Gravity effect on the flow between two coaxial rotating spheres

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Abstract

With general acceptance that the flow between two coaxial rotating spheres or spherical Taylor–Couette flow is highly structured and has come a wealth of literature dealing with instabilities in laminar–turbulent regime. The present work regarding new measurements point out the possible effects of gravity and free surface on the onset of instabilities in spherical Taylor–Couette flow. The phenomena were described by visualization techniques in such a way as Taylor number *Ta* increases according to the appearance of instabilities induced by means of an inclination α (i.e. gravity effect) of the system of the flow and this constrained to move step by step on the angular range $0° \le \alpha \le 90°$ from the vertical axis *(α=0°)*. It is observed that when the system is completely filled $H=H_{max}$ the angle of inclination has no effect. For a given height limitation of the flow $H<H_{max}$, the most significant result concerns the dependence on α of the onset wavy mode instability, say, $Tc_1=Tc_1(\alpha)$ while Tc_1 characterize the appearance of the Taylor vortex flow remains unchanged as α is increasing. Another significant result concerns the relaminarization of the flow as both values of the Taylor number *Ta* and α are increasing.

I. INTRODUCTION

The flow in a spherical annular space is one of the important problems in hydrodynamics physics. In particular, when it acts to better understanding certain natural phenomena in Physics of the Atmosphere [1] (meteorology), in astrophysics [2] (dynamic of stars and plantery), solid earth geophysics (dynamo effect) or to ensure the control of industrial operations particularaly, Centrifugation processing as crystalline growth and Tribology.

A that time, the majority of the work devoted to this geometry are carried out in case of a vertical system and completely filled. In theoretical works, V.S. Sorokin [3] has proposed an approximate solution followed by the prediction of K. Bratukhin [4] who obtained an approximate limit of linear stability for a radius report $\eta = R_1/R_2 = 0.5$, corresponding to the gap near to the unit $\delta = (R_2 - R_1)/R_2 \cong 1$. Moreover, the flow between eccentrics rotating spheres was approached analytically by B.R. Munson [5] for the basic laminar flow, by treating all the possible cases: rotating inner sphere, the outer fixed one; two Co-rotating spheres and in counter-rotation. G. Schrauf [6] examined the influence of on the first appearance of a pair of vortex of Taylor in calculating the axisymmetric stationary solutions but not necessarily stable.

The observations are frequently realized in spherical geometry where the inner sphere is rotating and the outer one is at rest. The measurements were carried out for small and medium gap. It is noted, in each case, that the vortex of Taylor exists as in the flow between coaxial cylinders [G.ITaylor [7]]. Thus, G.N. Khlebutin [8] is the first author to be discovered experimentally in 1968 the existence of the vortex of Taylor in this system of flow for a medium gap of *0.19.*

Other investigations on the instability of Taylor in the flow between coaxial spheres were realized by V.I Yakushin [9], B.R. Munson & M. Menguturk [10], M. Wimmer [11], I.M. Yavorskaya & Al [12], and K. Nakabayashi & al. [13] [14] [15] [16] and, P. Bar-Yoseph & al [17]. We note that in the case of a wide gap configuration, experimental observations experimental are very few. Thus B.R. Munson & M. Menguturk [10] and A.M. Waked & B.R. Munson [18] showed that the laminar basic flow can become unstable by direct transition regime to the turbulent flow whereas I.M. Yavorskaya & Al [12] have detected a large regime of transition until $\delta=0.54$, where secondary waves persist before the flow becomes turbulent. However, it appears that the description of the behavior of these waves seems not very detailed.

Actually, considering the great interest and the richness which this configuration has on the hydrodynamic, the researchers are also interested in mechanisms related to this type of movement to study the laminar–turbulent transition regime including various mode of instabilities in order to give some explanation to the phenomena of chaos and turbulence underthe influence of several effects for examples,

– Geometrical effects : Effect of gap δ [19] [20] [21]
– Heating effects [22] [23] [24]
– Dynamic effects : effect of the field magnetic [25], effect of axial flow [26] …

Up to now, many theorical and experimental investigations are carried out, without approaching, to our knowledge, the effect of the inclination α of the system linked to gravity effect. Our objective consists in analyzing by photometry technic the influence of the parameter α on the occurrence or the disappearance of certain structures of the flow in laminar–turbulent transition regime. On the basis of these results we try to establish phenomenological laws representing the evolution number of Taylor according to the inclination α.

