ALE DISCONTINUOUS GALERKIN SIMULATION OF COMPRESSIBLE FLOW IN TIME-DEPENDENT DOMAINS

Miloslav Feistauer, Václav Kučera, Jaroslava Prokopová
Charles University Prague, Faculty of Mathematics and Physics, Sokolovská 83, 186 75 Praha 8, Czech Republic

ABSTRACT

This work is concerned with the numerical solution of inviscid compressible fluid flow through a channel with moving walls. We present two formulations of the Euler equations describing compressible flow in the ALE (Arbitrary Lagrangian-Eulerian) form. These two formulations are discretized in space by the discontinuous Galerkin method. The time discretization is carried out with the aid of a semi-implicit linearized scheme resulting in only one linear system on each time level. Currently, the motion of the wall is prescribed by a given formula. As an example we present here the flow simualtion in a channel with a pulsation sinusoidal bump at the lower wall.

1. INTRODUCTION

In a number of problems of science and technology we meet the necessity to solve initial-boundary value problems in time dependent domains. The solution of such problems is very difficult and this is the reason that practically all works dealing with evolution partial differential equations consider problems in domains that are independent of time. Numerical simulation of processes in time dependent domains can be solved with the aid of the ALE (arbitrary Lagrangian-Eulerian) method proposed e.g. in Nomura and Hughes (1992). This method can be applied in the framework of various numerical techniques, but the solution of technically relevant problems requires the use of sufficiently accurate, robust and flexible method. A method suitable for the solution of complex problems describing compressible flow is the discontinuous Galerkin (DG) finite element method using advantages of the finite volume as well as finite element approaches and allowing to obtain schemes with a higher order accuracy in a natural way. The DG method is based on the idea to approximate the solution of an initial-boundary value problem by piecewise polynomial functions over a finite element mesh without any requirement on interelement continuity.

Here we present the ALE version of the DG method for the solution of inviscid compressible flow in time dependent domains. We start here from the paper Dolejší and Feistauer (2004), where a DG semi-implicit method is described, requiring the solution of a linear algebraic system on each time level. It was shown (Feistauer and Kučera (2007)) that this method is unconditionally stable and allows the solution of flows with all Mach numbers. In this paper we assume that the dependence of the domain on time is known, but it is only the first step to the solution of a complete coupled problem, when the shape of the domain is influenced by a moving fluid. The developed technique will be applied to the modelling of an air flow through the glottal space of a human vocal tract.

2. CONTINUOUS PROBLEM

We consider inviscid compressible flow in a bounded domain $\Omega_t \subset \mathbb{R}^2$ depending on time $t \in [0,T]$. Let the boundary of Ω_t consist of three different parts $\partial\Omega_t = \Gamma_I \cup \Gamma_O \cup \Gamma_{W_t}$, where Γ_I and Γ_O represent the inlet and outlet and Γ_{W_t} represents impermeable walls that may move in dependence on time.

We shall discretize the Euler equations written in the conservative form (Feistauer, Felcman and Straškraba (2003)):

$$\frac{\partial \boldsymbol{w}}{\partial t} + \sum_{s=1}^{2} \frac{\partial \boldsymbol{f}_s(\boldsymbol{w})}{\partial x_s} = 0, \text{ in } \Omega_t,\ t \in (0,T), \quad (1)$$

$$\boldsymbol{w} = (\rho, \rho v_1, \rho v_2, e)^{\mathrm{T}} \in \mathbb{R}^4,$$

$$\boldsymbol{f}_i(w) = (\rho v_i, \rho v_1 v_i + \delta_{1i} p, \rho v_2 v_i + \delta_{2i} p, (E+p) v_i)^{\mathrm{T}}.$$

We use the following notation: ρ - fluid density, p - pressure, $\boldsymbol{v} = (v_1, v_2)$ - velocity vector, E - total energy. This system is equipped with standard inlet and outlet boundary conditions. On

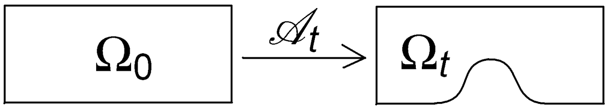

Figure 1: The ALE mapping $\mathcal{A}_t$.

