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STREAK PATTERNS IN BINARY GRANULAR MEDIA IN A ROTARY CLASSIFIER

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ABSTRACT

The Discrete Element Method (DEM) is used to understand the formation of radial streak patterns produced when binary granular material (which may differ either in size, density or shape) segregate in a slowly rotating drum. Our simulations show that initial streak formation requires temporal fluctuations in the particle bed's strength. This, in turn, creates fluctuations in the slope and shape of the upper surface of the bed which control the particle avalanches down the free surface. These ultimately lead to streak formation. We conjecture that growth and stabilisation of a regular streak pattern requires the two sets of particles to have significantly different angles of repose.

INTRODUCTION

Granular matter represents one of the scientifically least understood yet one of the industrially most important areas of physics (Duran, 2000, Bagnold, 1941, Jaeger and Nagel, 1992, Mehta, 1994, Ristow, 2000). Segregation in granular materials occurs when particles differ in some fundamental property such as size, density or shape. For example, the idea that the smaller particles can more easily percolate through interstitial volumes explains why it is that Brazil nuts that are predominantly found at the top of a jar of mixed nuts (Rosato *et al*, 1987). Vibrated beds lead to particle segregation, but it has been recently recognized an even better device for promoting segregation in granular media is a rotary classifier (Ottino and Khakhar, 2000, Metcalfe and Liffman, 2002, Meir *et al*, 2007) which consists of a cylindrical drum roughly half-filled with the granular media. The drum is placed with its cylindrical axis perpendicular to the gravitational field and then rotated slowly about this axis (one revolution per minute is a typical speed). After a revolution, particles tend to segregate. For example, in a binary granular medium (i.e., the media consists of two different particle types) the smaller and/or the denser particles segregate to the centre, while larger and/or less dense particles segregate to the periphery (see Fig. 1a). This segregation pattern, referred to as a "core" or sometimes as a "moon pattern" (Hill *et al*, 2004) has been studied quite extensively experimentally (Khakhar *et al*, 1997, Metcalfe and Shattuck, 1996, Metclafe *et al*, 1999), theoretically (by developing a continuum type model) and via simulations (Khakhar *et al*, 1997, Ristow, 1994, Pereira *et al*, 2009).

In this paper we focus on another, much less studied, exotic pattern known as a "streak pattern" (Khakhar *et al*, 2001, Khakhar *et al*, 2003, Jain *et al*, 2005) or sometimes as a "sun pattern" (Hill *et al*, 2004, Hill *et al*, 2005). Although there has been some experiments (Hill *et al*, 2004, Hill *et al*, 2005, Khakhar *et al*, 2001) and some continuum level modelling (Hill *et al*, 2004, Khakhar *et al*, 2001) of this pattern there still remains a gap in the basic understanding as to the reason for its formation. For example, some studies imply it is only the size and density of the particles which are important (Jain *et al*, 2005) while others have referred to differing angles of repose between the two sets of particles (Khakhar *et al*, 2001). In this work, we set out to understand the fundamental reasons for the genesis (and growth) of the streak pattern. We do this by exploring the interactions between particles which lead to streaks. At the particle scale, continuum modelling is not appropriate and experiments are difficult to visualize and control. However, first principles computer simulations such as the Discrete Element Method (DEM) (Mehta, 1994, Campbell, 1990, Walton, 1994) are ideal, since we can easily control and probe interactions at this scale.

DEM MODEL DESCRIPTION

The simulation method which we use (DEM) is now a well-established and mature technique which has been extensively developed by us for a wide variety of granular flows (Cleary, 1998a and b, 2004). Here we very briefly describe the important aspects of this technique, which are important for our purposes and refer the reader to more detailed descriptions elsewhere (Walton, 1994, Cleary, 1998b, 2004).

