FLOW-INDUCED VIBRATIONS OF A NON-BUOYANT SPHERE

Hyeok Lee
FLAIR, Department of Mechanical and Aerospace Engineering, Monash University, Melbourne, Victoria 3800, Australia

Mark C. Thompson
FLAIR, Department of Mechanical and Aerospace Engineering & Division of Biological Engineering, Monash University, Melbourne, Victoria 3800, Australia

Kerry Hourigan
Division of Biological Engineering & FLAIR, Department of Mechanical and Aerospace Engineering, Monash University, Melbourne, Victoria 3800, Australia

ABSTRACT

The current study explores the behaviour of a neutrally buoyant sphere in a uniform flow using well resolved numerical simulations and water channel experiments. It is found that there exist seven different flow regimes within the range of the Reynolds number = [50, 800] according to the sphere response. Regime I (Re = [50, 205]) and Regime II (Re = [210, 260]) are characterised by steady flow structure without body movement except the loss of axisymmetry in Regime II. The sphere starts to vibrate from Regime III (Re = [270, 280]). Regime IV (Re = [300, 330]) shows suppressed body oscillation and steep decrease of off-centered distance in the plane normal to streamwise direction (yz plane). In Regime V (Re = [335, 550]), the sphere oscillates around (0,0) in yz plane. The sphere of Regime VI (Re = [600, 800]) oscillates rather irregularily. The transitions are compared with those for a fixed sphere. A series of experiments is carried out in a water channel to complement and validate the numerical findings. The experiments recorded the position of the sphere and covered the range of the Reynolds number Re = [700, 4000], extending the range of the numerical study. Within the Reynolds range = [700, 800], where the results of the simulations and experiments overlap, and corresponds to the Regime VI, it is observed that the response of the sphere is irregular. This verifies the existence of Regime VI which has been found in the numerical study. For Re ≥ 2000*, it is also observed that the sphere motion reverts from highly irrgular to quasi-circular motion in the plane normal to incoming flow as the Reynolds number is increased further.*

1. INTRODUCTION

Flow-induced vibration (FIV) of structures is of practical interest to many fields of engineering; for example, it can cause vibrations of heat exchanger tubes, and it influences the dynamics of offshore risers. It is important to the design of civil engineering structures such as bridges and chimney stacks, as well as to the design of marine and land vehicles, and it can cause large amplitude vibrations of tethered structures in the ocean. In blood flows, cells such as platelets and leukocytes undergo tethering to vessel walls leading to flapping. The practical significance of FIV has led to a large number of fundamental studies, many of which are discussed in the comprehensive reviews of Sarpkaya (1979, 2004), Griffin and Ramberg (1982), and Williamson and Govardhan (2004). Here, new results for a non-buoyant tethered sphere are presented.

A major difference in the wake transition behaviour of the sphere and the circular cylinder wake is that the sphere wake becomes asymmetrical prior to a transition to unsteady flow, whereas the cylinder wake does not become asymmetrical until the wake goes unsteady (Williamson, 1988). For the sphere wake, the transition from attached to separated flow at the rear of the sphere has been found from direct numerical simulations to be $Re_1 = 20$ (Johnson and Patel, 1999; Tomboulides and Orszag, 2000). As the Reynolds number increased, the wake remains steady and axisymmetric up to $Re_2 = 211$ (Johnson and Patel, 1999).

The transition to asymmetry is through a regular bifurcation, i.e., steady to steady flow. Tomboulides and Orszag (2000) determined the transition to occur at $Re_2 = 212$. Johnson and

Patel (1999), experimentally and numerically, found the resulting wake to undergo a regular bifurcation through a shift of the steady recirculating bubble behind the sphere from the axis. The early dye visualizations of Magarvey and Bishop (1961) found that a double-threaded wake exists in the range of $Re = [200, 350]$. Since then, more accurate experiments and numerical simulations have refined this range considerably. The two threads of vorticity trailing downstream from the recirculation bubble has also been predicted numerically by Tomboulides and Orszag (2000).