the moving wall we impose the impermeability condition $\boldsymbol{v}\cdot\boldsymbol{n} = \boldsymbol{z}\cdot\boldsymbol{n}$, where $\boldsymbol{n}$ is the unit outer normal to Γ_{W_t} and $\boldsymbol{z}$ is the speed of the moving boundary Γ_{W_t} (see below). To close system (1), we add the relation for pressure derived from the equation of state:

$$p = (\gamma - 1)\,(E - \rho|\boldsymbol{v}|^2/2). \qquad (2)$$

Here $\gamma > 1$ is the Poisson adiabatic constant.

3. ALE FORMULATION

The dependence of the domain on time is taken into account with the aid of a regular ALE mapping from a reference domain Ω_0 onto the current configuration Ω_t:

$$\mathcal{A}_t : \overline{\Omega}_0 \to \overline{\Omega}_t, \text{ i.e. } \mathcal{A}_t : \boldsymbol{X} \mapsto \boldsymbol{x} = \boldsymbol{x}(\boldsymbol{X}, t). \quad (3)$$

Given such a mapping, we define the ALE velocity:

$$\tilde{\boldsymbol{z}}(\boldsymbol{X}, t) = \frac{\partial}{\partial t}\mathcal{A}_t(\boldsymbol{X}), t \in [0, T], \boldsymbol{X} \in \Omega_0, \ (4)$$
$$\boldsymbol{z}(\boldsymbol{x}, t) = \tilde{\boldsymbol{z}}(\mathcal{A}_t^{-1}(\boldsymbol{x}), t),\ t \in [0, T],\ \boldsymbol{x} \in \overline{\Omega}_t$$

and the ALE derivative of a function $f = f(\boldsymbol{x}, t)$ defined in Ω_t:

$$\frac{D^A}{Dt} f(\boldsymbol{x}, t) = \frac{\partial \tilde{f}}{\partial t}(\boldsymbol{X}, t)|_{\boldsymbol{X} = \mathcal{A}_t^{-1}(\boldsymbol{x})}, \qquad (5)$$

where

$$\tilde{f}(\boldsymbol{X}, t) = f(\mathcal{A}_t(\boldsymbol{X}), t),\ \ \boldsymbol{X} \in \Omega_0.$$

It is possible to show that

$$\frac{D^A f}{Dt} = \frac{\partial f}{\partial t} + \boldsymbol{z}\cdot\nabla f = \frac{\partial f}{\partial t} + \mathrm{div}(\boldsymbol{z} f) - f\,\mathrm{div}\boldsymbol{z}. \ (6)$$

This leads to two different formulations of the Euler equations in ALE form:

$$1)\ \frac{D^A \boldsymbol{w}}{Dt} + \sum_{s=1}^{2} \frac{\partial \boldsymbol{f}_s(\boldsymbol{w})}{\partial x_s} - \boldsymbol{z}\cdot\nabla\boldsymbol{w} = 0, \ (7)$$
$$2)\ \frac{D^A \boldsymbol{w}}{Dt} + \sum_{s=1}^{2} \frac{\partial \boldsymbol{g}_s(\boldsymbol{w})}{\partial x_s} + \boldsymbol{w}\,\mathrm{div}\boldsymbol{z} = 0,$$

where $\boldsymbol{g}_s, s = 1, 2$, are modified inviscid fluxes

$$\boldsymbol{g}_s(w) := \boldsymbol{f}_s(w) - z_s\boldsymbol{w}. \qquad (8)$$

4. SPACE SEMIDISCRETIZATION

Let $\mathcal{T}_h$ be a partition of $\overline{\Omega}_t$ into a finite number of triangles K_i numbered by an index set I. Let $\Gamma_{ij} = \partial K_i \cap \partial K_j$ be a common edge of two triangles or edges, which form the boundary $\partial\Omega$. We use such a numbering that for each $i \in I$ we can define an index set $S(i)$ that $\partial K_i = \cup_{i\in S(i)}\Gamma_{ij}$.

