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COMPUTER-AIDED ANALYSIS OF FLOW IN A ROTATING SINGLE DISK

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ABSTRACT

In this study a two dimensional axisymmetric, steady state and incompressible laminar flow in a rotating single disk is numerically investigated. The finite volume method is used for solving the momentum equations. The numerical model and results are validated by comparing it to previously reported experimental data for velocities, angles and moment coefficients.

It is demonstrated that increasing the axial distance increases the value of axial velocity and vice versa for tangential and total velocities. However, the maximum value of nondimensional radial velocity occurs near the disk wall. It is also found that with increase rotational Reynolds number, moment coefficient decreases.

NOMENCLATURE

a	*Disk radius (m)*
p	*Pressure (N m^{-2})*
r	*Radial distance (m)*
Re_Φ	*Rotational Reynolds number ($\rho\Omega a^2 / \mu$)*
v_r	*Radial velocity component (m s^{-1})*
v_z	*Axial velocity component (m s^{-1})*
v_ϕ	*Tangential velocity component (m s^{-1})*
v_T	*Total velocity (m s^{-1})*
z	*Axial distance (m)*
ζ	*Nondimensional axial distance*
μ	*Dynamic viscosity (N s m^{-2})*
v	*Kinematic viscosity (m^2 s^{-1})*
ρ	*Density (kg m^{-3})*
ϕ	*Tangential direction*
Ω	*Angular velocity (rad s^{-1})*

INTRODUCTION

Many technologies utilize rotating-disk systems. Such systems can be used to model the flow that occurs in the internal cooling air-systems of gas turbines or other rotating bodies, centrifugal pumps, viscometers and other devices (Owen and Rogers, 1989). Much work has been done to investigate fluid flow and heat transfer on a plate disk rotating with constant angular velocity. For such rotary systems thermal efficiencies may not be functions of only temperature. The reason for this phenomenon is the complexity of the flow and thermal fields and the lack of implicit analytical expressions for each field. It is important to know how the flow behaves at every stage to permit safe and effective operation of rotary-type machines. Since the governing equations, namely the momentum equations, are highly nonlinear and coupled, it is difficult to obtain an exact analytical solution (Arikoglu et al., 2008b).

A schematic diagram of the flow for a free rotating disk is shown in Figure 1. A free disk is defined as a plane disk rotating with constant angular velocity Ω about its polar axis in a quiescent environment. A boundary layer is formed on the surface of the disk in which fluid is entrained axially and pumped radially outward under the action of the centrifugal force. The flow in the boundary layer may be laminar or turbulent, the transition occurring when the local Reynolds number, Re_ϕ exceeds a critical value. Here, $Re_\varphi = \Omega r^2 / \nu$, where *r* denotes the radius and *v* the kinematic viscosity of the fluid. This type of flow was first investigated theoretically with an approximate method by von Karman (1921). Experimental work of Theodorsen and Regier (1944) and the later work of Gregory et al. (1955) measured velocity profiles in the boundary layer. Cobb and Saunders (1956) conducted experiments for a disk with uniform temperature, while McComas and Hartnett (1970) measured the heat transfer on a free disk. The effect of slip on entropy generation was investigated for flow over a rotating free disk by Arikoglu et al. (2008b). The steady laminar magnetohydrodynamics (MHD) flow of a viscous Newtonian and electrically conducting fluid over a rotating disk with slip boundary condition was studied by Frusteri and Osalusi (2007). Also heat transfer from a rotating disk in a parallel air crossflow was investigated by Wiesche (2007). The steady laminar flow of an elastico-viscous fluid near a rotating disk is considered by Ariel (2003). Convection to rotating disks with rough surfaces in the presence of an axial flow was studied by Axcell and Thianpong (2001).

In this paper, consequently, the flow structure for rotating single-disk systems is modeled and numerically assessed. The main objective is to understand the flow behavior in a rotating disk system e. g. velocity and streamline. A secondary objective of this work is to re-examine a numerical method which validated by comparing it to previously available data for momentum so as to provide information that can help analysts and designers in the proper evaluation of efficiencies and in the geometrical optimization of such rotating systems or systems having rotating parts.