II. EXPERIMENTS

All the different experimental set-ups are, however, basically the same. There is a rotating inner sphere *(1)* of $R_1=45mm$ radius and spherical outer shell *(2)* of $R_2=50mm$ radius, which is kepted at rest for all experiment; both made of Plexiglas. In this way one obtains the gap $\delta= (R_2-R_1)/R_1=0.11$*(figure 1)*. The inner sphere is driven by a variable-speed electric motor. The speed of rotation is controlled without friction by using a tachymeter (photo-optics). The temperature is measured in order to determine the viscosity of the fluid in annulus by digital thermometer.

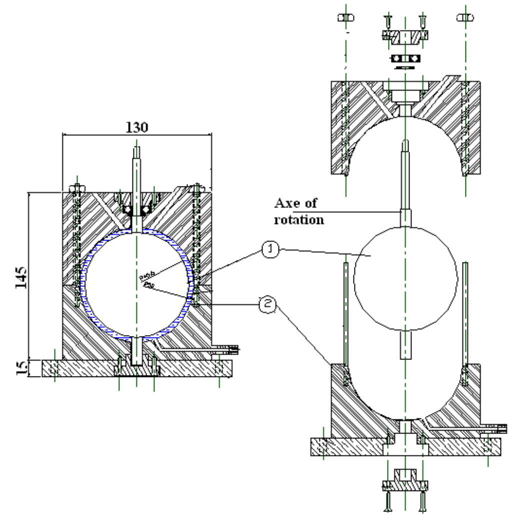


Figure 1: Flow system

We include to this device a new system *(figure 2)* in order to ensure the inclination of the apparatus in order to study the effect of the parameter α on the occurrence of structure within the flow.

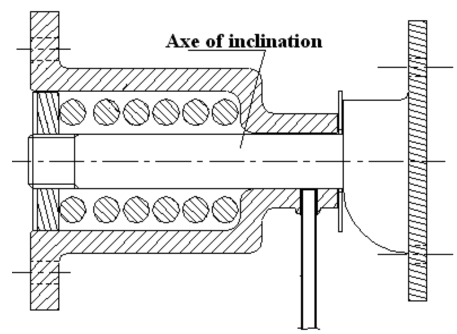


Figure 2 : Inclination system

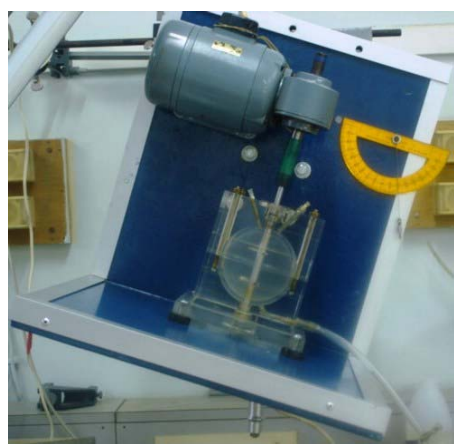


Figure 3 : Experimental apparatus

Essence (Similli) and Vaseline oil (with concentration 80% and 20% respectively) were used as the fluid. For the flow visualisation, a small amount of aluminium flakes (typical mean dimension *17 μm* and a concentration of about *2 g/ℓ* produces good signals) in suspended in the fluid; this amount is small enough not to influence the viscosity or the flow field.

III. CONDITION OF SETTING IN MODE OF SPEEDS

The geometrical characteristics are fixed and move neither in space nor in time, the value of the parameter of control *(Ta)* depends of velocity of the rotate sphere Ω_1, annular space $d=R_1-R_2$ and the kinematic viscosity ν of the fluid used

$$Ta = Re\sqrt{\delta} \quad \text{with} \quad Re = \frac{V_1\, d}{\nu} = \frac{R_1\, \Omega_1\, d}{\nu}$$

Where *Re* indicates the Reynolds number, $\delta = d/R_1$ denotes the gap, $V_1 = R_1\Omega_1$ the linear velocity of the inner sphere. As the bending flow, it is necessary to use the Froude number,

$$Fr = \frac{V_1}{\sqrt{g\,H\cos\alpha}} = \frac{R_1\,\Omega_1}{\sqrt{g\,H\cos\alpha}}$$

Featuring the influence of gravity effect via α.