The DG method uses the finite element space of discontinuous piecewise polynomial functions

$$S_h = S^{r,-1}(\Omega, \mathcal{T}_h) = \{v; v|_K \in P_r(K)\ \forall K \in \mathcal{T}_h\}, \qquad (9)$$

where $P_r(K)$ is the space of all polynomials on K of degree $\le r$. Moreover, we shall consider a finite dimensional space of vector-valued functions

$$\boldsymbol{S}_h = [S_h]^4. \qquad (10)$$

Let $\boldsymbol{n}_{ij}$ denote the unit outer normal to ∂K_i on the side Γ_{ij}. Functions $\boldsymbol{\varphi} \in S_h$ are in general discontinuous on interfaces Γ_{ij}. By $\boldsymbol{\varphi}_{ij} = \boldsymbol{\varphi}|_{\Gamma_{ij}}$ and $\boldsymbol{\varphi}_{ji} = \boldsymbol{\varphi}|_{\Gamma_{ji}}$ we denote the values of $\boldsymbol{\varphi}$ on Γ_{ij} considered from the interior and the exterior of K_i, respectively.

4.1. Formulation 1

We multiply system 1) in (7) by a test function $\boldsymbol{\varphi} \in [S_h]^4$ and integrate over $K_i \in \mathcal{T}_h$. With the aid of Green's theorem and summing over all $i \in I$, we obtain the discrete ALE formulation of the Euler equations

$$\sum_{K_i\in\mathcal{T}_h} \int_{K_i} \frac{D^A\boldsymbol{w}}{Dt}\cdot\boldsymbol{\varphi}\,dx + b_h^1(\boldsymbol{w}, \boldsymbol{\varphi}) \quad (11)$$
$$- \sum_{K_i\in\mathcal{T}_h} \int_{K_i} \sum_{s=1}^{2} z_s \frac{\partial\boldsymbol{w}}{\partial x_s}\cdot\boldsymbol{\varphi}\,dx = 0,$$

where we define the discrete convective form $b_h^1(\cdot,\cdot)$ as

$$b_h^1(\boldsymbol{w}, \boldsymbol{\varphi}) = \underbrace{\sum_{i\in I}\int_{K_i}\sum_{s=1}^{2}\boldsymbol{f}_s(\boldsymbol{w})\cdot\frac{\partial\boldsymbol{\varphi}}{\partial x_s}\,dx}_{T_1^1} \quad (12)$$
$$\underbrace{+\sum_{i\in I}\sum_{j\in S(i)}\int_{\Gamma_{ij}}\boldsymbol{H}_f(\boldsymbol{w}_{ij}, \boldsymbol{w}_{ji}, \boldsymbol{n}_{ij})\cdot\boldsymbol{\varphi}\,dS\,.}_{T_2^1}$$

In the term T_2^1, we have incorporated an approximation using a numerical flux $\boldsymbol{H}_f$, as known from the finite volume method. The approximate solution is defined as $\boldsymbol{w}_h \in [S_h]^4$ such that (12) holds for all $\boldsymbol{\varphi}_h \in [S_h]^4$.

4.2. Formulation 2

To obtain the second formulation of our problem, we multiply equation 2) in (7) by a test function $\boldsymbol{\varphi} \in [S_h]^4$ and integrate over $K_i \in \mathcal{T}_h$. Similarly as in the preceding, we obtain

$$\sum_{K_i \in \mathcal{T}_h} \int_{K_i} \frac{D^A \boldsymbol{w}}{Dt} \cdot \boldsymbol{\varphi} \, dx + b_h^2(\boldsymbol{w}, \boldsymbol{\varphi}) \quad (13)$$
$$+ \sum_{K_i \in \mathcal{T}_h} \int_{K_i} \mathrm{div} \boldsymbol{z} \; \boldsymbol{w} \cdot \boldsymbol{\varphi} \, \mathrm{dx} = 0,$$

where we define the discrete convective form $b_h^2(\cdot, \cdot)$ as

$$b_h^2(\boldsymbol{w}, \boldsymbol{\varphi}) = \underbrace{\sum_{i \in I} \int_{K_i} \sum_{s=1}^{2} \boldsymbol{g}_s(\boldsymbol{w}) \cdot \frac{\partial \boldsymbol{\varphi}}{\partial x_s} \, dx}_{T_1^2} \quad (14)$$
$$+ \underbrace{\sum_{i \in I} \sum_{j \in S(i)} \int_{\Gamma_{ij}} \boldsymbol{H}_g(\boldsymbol{w}_{ij}, \boldsymbol{w}_{ji}, \boldsymbol{n}_{ij}) \cdot \boldsymbol{\varphi} \, dS}_{T_2^2}.$$

Again, in term T_2^2, we incorporate a numerical flux $\boldsymbol{H}_g$.