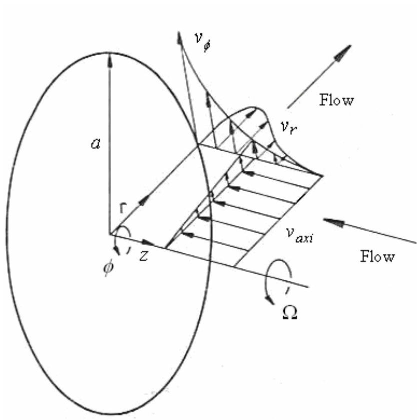


Figure 1: Schematic diagram of the flow for a rotating disk

MODEL DESCRIPTION

For the computational aspect of this research, a rotating single disk is modeled (see Fig. 2). The disk is taken to have a radius a, to be in the plane $z = 0$ and to be rotating with an angular velocity Ω about the *z*-axis. The integration domain is truncated at a distance *s* in the axial direction, with the axial distance s taken as $20(\nu/\Omega)^{1/2}$, where ν is the kinematic viscosity of the working fluid (Owen and Rogers, 1989). The rotating disk system is assumed to operate at steady state, be axisymmetric and involve incompressible flow.

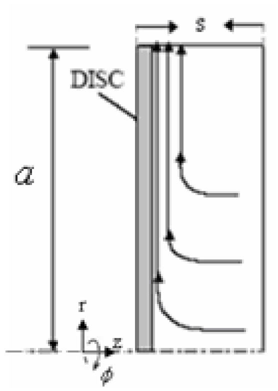


Figure 2: Physical model of a single disk

The relevant governing equations for continuity and momentum, assuming constant thermophysical properties, are as follows:

Continuity:

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \qquad (1)$$

Momentum:

$$v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\phi^{\,2}}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}(\nabla^2 v_r - \frac{v_r}{r^2}) \qquad (2)$$

$$v_r \frac{\partial v_\phi}{\partial r} + v_z \frac{\partial v_\phi}{\partial z} + \frac{v_\phi v_r}{r} = \frac{\mu}{\rho}(\nabla^2 v_\phi - \frac{v_\phi}{r^2}) \qquad (3)$$

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\nabla^2 v_z \qquad (4)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The boundary conditions for the problem are defined as follows:

- At the disk wall, i.e., $z = 0, 0 \le r \le a$: $v_r = 0$, $v_\phi = \Omega r$, $v_z = 0$
- At the end of integration domain, i.e., $z = s$, $0 \le r \le a$: $v_r = 0$, $v_\phi = 0$, $\partial v_z / \partial z = 0$
- At the center of the disk, i.e., $r = 0$, $0 \le z \le s$: $v_r = 0$, $v_\phi = 0$, $\partial v_z / \partial r = 0$
- At the circumference of the disk, i.e., $r = a$, $0 \le z \le s$: $\partial / \partial r (v_r / r) = 0, \partial / \partial r (v_\phi / r) = 0, \partial v_z / \partial r = 0$

Also, the moment coefficient is defined by:

$$C_m = \frac{|M|}{0.5 \rho \Omega^2 a^5} \qquad (5)$$

Here, M denotes the frictional moment and can be expressed as

$$M = -2\pi \int_a^b r^2 \tau_{\phi,o} dr \qquad (6)$$

Where $\tau_{\phi,o}$ is shear stress and can be calculated as :

$$\tau_{\phi,o} = \mu(\partial v_\phi / \partial z)_{z=0} \qquad (7)$$

NUMERICAL ANALYSIS

A computational fluid dynamics (CFD) code developed by the authors is used for solving the relevant mathematical expressions for the rotating disk system. This computer code is a finite-volume, steady, axisymmetric, elliptic and multigrid solver. The system of governing equations (1–4) is solved using a control-volume approach. The control-volume technique converts the governing equations to a set of algebraic equations that can be solved numerically. The control volume approach employs the conservation principles and physical relations represented by the overall governing equations on each finite control volume. A first-order upwind scheme is employed to discretize the convection terms, diffusion terms and other quantities in the governing equations. The grid schemes used are staggered, and velocity components are evaluated at the center of the control volume interfaces, while all scalar quantities are evaluated at the center of the control volume. Pressure and velocity are coupled using the Semi Implicit Method for Pressure Linked Equations (SIMPLE) of Patankar (1980).