For that, we adopted it even procedure for each test while proceeding systematically by increasing speed W1 so as to satisfaying the the inequality condition,

$$\frac{\Delta\Omega_1}{\Omega_1} \le 1\% \qquad (1)$$

This condition also appears necessary to ensure a good reproducibility of measurements which is besides enough near to experimental uncertainty on the angular velocity Ω_1.

The aspect ratio $\Gamma = \dfrac{H}{d}$ is defining the axial limitation such that the system is filled for a given value of height *H*.

Adopted procedure for the various tests is as follows:

From the rest, we increase the velocity of the interior sphere gradually by respecting the preceding inequality; we stops a few minutes order to allow the flow to stabilize itself then we observes the appearance of phenomena.

For chosen Ω_1, characterizing the appearance of a given phenomenon, measurements of characteristics of the structures considered are noted and we take an image of the state of the flow. The process thus described corresponds to a rigorous procedure that we calls, in thermodynamics, quasi-static mode allowing us to observe the conditions of reversibility associated with the movement.

III RESULTS AND DISCUSSION

The processing of photometric data leads to establish the law of variation of the critical Taylor number Ta_c bound with the appearance of instabilities according to the inclination α.

The observations carried outin case of the system is completely filled $H=H_{max}$ or $\Gamma=\Gamma_{max}= 20$, the inclination α is then without influence on the occurence of instabilities. On the other hand, for a system partially filled $\Gamma<\Gamma_{max}$ the inclination α play an important role. Examinating of the experimental results enabled us to analyze the effect of the inclination α, allowing to establish a phenomenological laws, obeying linear or exponential expressions,that one presents in the following general forms,

- Laws of the linear type: for the case of system completely filled

$$Tc(\alpha) = A$$

- Laws of the exponential type:
  - Exponentiel law

$$Ta_C(\alpha) = Tc_0 + B\exp(-\alpha/\alpha_0)$$

  - Gaussien law,

$$Tc(\alpha) = Tc_0 + C\exp(-D(\alpha-\alpha_0))$$

  - Boltzmaien law,

$$Ta_C(\alpha) = Tc_0 + \frac{E}{[1+\exp((\alpha-\alpha_0)/F)]}$$

All constants A, *B, C, D, E, F*, Tc_0 and α_0 are determined by adjustment on the experimental curves, présented in the following table,

The systematization of these tests lead to show the noticable influence of the inclination α of the system of flow on the conditions of appearances of instabilities that we summerize in the form of set of curves (Figure 4).

The laws of the linear type are valid for the case of a system completely filled $\Gamma=\Gamma_{max}= 20$. (Figure 5, 6)

For a system partially filled $(\Gamma=19, \Gamma=18, \Gamma=10)$, one could highlight the remarkable influence of the angle of inclination α. The interest of such an evolution is revealed by the existence of a very important particular value: for a critical angle $\alpha = \alpha_C = 30°$ correspondent to $H=H_C=90mm$ that is to say $\Gamma = \Gamma_C = 18$. One observes the destruction of any structure within the flow (Figure 7).

Generaly, the most effect of the inclination a is to retard the occurrence of the instabilities Tc_i $(i=1, 2, 3, 4)$ and we note that the disappearance of some modes for an other critical value of inclination a for example the desaperance of the Taylor vortex flow at $\alpha = \alpha^*_{c1} = 45°$ correspanding to aspect ratio $\Gamma=10$. and the desaperance of the regime of spiral mode and the wavy mode (Tc_3) For $\Gamma=\Gamma_c=10$. Figure 4.D

When the flow is in load, the aspect ratio is at maximum, one observes that the evolution of the critical parameters Tc_i $(i=1, 2, 3, 4)$ are insensitive with the angular effect α and consequantly with the gravity force (Figure 8.A). On the other hand, when Γ decrease we note that an evolution of the previous critical parameters Tc_i $(i=1, 2, 3, 4)$.(Figure 8.*B*, 8.*C*)

As the bending flow, it is necessary to represent the evolution of critical taylor number according to the Froude number *Fr* Featuring the influence of gravity effect via α.