5. SEMI-IMPLICIT TIME DISCRETIZATION

Schemes (12) and (14) represent a system of ordinary differential equations, which we must discretize with respect to time. Explicit time discretization is however undesirable due to a CFL-stability condition, which limits the time step proportionally to the Mach number. A fully implicit scheme presents us with the task of solving a large nonlinear system on each time level. We therefore use the method presented in [1] and adapt it to the ALE setting. A backward Euler method is used and the nonlinear terms in the scheme are linearized using their respective properties. The resulting systems are solved using block-Jacobi preconditioned GMRES or the UMFPACK direct solver.

5.1. Formulation 1

We consider a partition $0 = t_0 < t_1 < t_2 \ldots$ of the time interval $(0, T)$ and set $\tau_k = t_{k+1} - t_k$. We use the symbol $\boldsymbol{w}_h^k$ for the approximation of $\boldsymbol{w}(t_k)$. Moreover, we use the notation $\boldsymbol{z}^k = \boldsymbol{z}(t_k)$.

The ALE derivative can be approximated by the finite difference:

$$\frac{D^A \boldsymbol{w}}{Dt}(\boldsymbol{x}, t_{n+1}) \quad (15)$$
$$= \frac{\partial \tilde{\boldsymbol{w}}}{\partial t}(\boldsymbol{X}, t_{n+1})|_{\boldsymbol{X} = \mathcal{A}_{t_{n+1}}^{-1}(\boldsymbol{x})}$$
$$\approx \frac{\tilde{\boldsymbol{w}}^{n+1}(\boldsymbol{X}) - \tilde{\boldsymbol{w}}^{n}(\boldsymbol{X})}{\tau_n}.$$

In what follows we shall use the following notation:

$$\langle \boldsymbol{w}^n \rangle_{ij} = (\boldsymbol{w}_{ij}^n + \boldsymbol{w}_{ji}^n)/2 \quad (16)$$

and

$$\hat{\boldsymbol{w}}^n(x) = \boldsymbol{w}^n(\mathcal{A}_{t_n}(\mathcal{A}_{t_{k+1}}^{-1}(x))), \; x \in \Omega_{t_{k+1}}. \quad (17)$$

In the linearization of $b_h^1(\cdot, \cdot)$ we shall use the homogeneity of the Euler fluxes, which implies

$$\boldsymbol{f}_s(\boldsymbol{w}) = \boldsymbol{A}_s(\boldsymbol{w})\boldsymbol{w}, \text{ where } \boldsymbol{A}_s(\boldsymbol{w}) = \frac{D f_s(\boldsymbol{w})}{D\boldsymbol{w}}. \quad (18)$$

The term T_1^1 in (15) is linearized in the following way:

$$T_1 \approx \sum_{i \in I} \int_{K_i} \sum_{s=1}^{2} \boldsymbol{A}_s(\hat{\boldsymbol{w}}^k) \, \boldsymbol{w}_h^{k+1} \cdot \frac{\partial \boldsymbol{\varphi}_h}{\partial x_s} \, dx. \quad (19)$$