The CFD code solves the linear systems resulting from the discretization schemes using a point implicit (Gauss–Seidel) linear equation solver in conjunction with an algebraic multigrid method. During the iterative process, the residuals are carefully monitored. For all simulations performed in the present study, convergence is deemed to be achieved when the residuals resulting from iterative process for all governing equations (i.e., equations (1–4) are smaller than 10^{-6}.

The grid used in the present analysis is 40 × 20, with 40 volumes in the *r*-direction and 20 in the *x*-direction. Grids having 60 × 40 and 80 × 60 configurations are also tested. As all grid configurations yield similar values of velocity, the 40 × 20 grid is accepted as sufficiently fine. This grid size is also validated using experimental data in the next section.

RESULTS

Figures 3 and 4 show the axial variation with nondimensional axial distance ζ, where $\zeta = z\sqrt{\Omega/\nu}$, of velocity components ($v_\phi/\Omega r$, $v_r/\Omega r$, $v_z/(\Omega\nu)^{1/2}$) and the magnitude of the nondimensional total velocity ($v_T/\Omega r$ where $v_T = (v_r^2 + v_\phi^2)^{1/2}$) at $Re_\phi = 4 \times 10^4$.

In Fig. 3, the numerical results for the velocity components are shown. Increasing the nondimentional axial distance is seen in this figure to increase the value of nondimensional axial velocity and vice versa for nondimensional tangential velocity. However, the maximum value of nondimensional radial velocity occurs near the disk wall. The computational results for the total velocity is compared with the experimental results of Gregory et al. (1955) and Cham and Head (1969) in Fig. 4 . The comparison shows the agreement between the results obtained with CFD and those obtained via experimental methods.

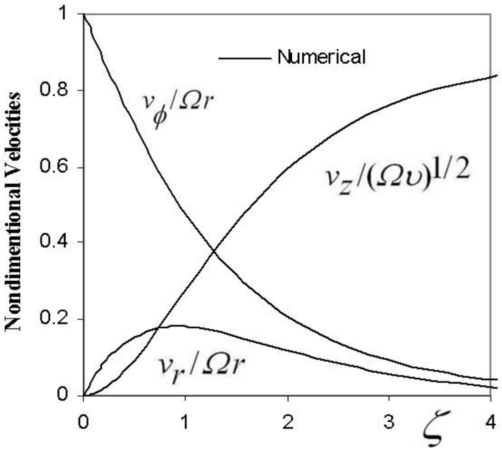


Figure 3: Axial variation of velocity components near a disk for $Re_\Phi = 4 \times 10^4$

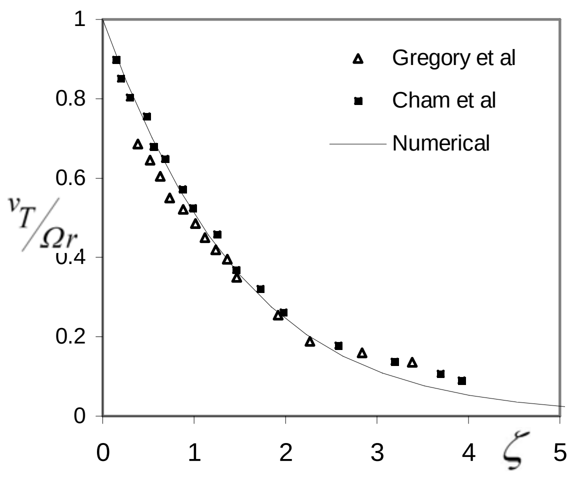
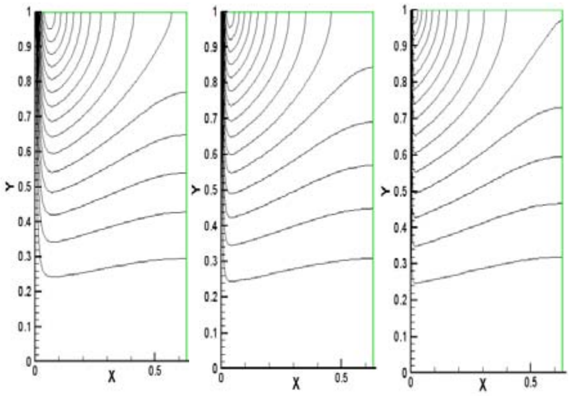