The steady asymmetric wake undergoes a further transition to unsteady flow at $Re = 277.5$ as determined by stability analysis (Natarajan and Acrivos, 1993). Tomboulides and Orszag (2000) and Johnson and Patel (1999) support this bifurcation scenario, with unsteady wakes being observed for $Re > 280$. In all cases, the unsteady wake consisted of hairpin-shaped vortex loops shedding downstream from the sphere, in the same plane as that of the initial steady asymmetric structures. The periodic wake of the sphere remains planar-symmetric up to $Re \approx 375$, as observed numerically by Mittal (1961).

The majority of early work on tethered spheres was concerned with the action of surface waves on tethered buoyant structures (Harlemann and Shapiro, 1961; Shi-Igai and Kono, 1969). They employed empirically obtained drag and inertia coefficients for use in Morison's equation. The tethered sphere was found to vibrate vigorously due to the waves as expected. However, the coupling of the wave motion and the dynamics of the sphere made it difficult to understand the underlying dynamics of the sphere motion.

Results of research concerning fully submerged tethered bodies was first published by Govardhan and Williamson (1992) who found that a tethered sphere does indeed vibrate in a uniform flow. In particular, they found that it will oscillate vigorously at a transverse peak-to-peak amplitude of about two diameters. The transverse oscillation frequency was at half the frequency of the in-line oscillations, although the natural frequencies of both the in-line and transverse motions were the same. In the Reynolds number range of their experiments ($Re < 12,000$), the response amplitude was a function of the flow velocity. However, conclusions regarding the synchronization of natural and vortex formation frequencies were lacking due to the large scatter in the literature of the vortex formation frequency in the wake of a sphere. Govardhan and Williamson (1992) noted that the maximum root-mean-square (RMS) amplitude was approximately 1.1 diameters, regardless of the mass ratio. It was further found that the vortex shedding frequency for a fixed sphere matched the natural frequency for the tethered sphere at the same reduced velocity, $U^* \approx 5$, at which the local peak in the RMS response occurred. This suggests that the local peak in the RMS response is caused by a resonance between the natural frequency of the tethered body and the wake vortex shedding frequency, and is known as Mode I response. For high mass ratios (typically $m^* >> 1$), the oscillation frequency at high U^* tended toward the natural frequency. However, it is interesting to note that the oscillation frequency for low mass ratios ($m^* < 1$) at high $U*$. did not correspond to either the natural frequency or the vortex shedding frequency for a fixed sphere. Through wind tunnel experiments, Jauvtis et al (2001) were able to study mass ratios between $m^* = 80$ and 940 and reduced velocities in the range of $U^* = [0, 300]$. For the sphere of $m^* = 80$, they found a new mode of vibration (which they define as Mode III) and which extends over a broad regime of U^* from 20 to 40. Govardhan and Williamson (2005) extended their previous study on sphere vortex-induced vibration and found that the body oscillation frequency (f) is of the order of the vortex shedding frequency of the fixed body (f_{vo}), and that there exist two modes of periodic large-amplitude oscillation, defined as Modes I and II (Govardhan and Williamson, 1992; Williamson and Govardhan, 1997), separated by a transition regime exhibiting non-periodic vibration.

In the case of a very light tethered body, the transition between modes is quite distinct, especially when the response amplitude is plotted versus the parameter f_{vo}/f, where a jump between modes is clearly exhibited They noted that the phase of the vortex force relative to sphere dynamics is quite different between modes I and II. This difference in the phase of the vortex force is consistent with the large difference in the timing of the vortex formation between modes, which was observed from the vorticity measurements for the light sphere vibrations. Mode III cannot be explained as the classical lock-in effect, since between 3 and 8 cycles of vortex shedding occurs for each cycle of sphere motion.

For reduced velocities beyond the regime of Mode III, another vibration mode was discovered that grew in amplitude and persisted to the limit of flow speed in the wind tunnel (Jauvtis et al, 2001). The sphere dynamics of this Mode IV were characterized by intermittent bursts of large-amplitude vibration, in contrast to the periodic vibrations of Modes I, II and III. In previous numerical studies, Pregnalato (2003) found that a buoyant tethered sphere oscillates at large amplitude over a wide range of reduced velocity, which is similar to the previous studies (Govardhan and Williamson, 1992; Williamson and Govardhan, 1997). He adopted a spectral element method and a coordinate transform to solve the combined fluid-structure system. Even though the flow was within laminar regime ($Re = 500$), he observed that Modes I, II and III similar to those found experimentally at higher Re.