As for the term T_2^1, the Vijayasundaram numerical flux is chosen, since it is suitable for the linearization. This numerical flux has the form

$$\boldsymbol{H}_f(\boldsymbol{w}_L, \boldsymbol{w}_R, \boldsymbol{n}) \quad (20)$$
$$= \boldsymbol{P}^+ \left(\frac{\boldsymbol{w}_L + \boldsymbol{w}_R}{2}, \boldsymbol{n} \right) \boldsymbol{w}_L$$
$$+ \boldsymbol{P}^- \left(\frac{\boldsymbol{w}_L + \boldsymbol{w}_R}{2}, \boldsymbol{n} \right) \boldsymbol{w}_R,$$

where $\boldsymbol{P}^+$ and $\boldsymbol{P}^-$ are a positive and negative splitting of the matrix $\sum_{s=1}^2 \boldsymbol{A}_s(\boldsymbol{w}) n_s$. We therefore linearize T_2^1 using

$$\boldsymbol{H}_f(\boldsymbol{w}_{ij}^{k+1}, \boldsymbol{w}_{ji}^{k+1}, \boldsymbol{n}_{ij}) \quad (21)$$
$$\approx \boldsymbol{P}_f^+ \left(\langle \hat{\boldsymbol{w}}^k \rangle_{ij}, \boldsymbol{n}_{ij} \right) \boldsymbol{w}_{ij}^{k+1}$$
$$+ \boldsymbol{P}_f^- \left(\langle \hat{\boldsymbol{w}}^k \rangle_{ij}, \boldsymbol{n}_{ij} \right) \boldsymbol{w}_{ji}^{k+1}.$$

The remaining terms in (12) due to the ALE formulation are linear with respect to $\boldsymbol{w}$ and can be treated implicitly. If we introduce the form

$$
\begin{aligned}
&b_h(\hat{\boldsymbol{w}}^k, \boldsymbol{w}^{k+1}, \boldsymbol{\varphi}) \qquad (22)\\
&= \sum_{i\in I}\int_{K_i}\sum_{s=1}^{2}\boldsymbol{A}_s(\hat{\boldsymbol{w}}^k)\,\boldsymbol{w}^{k+1}\cdot\frac{\partial\boldsymbol{\varphi}}{\partial x_s}\,dx\\
&-\boldsymbol{P}_f^{+}\left(\langle\hat{\boldsymbol{w}}^k\rangle_{ij}, \boldsymbol{n}_{ij}\right)\boldsymbol{w}_{ij}^{k+1}\\
&-\boldsymbol{P}_f^{-}\left(\langle\hat{\boldsymbol{w}}^k\rangle_{ij}, \boldsymbol{n}_{ij}\right)\boldsymbol{w}_{ji}^{k+1}\\
&-\sum_{K_i\in\mathcal{T}_h}\int_{K_i}\sum_{s=1}^{2} z_s^{k+1}\frac{\partial\boldsymbol{w}_h^{k+1}}{\partial x_s}\cdot\boldsymbol{\varphi}\,dx,
\end{aligned}
$$

then the resulting semi-implicit scheme can be formulated in the following way: For each $k\geq 1$ find $\boldsymbol{w}_h^{k+1}$ such that

$$
\begin{aligned}
&\text{a)}\qquad \boldsymbol{w}_h^{k+1}\in\boldsymbol{S}_h, \qquad (23)\\
&\text{b)}\qquad \frac{\boldsymbol{w}_h^{k+1}-\hat{\boldsymbol{w}}_h^k}{\tau_k}+b_h(\tilde{\boldsymbol{w}}_h^k, \boldsymbol{w}_h^{k+1}, \boldsymbol{\varphi}_h)=0\\
&\qquad\quad \forall\,\boldsymbol{\varphi}_h\in\boldsymbol{S}_h,\ k=0,1,\dots,\\
&\text{c)}\qquad \boldsymbol{w}_h^0=\Pi_h\boldsymbol{w}^0,
\end{aligned}
$$

The condition (23) is equivalent to a linear algebraic system. We solve it either by the direct solver UMFPACK (Davis and Duff (1999)) or we apply the GMRES method with block Jacobi preconditioning.