Figure 4: Axial variation of nondimensional total velocity for $Re_\Phi = 4 \times 10^4$

Figure 5 shows the predictions of streamline patterns for the rotating disk. As illustrated in this figure, the centrifugal forces created by the rotating disk cause a radial outflow of fluid within the boundary layer. Since the radial component of velocity is zero both on the disk and in the free stream, external fluid is entrained axially into the boundary layer. The radial outflow is often referred to as the free disk pumping effect.

A boundary layer on each side of the disk develops, through which the tangential component of velocity, v_ϕ is sheared from the value Ωr at the surface to the value zero in the "free stream" outside the boundary. The boundary-layer thickness over the disk is defined to be that height where the azimuthal flow velocity v_ϕ has the value $v_\phi = 0.01\Omega r$.



(a) (b) (c)

Figure 5: Predictions of streamline pattern for (a) $Re_\Phi = 1 \times 10^4$, (b) $Re_\Phi = 4 \times 10^4$ and $Re_\Phi = 2 \times 10^5$

The axial variation of angle between the direction of flow and the radius vector $\psi^\circ = \tan^{-1}(v_r/v_\phi)$ is shown in Fig. 6. It is observed in this figure that the maximum value of ψ occurs at the maximum value of ζ. Also, the agreement is less satisfactory for the experimental and numerical results. This was probably due to the experimental errors arising from the inaccuracy of the yawmeter.

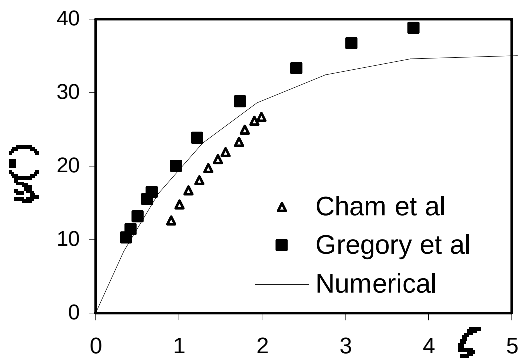


Figure 6: Axial variation of angle between the direction of flow and the radius vector

The variation of moment coefficient with rotational Reynolds number given by numerical solution is compared in Fig. 7 with the experimental results of Theodorsen and Regier (1944). It can be seen that the agreement between the experimental and numerical results is satisfactory.

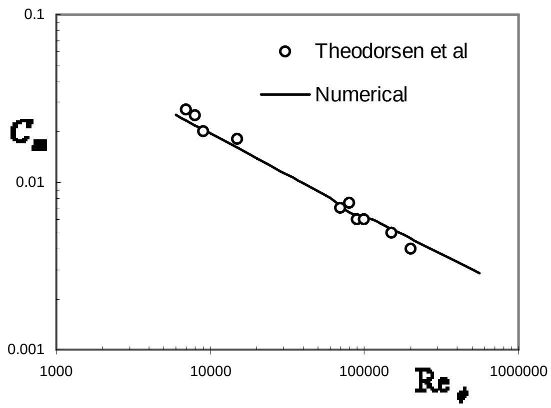


Figure 7: variation of moment coefficient with rotational Reynolds number

CONCLUSION

A rotating single disk with constant angular velocity Ω about its polar axis in a quiescent environment which causes a boundary layer is investigated using computer aided analysis. The common feature observed is that there is a good agreement between the results obtained with CFD and those obtained via experimental methods. The flow fields are computed numerically using the finite volume method. Axial, tangential, radial and total velocities are computed and their behaviors discussed. The results demonstrated that increasing the axial distance increases the value of axial velocity and vice versa for tangential and total velocities. However, the maximum value of nondimensional radial velocity occurs near the disk wall. It is also found that with increase rotational Reynolds number, moment coefficient decreases. The results are expected to be useful to those involved in the design of engineering systems incorporating rotating disks.

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