DEM models particulate systems whose motions are dominated by collisions. It follows the motion of every particle and object in the flow and models each collision between particles and between particles and objects (i.e., inner surface of rotating drum). All forces and torques on each particle and object are summed and the resulting equations of motion are integrated to give the resulting motion of these bodies. The collisions between particles and/or objects are modelled such that they are allowed to overlap. The amount of overlap and relative velocities between particles determine the collisional force via a contact force law. We use a linear spring and dashpot model to predict the collision dynamics. Other important parameters such as friction and coefficient of restitution are included in the model and specific values of all these can be found in a recent study (Pereira *et al*, 2009). The simulations reported in this study are carried out at angular rotations of the cylinder of about one revolution every 60 to 80 seconds. This puts these simulations on the border of avalanching and rolling regimes for granular flows (Meir *et al*, 2007). Initially, we shall use two different spherical particle types (i) Smaller, denser particles and (ii) Larger, less dense particles. After reporting on these simulations, we continue on to consider particles of non-spherical shape.

SIMULATIONS WITH SPHERICAL PARTICLES

As we shall argue, an important contribution to the initiation of streaks is a strong bed microstructure which results from jamming and/or a tighter packing of particles in the bed. To demonstrate this we consider two scenarios which are the same in all ways except that in case (a) we have periodic boundaries in the axial direction while in case (b) end walls present are 3-5 particle diameters apart. With periodic boundaries, particles exiting on one side (along the axial direction) re-enter on the opposite side and thus this simulation represents a long cylinder.

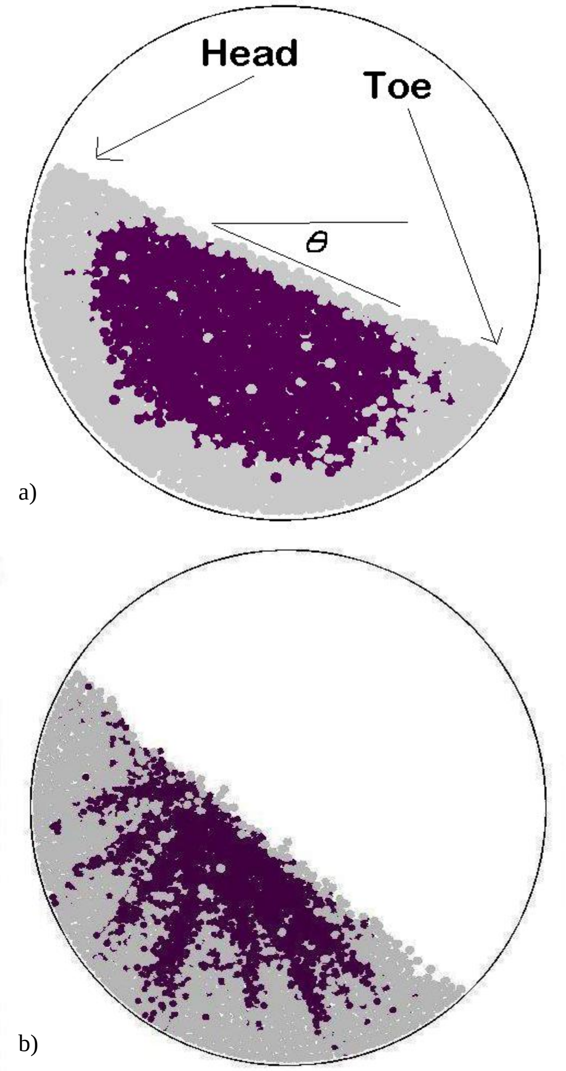


Figure 1: Comparison of stable, equilibrium particle distribution for (a) a periodic slice, and (b) short cylinder. The smaller, denser particles are dark grey while the larger, less dense particles are light grey. The cylinder rotates clockwise. In (a) the slope angle of the top surface is labelled (θ), which is equal to the angle of repose. The "head" and "toe" of the bed are also marked.

Case (b) represents the situation used in recent experiments (Khakhar et al, 2001, Jain et al, 2005, Hill et al, 2004) where streak formation was obtained. The main consequence of the length of the cylinder is that particles have a greater degree of freedom in (a) compared to (b). For short cylinders a significant part of the particle bed's weight is transmitted to the end-walls rather than the shell. As a result there is a net outward force on the particles towards the end-walls which consequently strengthens the microstructure. In turn this means particles will tend to have a tighter packing structure in (b) and hence the bed can withstand greater stresses (at certain instances in time.)