5.2. Formulation 2

We proceed similarly as in the previous case. In T_1^2 we use the approximation

$$
\begin{aligned}
\boldsymbol{g}_s(\boldsymbol{w}^{k+1}) &= (\boldsymbol{A}_s(\boldsymbol{w}^{k+1})-z_s^{k+1}\boldsymbol{I})\boldsymbol{w}^{k+1} \qquad (24)\\
&\approx (\boldsymbol{A}_s(\hat{\boldsymbol{w}}^k)-z_s^{k+1}\boldsymbol{I})\boldsymbol{w}^{k+1}.
\end{aligned}
$$

In T_2^2 we use the Vijayasundaram numerical flux for $\boldsymbol{g}$ defined using the positive and negative splitting of the matrix

$$
\boldsymbol{P}(\boldsymbol{w},\boldsymbol{n})=\sum_{s=1}^{2}\boldsymbol{A}_s(\boldsymbol{w})n_s-\boldsymbol{z}\cdot\boldsymbol{n}\,\boldsymbol{I}. \qquad (25)
$$

Thus we can obtain a semi-implicit linearized scheme similarly as above.

5.3. Boundary conditions and stability

On the inlet and outlet it is necessary to use nonreflecting boundary conditions transparent for acoustic effects coming from inside of Ω. Therefore, characteristics based boundary conditions described in Feistauer and Kučera (2007) are used. On the moving wall, we prescribe the normal component of the velocity $\boldsymbol{v}\cdot\boldsymbol{n}=\boldsymbol{z}\cdot\boldsymbol{n}$, where $\boldsymbol{n}$ is the unit outer normal to the moving boundary. Other quantities, i.e. the tangential velocity, pressure and density are extrapolated.

In order to guarantee the stability of the proposed method, we consider the CFL-stability condition

$$
\begin{aligned}
&\tau_k\max_{K_i\in\mathcal{T}_h}\frac{1}{|K_i|}\left(\max_{j\in S(i)}|\Gamma_{ij}|\lambda^{\max}_{\boldsymbol{P}(\boldsymbol{w}_h^k|_{\Gamma_{ij}},\boldsymbol{n}_{ij})}\right) \qquad (26)\\
&\leq\ \text{CFL},
\end{aligned}
$$

where $|K_i|$ denotes the area of K_i, $|\Gamma_{ij}|$ the length of the edge Γ_{ij}, CFL is a given constant and $\lambda^{\max}_{\boldsymbol{P}(\boldsymbol{w}_h^k|_{\Gamma_{ij}},\boldsymbol{n}_{ij})}$ is the spectral radius of the matrix $\boldsymbol{P}(\boldsymbol{w}_h^k|_{\Gamma_{ij}},\boldsymbol{n}_{ij})$ (see (25)). The maximum is taken over Γ_{ij}.

Numerical experiments (cf., e.g. Dolejší and Feistauer (2004), Feistauer and Kučera (2007)) however show that the method is practically unconditionally stable. Usually, in the begining of the computational process we use a smaller CFL number, for example CFL$\approx$ 10 and then, during the computational process, CFL is successively increased.

6. NUMERICAL EXAMPLE

The method using piecewise quadratic elements (i.e. $r=2$) was applied to the solution of compressible inviscid flow through a channel with one fixed wall of the form of a straight line, and a periodically oscillating wall with time period 4π. We assume that parts of the walls are time independent near the inlet and outlet. We consider the inlet and outlet as straight segments given by the conditions $X_1=-2$ and $X_1=2$, respectively. Further, we assume that the upper wall is given by the condition $X_2=1$ and that the ALE mapping is equal to the identity in the sets $[-2,-1]\times[0,1]$ and $[1,2]\times[0,1]$. Otherwise we construct the ALE mapping so that lower wall is represented at time t by the graph of the smooth function

$$
\begin{aligned}
&X_2=\sin(0.5\,t)\,(\cos(\pi\,X_1)+1)/4, \qquad (27)\\
&X_1\in[-1,1],\ t\geq 0.
\end{aligned}
$$

This movement is interpolated inside the domain, which results in the ALE mapping. Then the domain velocity is computed. We start from a constant initial condition prescribed in the straight channel Ω_0. The initial and inlet Mach number is 0.1. During the computational process the Mach number achieves the maximal value 0.395. The time step used in the numerical experiment was $\tau = 0.05$ and in the stability condition we use CFL=83.

