Development of helical vortex theory

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Recent progress in the theory of helical vortices is addressed. New ideas for studying the classical helical vortex problem were due to by Hardin[1] who introduced Biot-Savart law in the form of Kapteyn series from twisted products of the modified cylindrical functions to describe velocity field induced by a helical filament of infinitesimal thickness. Ricca[2] later employed numerical simulations of the Kaptayn series to determine the influence of the helix torsion on the self-induced motion of thick helical vortices. The representation via Kapteyn series for a helical filament bounded on a circular cylinder has been considered by Okulov[3] who developed an efficient analytical technique for a singularity separation the series (see also Kuibin & Okulov[4]). Boersma & Wood[5] used an integral representation of the Kapteyn series and from that made an asymptotic as well as numerical analyses of the velocity. In addition to this, Wood & Boersma[6] expanded the same technique to describe the motion of a system of *N* helical vortices.

Using the singularity-separation technique, the algebraic approximations for the velocity of self-induced motion of a single helical vortex and the total velocity field of an array consisting of *N* helical vortices was obtained[7,8] for the first time. Furher, a linear stability analysis of a regular *N*-gon of point vortices from equilibrium was generalized to be valid for an array of *N* helical vortices placed in free space[7] as well as for an array embedded in an axisymmetric helical vortex field[8]. The influence of finite-core thickness on the velocity field around a helical vortex tube was investigated by Fukumoto & Okulov[9].

The purpose of the present work is partly to review and summarize the last achievements in the theory of helical vortices and partly to report on some new important mathematical aspects. Exact solutions of the motion of multiple helical vortices are compared to the approximate formulas of Boersma & Wood[5] and Okulov[7] to estimate the domaine of applicability of both approximations. The influence of different types of assigned flows and the question of right or left-handed helical symmetry of the vortices on the stability properties are also discussed.

References

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