Figure 1(a) shows the equilibrium configuration for case (a). Strong radial segregation has occurred with the smaller and denser particles migrating inwards to form a dense core. The core is broadly semi-circular with a degree of noise in the boundary. This is the classical interfacial boundary shape for particle segregation in a rotary cylinder and we now understand quite well how and why this shape appears (Khakhar et al, 1997, Metcalfe et al, 1996, Metcalfe et al, 1999, Pereira et al, 2009). It is important to note that we always obtained this structure in all our simulations with axial periodic boundaries.

A detailed description of the density segregation process is given in Pereira et al (2009), but we briefly describe the process here since it is useful to contrast with the behaviour in case (b). Rotation of the cylinder causes particles which are adjacent to the cylinder walls to rigidly rotate with the cylinder. Once they reach the head of the bed they now can move independently. The layer close to the free surface is termed the "active layer". Here there is much greater dilatency and mobility than in the rest of the bed. As a result segregation occurs in this layer. Heavy particles tend to sink into the bed (since they have a greater mass than the average particle surrounding them) in the direction of the gravity vector and begin to form a core. Lighter particles, on the other hand, roll along the top surface and collect at the toe. From there they once again undergo rigid body rotation. The toe region becomes tightly packed and only particles rolling along the top surface can enter this region. This process continues until a core has formed at which point segregation ceases because the average density difference between particles in the core and particles entering the core becomes negligible. Segregation is also inhibited by diffusion (due to collisions between particles) leading to an interface between different particle types that is not precisely semi-circular but has some superimposed noise. We have talked about segregation in terms of density differentials, but similar segregation patterns also occur for different sized particles (of the same density) due to percolation of smaller particles through the dilated active layer.

Now consider the case where walls are placed at the ends of the cylinder. The equilibrium particle distribution is shown in Fig. 1(b). Once again segregation occurs with smaller and denser particles moving towards the centre. However, now the interface between small and large particles is no longer semi-circular with long, almost radial streaks (or fingers) radiating from the central core. These streaks are somewhat irregular in shape and position in the bed. This figure shows the structure at a specific instant in time. It is not stable, with the boundary shape constantly changing, Streaks tend to appear and disappear more or less randomly over time. We now explore the physics behind the formation of these streaks.

Underlying physics of streak formation

Let us accept the premise that the presence of end-walls strengthens the particle bed (in comparison to the case without end-walls). A stronger particle bed structure implies that as the bed rotates the slope angle of the top-surface (shown in Figure 1a) can exceed the (free) angle of repose. However, we must be careful - the bed's microstructure (strength) fluctuates in time. Once the angle of repose has been exceeded the slope angle increases until failure of the top surface occurs. Particles then avalanche down the top surface and subsequent collisions from this avalanche impart a significant amount of momentum into the bed which, in turn, disturbs the bed's structure. As the avalanche runs out, the slope of the top surface decreases rapidly to the angle of repose. It is apparent that the bed strength is not spatially uniform.

We now consider the formation of a streak in more detail. Figure 2 shows a sequence of snap-shots which demonstrate the ideas outlined in the paragraph above. In Fig. 2a the top surface is (comparatively) flat and the slope angle of this surface is close to the angle of repose – around 35.5^{o}. Larger particles (light grey) travel at their normal rate down the surface. Note that a dark streak is adjacent to the top free surface (near the head of bed). In Fig. 2b one can see a bulge in the surface near the head of the bed - particles are mounting up in this region. This grows until the slope in this top one-third region has increased appreciably. This occurs, because this part of the bed has become more tightly packed and can support additional mass. The angle of failure is being rapidly approached.

In Fig. 2c the top one-third of the bed has just collapsed (the angle of failure has been exceeded) and particles avalanche down the surface. This, in turn, implies these particles have much larger velocities, *especially velocity components in the direction parallel to the angle of slope of the top surface*. Recall that in the case where we formed a semi-circular core of denser particles, these particles primarily moved in the direction of the gravity vector after reaching the head of the bed. This is a major difference between the two cases. As a result, denser particles travel further down the free surface and come to rest (relative to nearby particles) at a larger radial distance. As a result a larger streak is formed on the opposite side of the bed (near the toe). Note, that during this process the interface shape is not flat. As the bed becomes more tightly packed the top surface takes the shape of reclining chair (i.e., top one-third section of bed has a larger slope angle than remainder).