2. METHODS

Both experiments and numerical studies have been undertaken as part of this research program.

2.1. Computational Methods

In the present study, the dynamics of a tethered sphere were investigated for mass ratio of $m^* = 1.0$, i.e., neutrally buoyant (see schematic in Figure 1). A high-order, three-step, time-splitting scheme was employed to evolve the velocity and pressure field in time. For the spatial discretisation, a spectral-element method was used, with a global Fourier spectral discretisation in the third dimension, which is the azimuthal direction in the present study (see sample mesh in Figure 2). This approach has been employed previously for the case of the flow past a circular cylinder by Karniadakis and Triantafyllou (1992) and Thompson et al (1996) (see Thompson et al (2006) for more information regarding the method). For this problem, the sphere moves under the influence of fluid forcing. Determining the sphere motion was achieved by solving the equations of motion for the sphere, using the total pressure and viscous force acting on it, simultaneously with the flow field integration. A non-inertial frame was used in which the sphere was stationary.

2.2. Experimental Methods

The experiments were conducted in a recirculating free surface water channel. Water was recirculated through the channel using a centrifugal pump controlled by an electronic controller to give flow speeds between 0.047 and 0.456m/s in the glass working section. The working section had a width of 0.6 m, a height of 0.8m and a length of 4.0m and was positioned between two large tanks of water. Upstream of the working section, water flowed through a honeycomb and thin wire mesh before going through a 9:1 contraction to the working section. The combination of the screens and contraction yielded a turbulence level of less than 1.0%. A sphere made of Perspex with a diameter of 16mm was used for the experiments. The sphere was manufactured so that it could be separated into halves. Each half was hollow to allow its buoyancy to be adjusted by adding material inside. For the experiments, sponge was inserted to adjust its buoyancy and mass distribution. A thin string with the diameter of 0.1mm was connected to the sphere as a tether, and then the tether was attached to 0.315mm wire, which was under vertically tension, between the bottom of the working section and the ceiling directly above it as a support. A sequence of images was captured using a Pixelfly camera to locate the centre of the sphere as a function of time. Each image had 1360x1024 pixels. Capture rates of 2fps (frames per second), 4fps and 8fps were used to record the images, with 4fps used in the majority of cases.

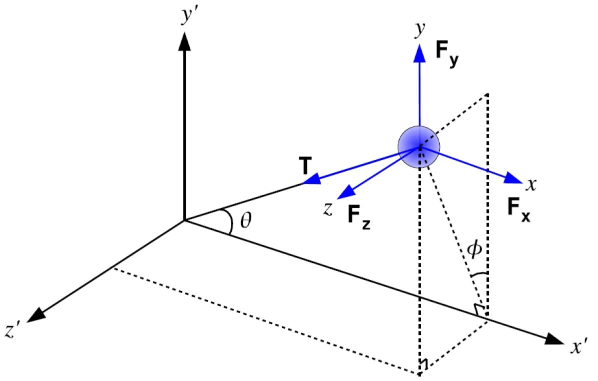

Figure 1: *Coordinate system and geometry of tethered sphere and forces (flow forces* **F** *and tether tension* **T***).*

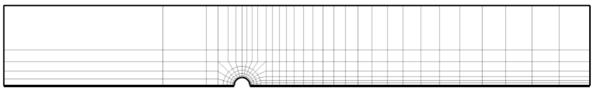

Figure 2: *A sample mesh used for a tethered sphere system, showing macro elements. The mesh is extended along the azimuthal direction with 24 Fourier modes.*

3. RESULTS

It was found that as the Reynolds number, Re, based on the mean flow velocity and sphere diameter, D, was increased, the tethered sphere ex-

