cantilever beam and modifying only some terms of unsteady aerodynamic model can be the other sources of error. In table 2 where the numerical results of governing equations developed using either modified or unmodified models are given, the effectiveness of our modification in aerodynamic model is obvious. It is shown that our modification in aerodynamic model lowers the error of the results for flutter speed.

Case	Experimental flutter speed (m/sec)	Calculated flutter speed (m/sec)	Error (percentage)
40A-2	69.291	71.953	+3.84
40D-2	79.126	79.264	+0.17
62-3	78.232	83.896	+7.24
63-4	80.020	84.747	+5.91
72-1	88.067	86.426	-1.86
73-1	86.279	90.046	+4.37
75-1	80.914	86.044	+6.34

Table 1: Calculated and experimental results for flutter speed

Case	Calculated flutter speed using modified aerodynamic model (m/sec)	Calculated flutter speed using unmodified model (m/sec)
40A-2	71.953	78.482
40D-2	79.264	87.730
62-3	83.896	92.153
63-4	84.747	93.382
72-1	86.426	97.299
73-1	90.046	101.17
75-1	86.044	94.071

Table 2: Calculated results for flutter speed using modified and unmodified aerodynamic models

Using this formulation and applying proper mathematical model instead of $G(\xi_1)$ and $M(\alpha_1)$, it is very easy to treat with the concentrated structural nonlinearity such as cubic and freeplay nonlinearities. Here, these structural nonlinearities are considered and some phenomena like limit cycle oscillations and divergence below linear flutter speed are observed. For wing containing hardening cubic nonlinearity (figure 4), $\beta_{\alpha^3} = 3$ and $\beta_{\xi^3} = 0$, LCOs are seen when the free stream speed is beyond linear flutter speed. Due to the difference in flutter speed prediction between modified and unmodified equations, the unmodified equations predict different aeroelastic behaviors when aeroelastic nonlinearities are considered. For instance, in a case containing cubic nonlinearity (figure 4), in a flow condition where LCO must be predicted, unmodified equations predict stable oscillations. This is also true for cases with softening cubic nonlinearity.

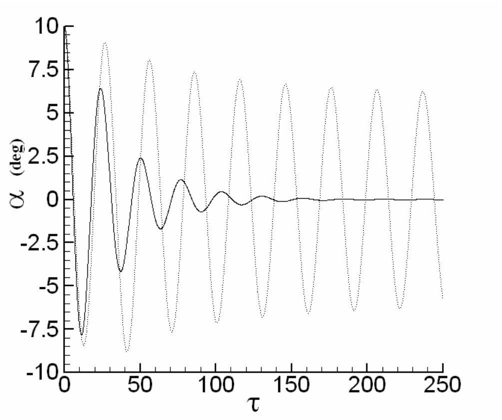

Figure 4: *Wing tip pitch displacement versus τ for case 40D-2 containing hardening cubic nonlinearity at $U = 80.057$ m/sec with different aerodynamic models ; ..... modified, ———— unmodified.*

Case	Calculated nonlinear flutter speed using modified aerodynamic model (m/sec)	Calculated nonlinear flutter speed using unmodified aerodynamic model (m/sec)
40D-2	77.235	85.423
63-4	83.103	91.861
72-1	85.112	95.901

Table 3: Calculated results for nonlinear flutter speed of swept wing containing softening cubic nonlinearity using modified and unmodified aerodynamic models

As seen in table 3 for tested cases with $\beta_{\alpha^3} = -3$, the results for nonlinear flutter speeds differ considerably from modified ones.

In this paper, different methods are used in nonlinear analyze of derived equations. For instance, see figures 5 to 7 for LCO frequency, pitch and plunge amplitudes, respectively. Results of harmonic balance (HB) method are in a close agreement with numerical results. The higher the order of HB method, the more accurate results are obtained. In this figures, it is obvious that the error of the results obtained from unmodified equations is considerable in comparison with modified equations solution. Furthermore, this conclusion is true for swept wings containing freeplay nonlinearity in a subsonic compressible flow as case 40D-2 containing freeplay with $\alpha_f = M_0 = 0.25^\circ$, $\delta = 0.5^\circ$ and $M_f = 0$ in figure 8. Again, the unmodified equations predict stable situation while modified ones predict LCO. As a conclusion, it can be noted that the effect of this modification is of great importance.