Not every collapse of the top surface results in the formation of a streak. A streak forming requires the coincidence or synchronisation of two events. The first is the presence of a large number of denser/smaller particles adjacent to the top surface near the head *and* the second is the angle of failure just being exceeded. If there are not simultaneously a larger number of denser particles near the head when the angle of failure is approached, only lower density/larger particles will avalanche down the slope and a streak will not form. To confirm this we need to verify that (i) there is an increase in stress capacity of the bed especially in region near the head, (ii) an increase in the angle of the top one-third of the slope of the top surface, and (iii) an increase in particle velocity as they travel down the top surface (especially in middle region) which enables smaller particles to travel further down the slope and form a streak.

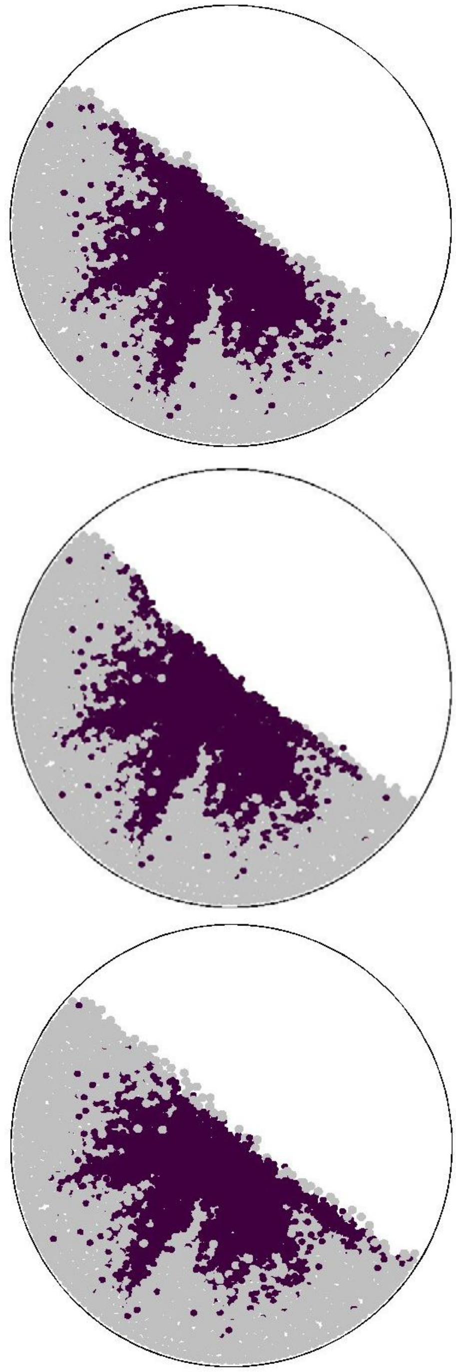
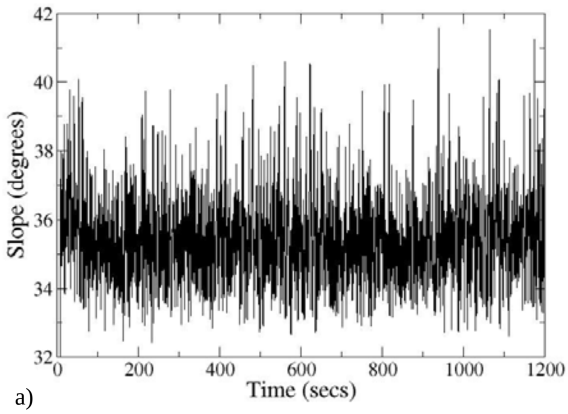
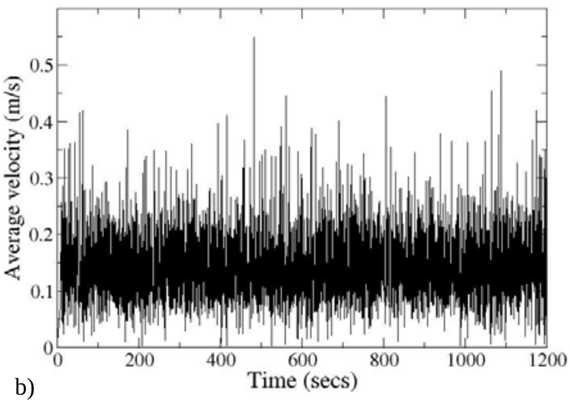

a)
b)
c)