Figure 11 show partially cures of the variation of critical taylor number according to the Froude number *Fr* Tc_i $(i=1, 2, 3, 4)$ we expand our invistigation to other values of aspect ratio G for the occurrence of the Taylor vortex flow.(figure 12) [27][28]

Ta_c / Γ	Tc_1	Tc_2	Tc_3	Tc_4
20	linear Laws $A = 43,30$ $B = 0$	linear Laws $A = 47,47$ $B = 0$	linear Laws $A = 54,74$ $B = 0$	linear Laws $A = 61,50$ $B = 0$
19	Gaussien law $Tc_0 = 44.01$ $\alpha_0 = 75.62$ $C = 3.46$ $D = 0.0031$	Boltzmaien law $Tc_0 = 47.86$ $\alpha_0 = 87.05$ $E = -392.92$ $F = 2.97$	linear Laws $A = 55,77$ $B = 0,15$	Gaussien law $Tc_0 = 65.17$ $\alpha_0 = 83.07$ $C = 34.20$ $D = 0.0026$
18	Gaussien law $Tc_0 = 43.25$ $\alpha_c = 84.57$ $C = 11.35$ $D = 0.0003$	Exponential law $Tc_0 = 49.95$ $B = 0.032$ $\alpha_0 = 10.16$	Gaussien law $Tc_0 = 54.88$ $\alpha_0 = 39.06$ $C = 7.27$ $D = 0.0043$	Gaussien law $Tc_0 = 63.96$ $\alpha_0 = 75.48$ $C = 42.82$ $D = 0.0023$
10	Gaussien law $Tc_0 = 42.72$ $\alpha_0 = 49.69$ $C = 83.72$ $D = 0.0022$	Gaussien law $Tc_0 = 59.12$ $\alpha_0 = 62.56$ $C = 198.74$ $D = 0.0016$	—	Gaussien law $Tc_0 = 52.32$ $\alpha_0 = 28.57$ $C = 156.80$ $D = 0.0060$

Table 1 : value of constants of the phenomenological law of the evolution of the critical Taylor number $Tc_I(i=1..4)$ according to the inclination α