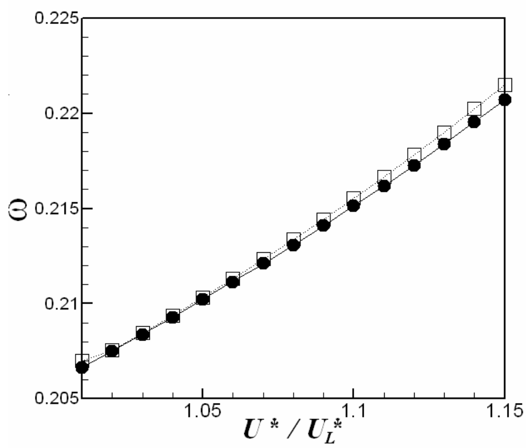

Figure 5: *Wing tip LCO frequency (Case 40D-2): ————, numerical result; ⋯□⋯, HB1; ●, HB3.*

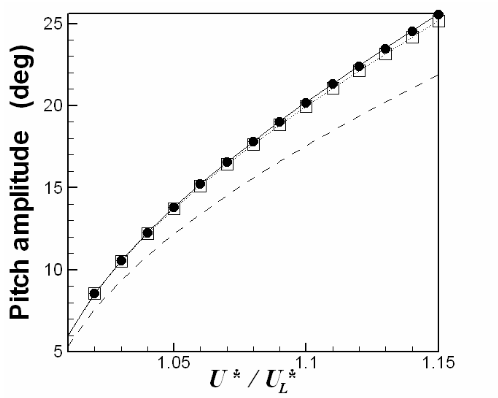

Figure 6: *Wing tip LCO pitch amplitude for case 40D-2:* ——, *numerical result;* ⋯□⋯, *HB1;* ●, *HB3;* — — —, *unmodified model*

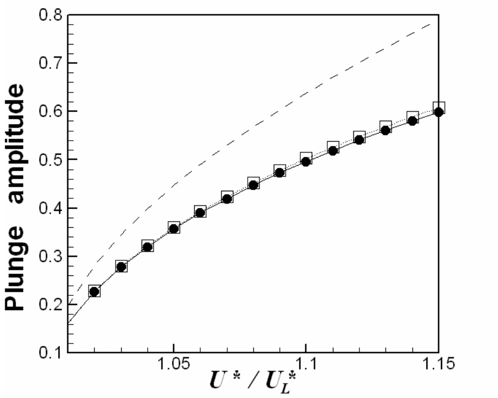

Figure 7: *Wing tip LCO plunge amplitude for case 40D-2:* ——, *numerical result;* ⋯□⋯, *HB1;* ●, *HB3;* — — —, *unmodified model*

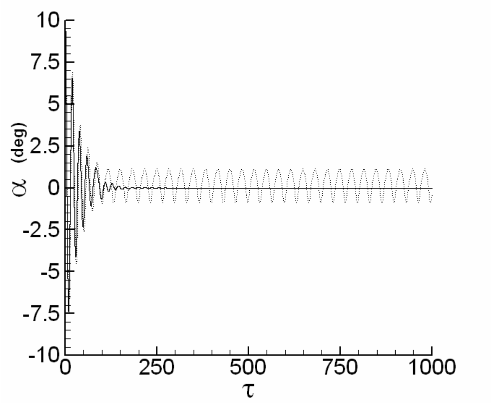

Figure 8: *Wing tip pitch displacement versus nondimessional time for case 40D-2 containing freeplay nonlinearity at* $U = 67.374$ *m/sec with different aerodynamic models ;*
..... *modified,* —— *unmodified.*

7. CONCLUSION

In the current work, governing equations of a two degree of freedom swept wing in compressible flow were derived through modifying a previously developed aerodynamic model for incompressible flow using Prandtl-Glaurt compressibility correction factor. The presented results in this paper clearly showed the effectiveness of this modification to obtain more reliable solutions. This is not only true for linear analysis but also for nonlinear one.

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