Figure 2: Changes in shape of top surface during streak formation. (a) At t=500s the interface is initially flat with slope angle equal to angle of repose. (b) At t=502s the slope of free surface increases in region near head of bed. (c) At t=503s the angle of failure is exceeded and the surface near the head collapses with particles avalanching down the free surface. The streak now appears at toe of bed.

Fig 3(a) shows the average angle of the top one-third of the bed as a function of time while Fig. 3(b) shows the average velocity of particles in the active avalanching layer of the bed (in the region between the toe and head). The important points to note are the “spikey nature” of these quantities and large variations in slope angle and average velocity. In contrast, for the case with periodic boundary conditions these quantities are much more uniform and have significantly smaller variations. The variation in slope angle is about 5^o for the periodic cylinder compared to about 10^o for the short cylinder shown in Fig. 3(a). The peak velocities vary up to only 0.2 for the periodic case compared to 0.5 for the short cylinder cane in Fig. 3(b). Both quantities are more than doubled for the case with end-walls.



a)



b)

Figure 3: (a) Slope angle for the thin cylinder (in Fig. 2) over the simulation duration. (b) Average particle velocity in the surface layer.

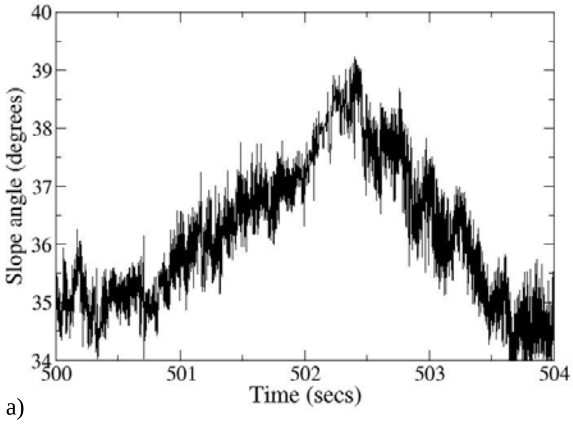
The spikes in slope, match quite closely with the spikes in velocities, so we are quite sure that it is the avalanching of particles, after collapse once the angle of failure has been exceeded, that is the cause of the large velocities. To further demonstrate this sequence of events we focus the slope angle and average velocity measures on the time around the sequence of snap-shots shown in Fig. 2. Fig. 4 shows that as the time approaches 502.5 s the slope angle has increased up to around 39.4^o. At this point the particle bed in this region fails and the slope angle rapidly diminishes. Correspondingly, just after 502.5 s, the particle velocity increases to a maximum of 0.35 m/s and thereafter decreases back to a plateau value.

SIMULATIONS WITH BLOCKY PARTICLES

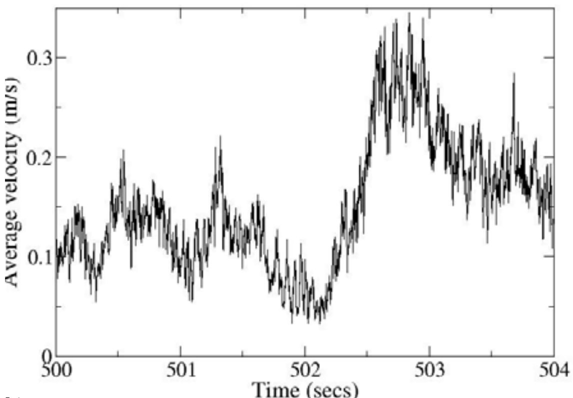
We have shown now that streaks can be formed with spherical particles. Particle shape, however, is well known to affect the flow of granular materials (Cleary and Metcalfe, 2002, Cleary *et al*, 1998, Debroux and Cleary, 2001). Blocky (or angular) particles tend to have a higher resistance to shear compared to spherical particles since neighbouring particles lose their ability to freely roll over each other. With this in mind, we now consider similar particles to those just discussed, in terms of density and average size but with a more block-like shape. We refer to such particles as *super-quadrics* and they obey a shape equation