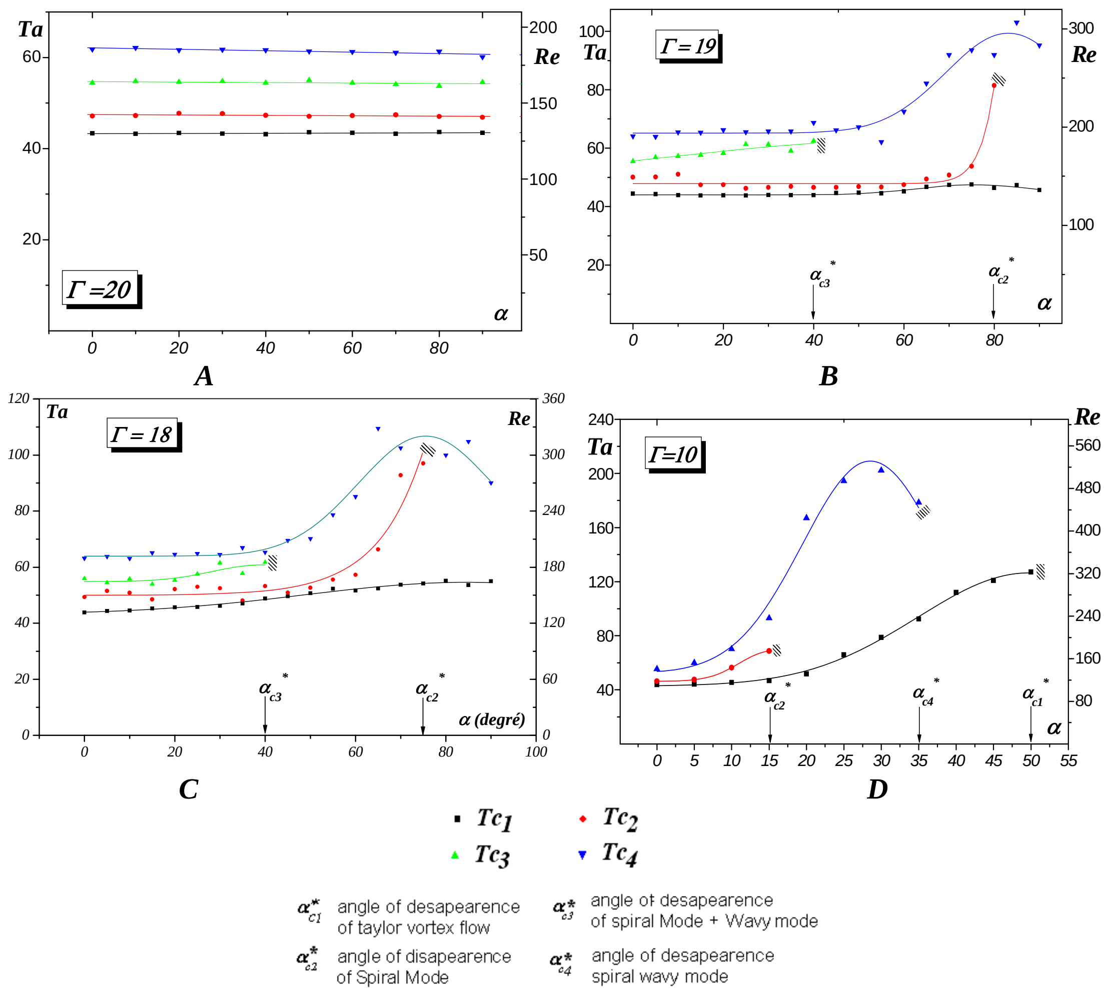
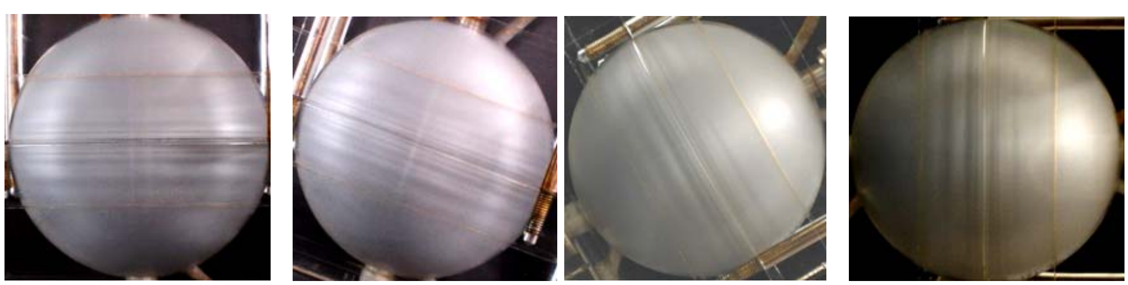
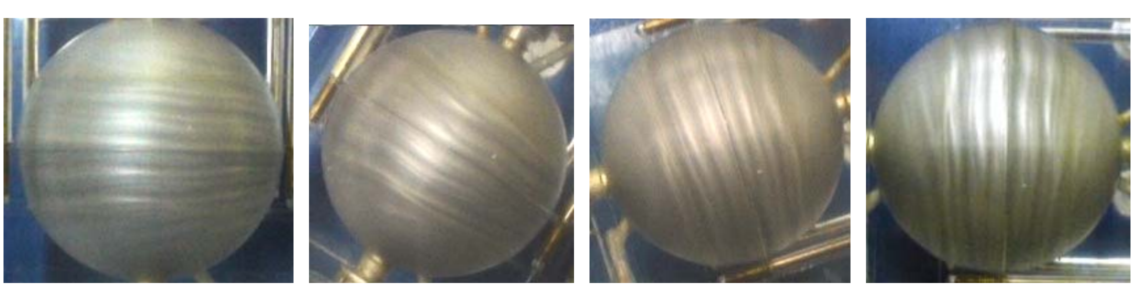


Figure 4 : Evolution of the critical Taylor number $Tc_I(i=1..4)$ according to the inclination α



$\alpha = 0°$ $\alpha = 20°$ $\alpha = 60°$ $\alpha = 90°$

Figure 5: Effect of the inclination α on the vortex of Taylor $Ta=45$, $\Gamma = 20$