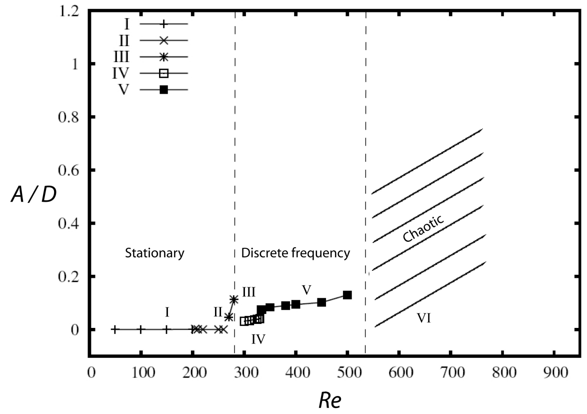

Figure 3: *Amplitude of radial oscillations, A.*

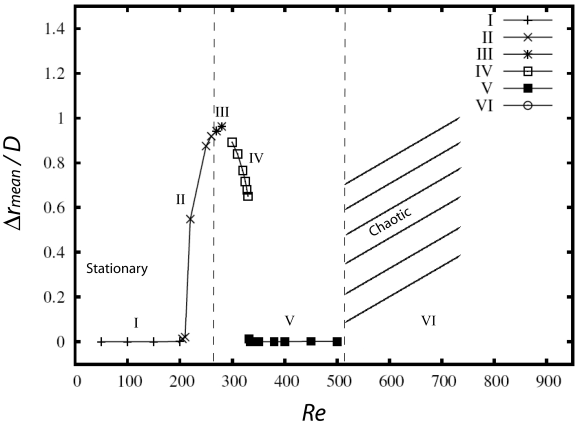

Figure 4: *Time-mean radial offset, Δr_{mean}.*

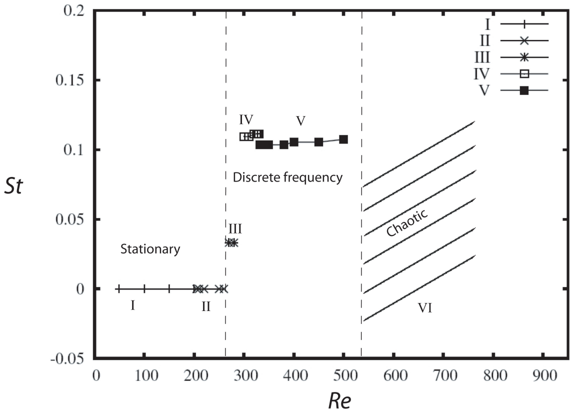

Figure 5: *Strouhal number of oscillations, St.*

perienced six different regimes of wake structure and body motion, which had some similarity to those of the fixed sphere (see Figures 3, 4 and 5). The regimes shown are defined mainly by the amplitude and the frequency of the body oscillation. In addition, the radial offset, Δr_{mean}, from the streamwise axis of the time-mean position of the sphere is used to identify the first and the second flow regimes. Regime I, over $Re = 6 - 205$, is characterised by a steady axisymmetric flow structure without body movement.

The sub-states of the separation bubble forming at the rear of the body are included in this regime because they maintain an axisymmetrical flow field. The second regime (Regime II) is also steady but with the loss of axisymmetry, observed within the range Re = 210-250. Here, planar-symmetry emerges with the appearance of the double-threaded wake, similar to the case of the fixed sphere. In Regime II, the sphere is steady and the radial off-set, Δr_{mean}, increases with Re. The sphere starts to vibrate from $Re = 270$, initiating Regime III; here the Strouhal number is 0.034 and Δr_{mean} continues to increase slightly. This critical Reynolds number is similar to that for a fixed sphere, $Re = 272-3$ (Ghidersa and Dusek, 2000; Thompson et al, 2001).

Regime IV begins at $Re = 300$, showing a decreased body oscillation amplitude, A, and a steep decrease of Δr_{mean}. The time-mean position of the sphere in Regime V is on the streamwise axis, the Strouhal number is approximately 0.11 (still less than $St = 0.132$ for a fixed sphere), and the oscillation amplitude increases slightly from Regime IV. In Regime VI, the vibrations become chaotic and the sphere undergoes chaotic wandering, having no restoring buoyant forces; here, the oscillation amplitude becomes less meaningful and very long integration times will be required to determine more precisely the time-mean position of the sphere.