$$x^n + y^n + z^n = 1 \ , \qquad (1)$$

where n defines the amount of blockiness of a particle with n = 2 representing a spherical particle and $n \to \infty$ representing a cube. Blockiness leads to a stronger packing of the particle bed and hence a more non-continuous particle flow. Figure 5 shows typical snap-shots of the particle distribution using n values of 2.2, 2.5 and 3.0. It is quite clear that an increase in n causes the streaks to become thinner and longer. This relates directly to the fact that with increasing n the bed becomes stronger and can reach higher failure angles. This, in turn, means particles collapse with larger momentum, leading to longer streaks. Since the volume of dark particles remains the same then these streaks also become thinner.



a)



b)

Figure 4: (a) Slope angle for the simulation in Fig. 2 for the period around 502 s, and (b) average velocity of particles in surface layer in region around 502 s.

Experiments are often carried out where there is some moisture present in the particle bed. This can have a number of effects on particle flow. The most important consequence of moisture is that particles tend to stick together and that moisture can absorb a certain amount of energy from the system and hence damp out vibrations. Simulations using liquid bridge cohesion between particles also produced streaks similar to those shown in Figs. 1 and 2. As the liquid layers become larger, the particles stick much more so that they form clumps of particles rather than individual entities. Consequently the discrete particle nature of the flow is lost. Hence we did not use any larger values of cohesion.

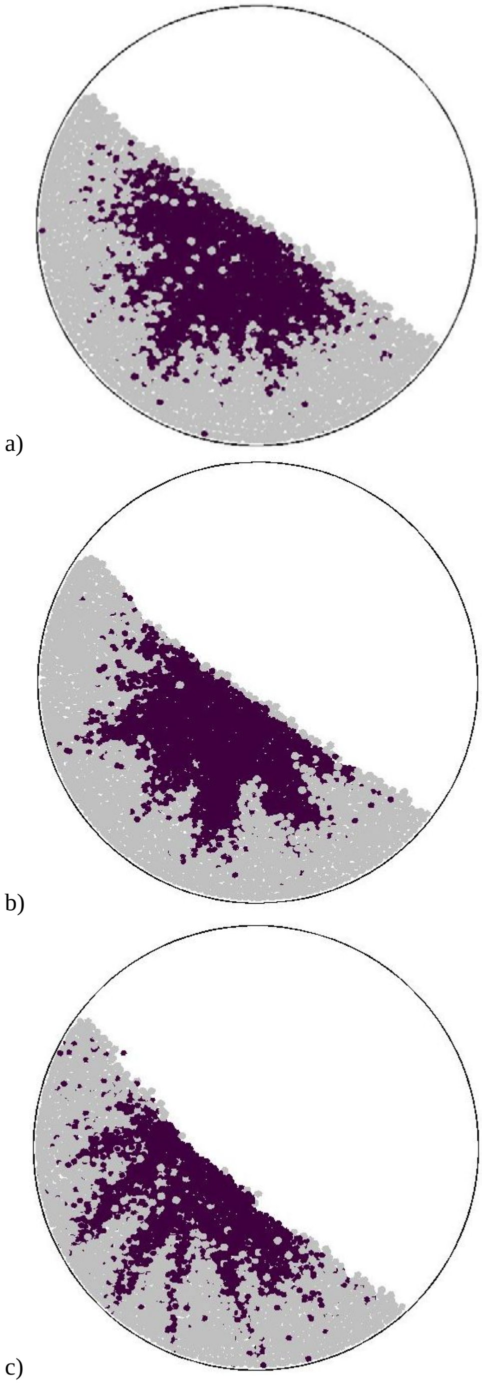


Figure 5: Snap-shots of simulations with super-quadric particles for various values of *n*. (a) $n = 2.2$, (b) $n = 2.5$ and (c) $n = 3.0$. All other parameters are exactly the same for all three simulations.