In Figure 2 we show the flow patterns in the channel represented by pressure isolines at time instants $t = 12.773$, $t = 15.973$, $t = 17.173$, $t = 19.773$, $t = 21.973$. The computed solution is periodic and although the flow is inviscid, a vortex is formed after the lower wall starts to descend. This vortex is then convected out of the domain through the outlet.

Acknowledgements

This research is a part of the research project MSM 0021620839 financed by the Ministry of Education of the Czech Republic (MSMT). It was also partly supported by the grant No. 201/08/0012 of the Grant Agency of the Czech Republic (M. Feistauer), the Nečas Center for Mathematical Modelling, project LC06052, financed by MSMT (V. Kučera) and the research project No. 48607 of the Grant Agency of the Charles University (J. Prokopová).

7. REFERENCES

Davis, T.A., Duff, I.S., 1999, A combined unifrontal/multifrontal method for unsymmetric sparse matrices. *ACM Transactions on Mathematical Software* **25**: 1–19.

Dolejší, V., Feistauer, M., 2004, A semi-implicit discontinuous Galerkin finite element method for the numerical solution of inviscid compressible flow. *J. Comput. Phys.* **198**: 727-746.

Feistauer, M., Kučera, V., 2007, On a robust discontinuous Galerkin technique for the solution of compressible flow. *J. Comput. Phys.* **224**: 208-221.

Feistauer, M., Felcman, J., Straškraba, I., 2003, *Mathematical and Computational Methods for Compressible Flow*. Oxford University Press, Oxford.

Nomura, T., Hughes, T. J. R., 1992, An arbitrary Lagrangian-Eulerian finite element method for interaction of fluid and a rigid body. *Comp. Methods Appl. Mech. Engrg.* **95**: 115-138.

Sváček, P., Feistauer, M., Horáček, J., 2007, Numerical simulation of flow induced airfoil vibrations with large amplitudes. *J. of Fluids and Structures* **23**: 391-411.

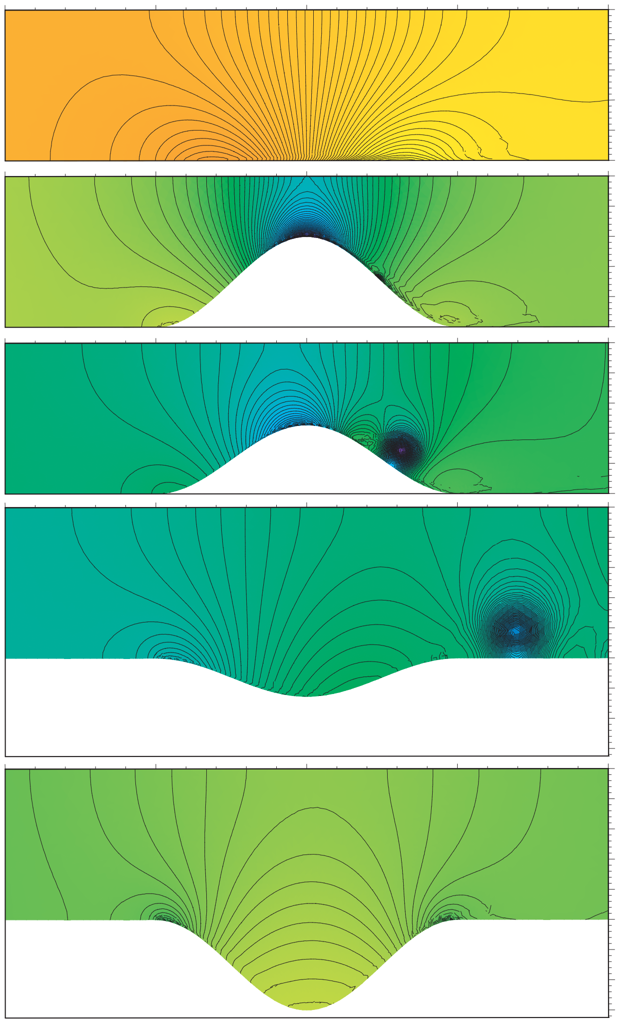

Figure 2: Pressure isolines in the channel at time instants $t = 12.773$, $t = 15.973$, $t = 17.173$, $t = 19.773$, $t = 21.973$