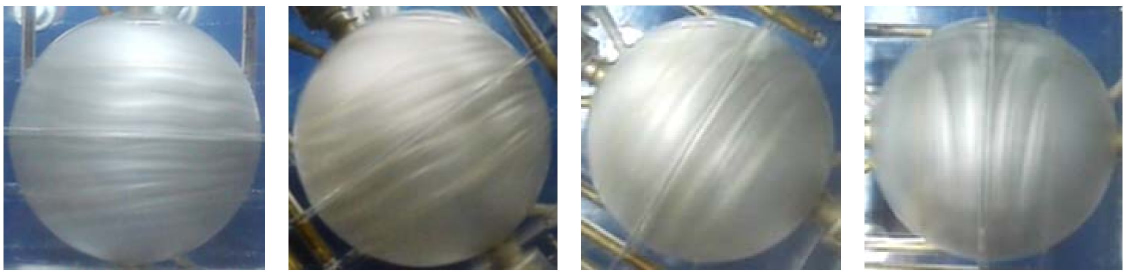


$\alpha = 0°$ $\alpha = 30°$ $\alpha = 60°$ $\alpha = 90°$

Figure 6: Effect of the inclination α on the Spiral Wavy Mode $Ta=64$, $\Gamma = 20$

α ***= 0°*** α ***= 30°*** α ***= 80 °*** α ***= 90°***

Figure 7: Effect of the inclination α on vortex of Taylor *Ta=45, Γ= 19*



α ***= 0°*** α ***= 40°*** α ***= 60°*** α ***= 90°***

Figure 8: Effect of the inclination α on the Spiral Wavy Mode *Ta=77, Γ= 19*

α ***= 0°*** α ***= 20°*** α ***= 30°*** α ***= 90°***

Figure 9: Effect of the inclination α on the vortex of Taylor *Ta=45, Γ= 18*
(Relaminarization $\alpha = \alpha_C$ *= 30°)*