Small, blocky and large, spherical particle simulations

Although we have produced streaks, these are still not uniformly sized or regularly spaced streaks which seem to occur in some experiments (Hill *et al*, 2004, Jain *et al*, 2005). The reason that we do not appear to have regular streaks is that, in general, we do not regularly have a coincidence of the two events (i) the top surface failing and (ii) a large number of denser/smaller particles in the region near the head (surface) of the bed. There must be a mechanism in the experimental system that ensures the synchronisation of the two event types.

If the two sets of particles had different angles of repose then this scenario could possibly be realized. A previous study of granular mixtures (composed of two different particles shapes) being poured into heaps has shown that shape differences can also lead to segregation and stratification (Makse, *et al*, 1997).

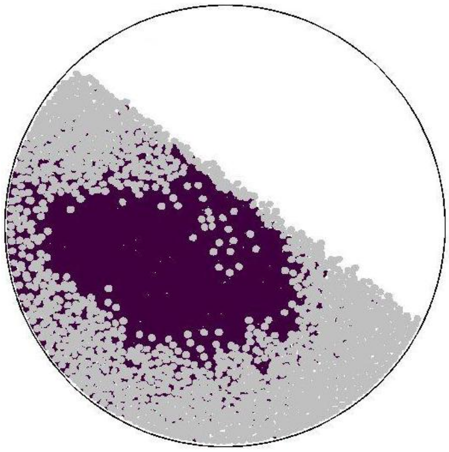


Figure 6: Snap-shot of simulation for case where we have spherical/larger/less dense particles and super-quadric ($n = 3$)/smaller/denser particles.

We therefore consider a combination of blocky and spherical particles. Since the blocky particles can sustain higher failure angles and since we require the smaller/denser particles to form the streaks, we make the smaller/denser particles blocky, with an $n = 3$, and keep the larger/less dense particles as spheres. Figure 6 shows a snap-shot of the particle distribution after about five revolutions of the cylinder. Although a core of smaller particles has formed, there is no indication (after about 5 rotations of the cylinder) of streak formation and certainly throughout the simulation we never observed uniformly sized and regularly spaced streaks.

There are number of reasons why we have not observed streaks in this simulation. It is clear that streak formation is a subtle effect and we have just simulated the system for one set of parameters (i.e. particles sizes, shapes and densities, fill levels etc). There is a large parameter space and we have by no means fully investigated this space. This will require a deeper understanding of the physics and further simulations which will be carried out in the future. The results presented here are preliminary, but are encouraging in that we have produced streaks, albeit not regular spaced and uniform sized streaks.

CONCLUSION

In this study we have sought to gain fundamental understanding of the formation of streaks in rotating drums for binary granular matter. The particles that make up the granular medium may differ in size, density or shape. The main findings from this study have been:

- Denser and/or smaller particles segregate to the centre of the particle bed. The reasons for this are similar to those which have been discussed in previous works for long cylinders – buoyancy or percolation type effects. For long cylinders there was no indication of streak formation with the classical moon-shaped core structure being found instead.

- Streak formation occurs only in thin cylinders, where the diameter of the cylinder is much larger than the length. Segregation occurs in the active layer, but now failure and discrete avalanches of the top surface is also important.
- For the short cylinders, particle motion is more restricted and the particle bed tends to pack much more tightly than for longer cylinders. Thus the slope angle of the top surface is no longer roughly constant at the angle of repose, but fluctuates from the angle of repose to a higher angle of failure. This variation predominantly occurs near the head of the bed.
- Once the angle of failure has been exceeded, the bed collapses and particles avalanche down the free surface. The extra momentum gained due to collapse (rather than the normal rolling) means particles come to rest (relative to other nearby particles) much further down the free surface.
- If this collapse coincides with a large number of denser/smaller particles adjacent to the head of the bed, a streak will result. Because the coincidence of these two conditions was not regular, we only observed random streaks.
- More blocky particle shape tends to produce streaks which are longer and thinner.

Future work will focus on obtaining the conditions for uniformly sized and regularly spaced streaks.

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