The streamwise vorticity in the wakes of each regime is shown in Figure 6 (except for Regime I for which the wake is axisymmetric). In Regime II, the sphere is off-centred from the axis of symmetry although the flow is steady. This is due to the asymmetry of the double-threaded vortex loops. In Regime III, the vortices start to shed periodically; however, the hairpin-shaped vortex loops do not appear until Regime IV, where they are found to maintain their planar-symmetry. These loops begin to lose their planar symmetry within Regime V. In Regime VI, the wake shows irregular behaviour, coupled with chaotic sphere wandering.

Constraints imposed by the experimental apparatus currently fix the lower Reynolds number limit to be 700. This overlaps with the upper end of the numerical prediction range. From the experimental study over the Reynolds number range [700, 4000], it is verified that there exists Regime VI which shows chaotic behaviour of the body. Moreover, it is found that there is another regime, Regime VII, which shows quasi-circular motion as the Reynolds number is increased above about 2000. Note that Provansal et al (2004) found that a vertically suspended tethered sphere with $m^* = 2.43$ in a vertical water channel could undergo elliptic orbits, albeit at lower Reynolds numbers (Re=600 - 800). Sample trajectories showing the behaviour of the sphere motion at different Reynolds numbers from current predictions and experiments are shown in Figure 7.

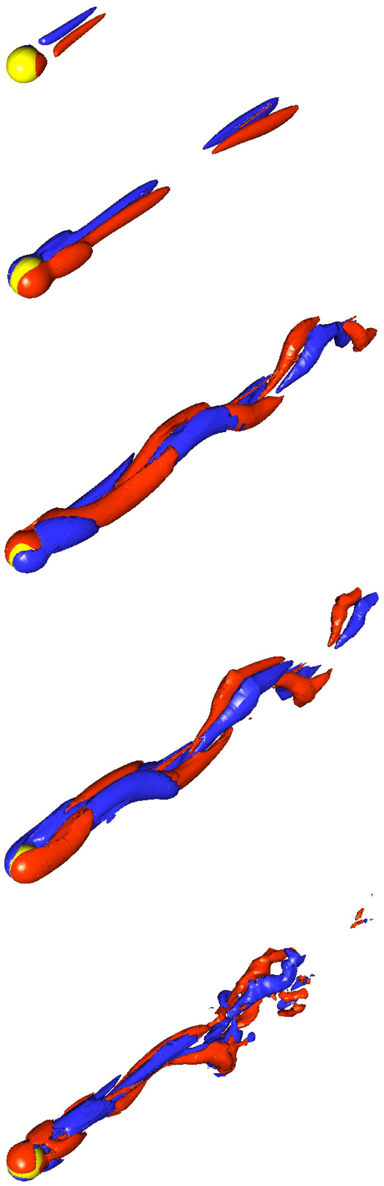

Figure 6: *Streamwise wake vorticity. Plots represent Regimes II, III, IV, V and VI, successively, from the top. Flow is from lower left to upper right*

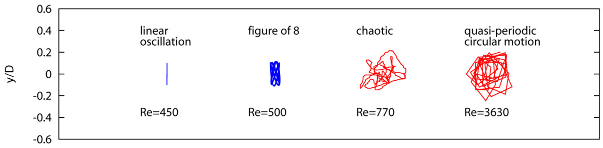

Figure 7: *Sample trajectories of the sphere in the plane normal to the stream.*

4. CONCLUSIONS

Both numerical and experimental studies have been undertaken of the wake and motion of a tethered neutrally buoyant sphere in a uniform flow. Seven regimes showing distinct wake structure and/or body motion have been found. Regime I (Re = [50, 205]) showed steady axisymmetry flow structure without body movement. Regime II (Re = [210, 260]) was also characterised by steady flow structure except the loss of axisymmetry. The sphere started to oscillate at Regime III (Re = [270, 280]). Regime IV (Re = [300, 330]) showed suppressed body oscillation and the off-centered distance decreases rapidly. In Regime V (Re = [335, 550]), the sphere vibrated around the centre of the plane normal to the stream . In Regime VI (Re = [600, 800]), the sphere oscillated rather irregularly at larger amplitude than other regimes. In Regime VII ($Re > 2000$), the oscillation transformed from an irregular pattern to become quasi-circular.

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