α ***= 0°*** α ***= 30°*** α ***= 60°*** α ***= 90°***

Figure 10: Effect of the inclination α on the Spiral Wavy Mode *Ta=80, Γ= 18*

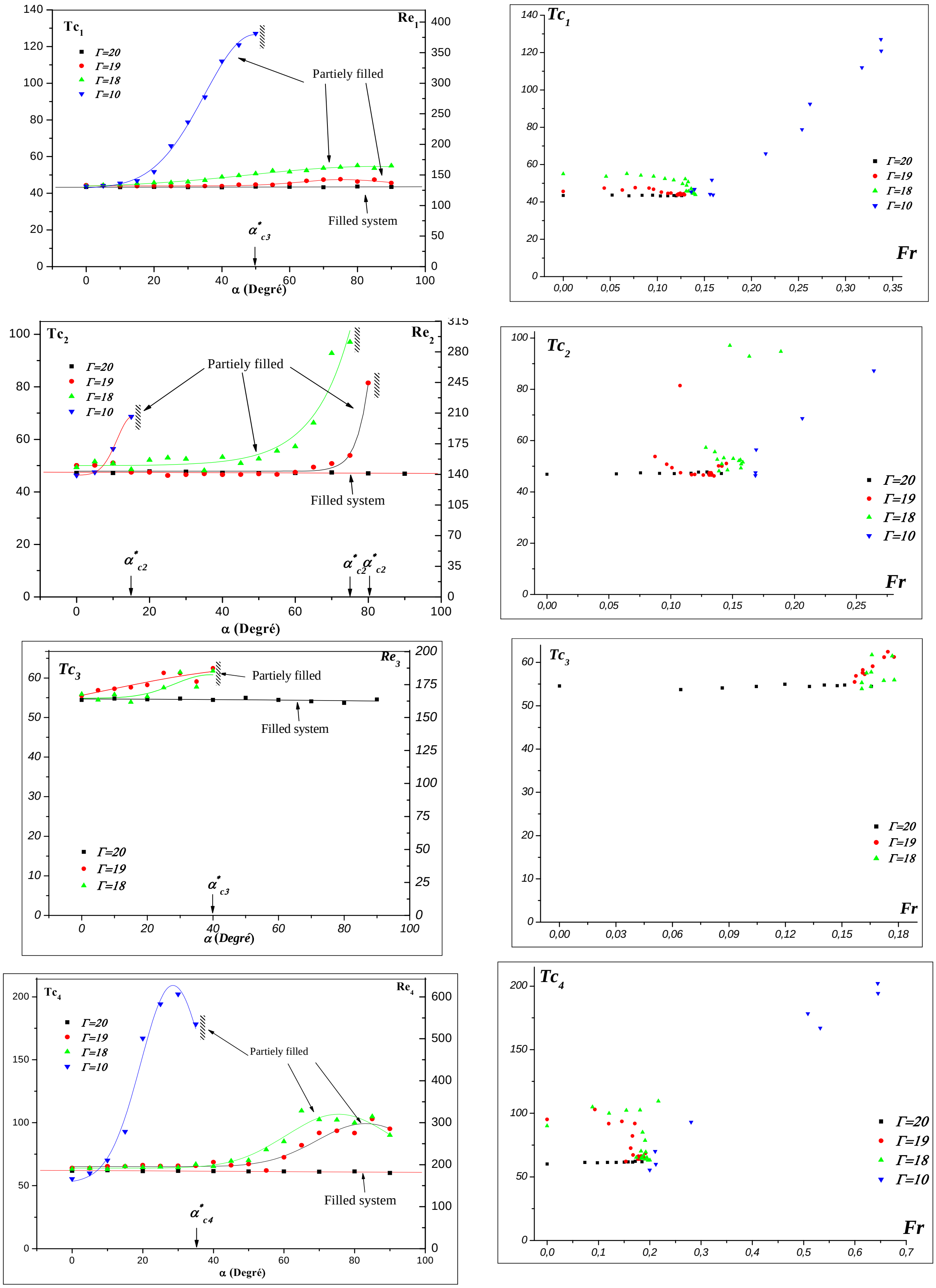


Figure 11 : Evolution of the critical Taylor number $Tc_I(i=1..4)$ according to the inclination α (at right) and according to Froud number Fr (at left)

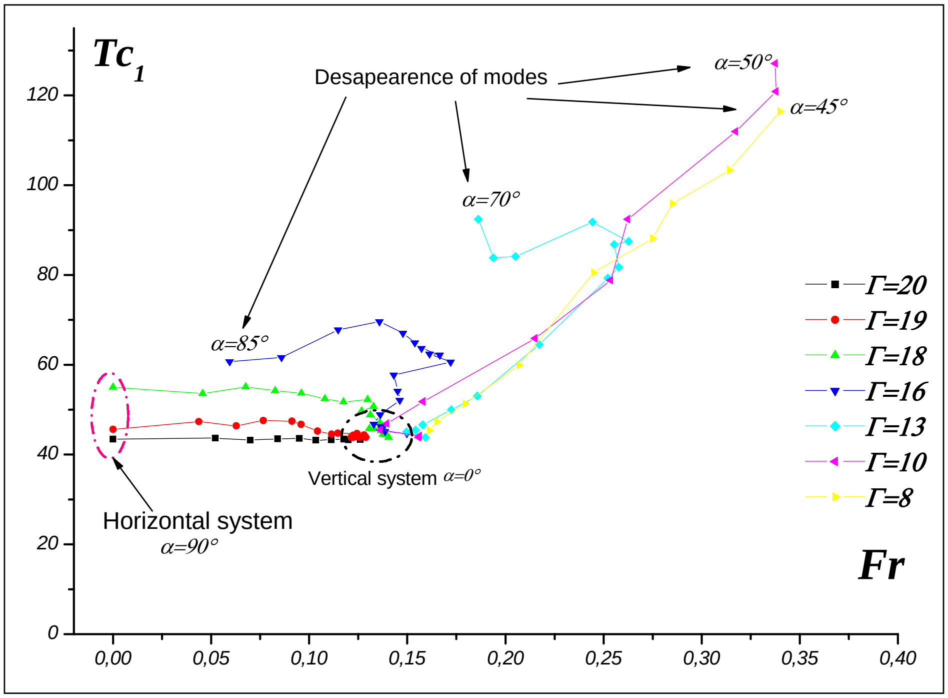


Figure 12 : Evolution of the critical Taylor number Tc_1 according to Froud number *Fr*

IV CONCLUSION

The present experimental study made it possible to highkight the influence of the inclination α of the system of flow on release of the phenomena of instabilities such as the Taylor vortices, Spiral Mode and Spiral Wavy Mode etc….

Now, we may say that the angle of inclination α does not have any effect on the occurence of the instabilities in a system completely filled ($\Gamma=\Gamma_{max}=20$). On the other hand, the inclination α play a major part in a system partially filled, $\Gamma<\Gamma_{max}$ giving place to significant modifications of the movement. In Particular, we are focusing the effect of relaminarization of the flow for a critical value of the angle of inclination α_c.

These measurements require necessaraly further investigation to examine with alternatively, each one of the effects or their combination on each structure. Also, it would be interesting to extend them systematically to the chaotic regime until to completely developed turbulence

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