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Adverse Pressure Gradient Turbulent Boundary Layer Flows: Part 2: Scaling of Reynolds Stresses

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Abstract

A fully developed state has been defined for APG boundary layer development corresponding to an equilibrium state. The Reynolds stresses data are examined within the fully developed region. A new set of scales are examined for mean deficit and Reynolds stress profiles.

Introduction

As shown in Part 1 of this paper, the mean deficit profiles of developing adverse pressure gradient (APG) flows can be scaled with $U_e\delta^*/\delta$ of Zagarola and Smits [10, 11] as suggested by Castillo and George [4]. It is shown in Part 1 that as a result, it is possible to define a fully developed equilibrium state for these flows. In Part 2, the corresponding scale is examined for Reynolds stresses.

Flow Details

The flows presented here are the same as the ones used in Part 1, namely Flows 2500 and 3300 of Bradshaw [1, 2], and Flows 2200 and 2300 of Clauser [5] and Flow 3500 of Newmann [8]. The flow classification number is as reported at the 1968 Stanford Conference [6]. Two new flows generated in the Victoria University Wind Tunnel are also used, Flows A and B as described in Part 1. The increasingly adverse pressure gradient flow of Samuel and Joubert [9] denoted Flow 141 at the 1980 Stanford Conference [7], is also used.

Scaling Velocity Deficit

Applying the Zagarola and Smits scaling of $U_e\delta^*/\delta$ to mean velocity deficit profiles can remove the Reynolds number dependence, and it is expected to result in a collapse of profiles [4]. The collapse is dependent on the pressure gradient, and three possible collapses have been reported, namely, one each for APG, favorable pressure gradient (FPG) and zero pressure gradient (ZPG) flows. In Part 1 of this paper, it is shown that this collapse could only be expected in fully developed flows.

In Figure 1, the mean velocity deficit profiles are scaled with $U_e\delta^*/\delta$ for all flows listed in the previous section. As discussed in Part 1, whilst all these flows are APG flows, FPG behavior is also evident. Flow development is causing this spread. Only after the flow reaches equilibrium, a good collapse is produced. In Part 1 the pressure parameter Λ_θ is used to further confirm this equilibrium state.

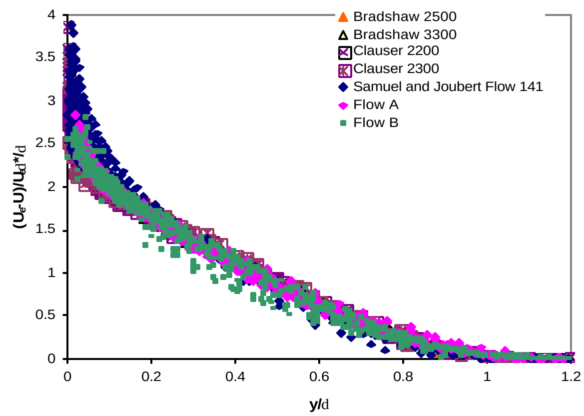

Figure 1 Mean velocity deficit scaled with $U_e\delta^*/\delta$ for Flows 2500, 3300, 2200, 2300, 3500, 141, A and B.

New Scaling

The similarity analysis technique of Castillo and George [4] has been applied to the mean continuity and x-momentum equations using δ^*/δ as the similarity parameter. The result is a set of new mean deficit and Reynolds stress scales which include the term δ^*/δ, as well the pressure parameter, Λ_θ. The solution is valid for the outer flow (typically $y/\delta > 0.1$) of a 2-D, incompressible boundary layer that is statistically steady in the mean. With this process, the outer scale for the mean velocity deficit becomes, $U_{so} = U_e(\delta^*/\delta)^{1-\Lambda_\theta}$ whilst the outer scale for the normal Reynolds stresses, u' and v' is $R_{su} = R_{sv} = U_e^2(\delta^*/\delta)^{1-\Lambda_\theta}$. For the shear stress, uv, the scale is $R_{suv} = U_e^2(\delta^*/\delta)^{1-\Lambda_\theta}.(d\delta/dx)$.

In Figure 2 the new scale of $U_e(\delta^*/\delta)^{1-\Lambda_\theta}$ is applied to the mean velocity deficit for the same flows as in Figure 1. When compared with the Zagarola and Smits [10, 11] scale of $U_e(\delta^*/\delta)$, the new velocity deficit scale produces less scatter. The developing region of the flow scales to the same profile as that of the fully developed flow.

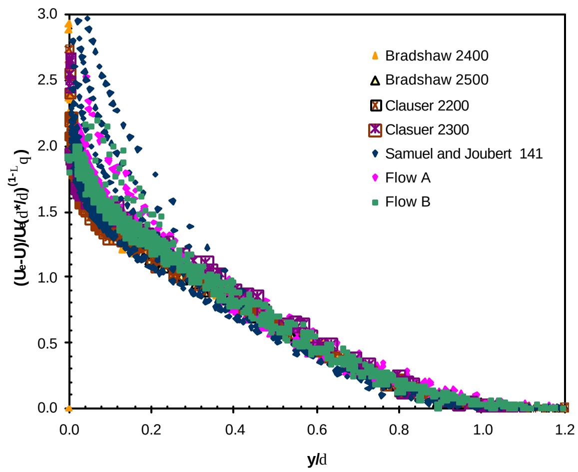

Figure 2. Mean velocity deficit profiles scaled with $U_e(\delta^*/\delta)^{1-\Lambda_s}$ for all flows.

Scaling Reynolds Stresses

In Figure 3 the normal Reynolds stresses, u'^2 and v'^2 are scaled with U_e^2 for Flows A and B, and uv is scaled with $U_e^2\, d\delta/\delta x$. These scales correspond to the Zagarola and Smits scale of $U_e\delta^*/\delta$ for the mean deficit. Although some clustering of the data is evident, a complete collapse is not obtained. For u'^2 and v'^2, the arrow on the figure indicates the flow direction. The profiles increase in magnitude downstream. The shading in this figure highlights the region of equilibrium as defined in Part 1. There is better clustering of the data within this region than outside of it. Even in this region the profiles do not collapse.

In Figure 4, the new scales are applied to the Reynolds stress data from Flows A and B. In comparison with Figure 3, the scatter is reduced. Similar to the comparison between Figures 1 and 2, better collapse of data is obtained both within and outside of the fully developed region.

Conclusions

The Zagarola and Smits scaling is applied to Reynolds stress data of Flows A and B. The resulting collapse is not as satisfactory as for the mean deficit profiles. A new set of scales is being developed. The results are encouraging.

Acknowledgments

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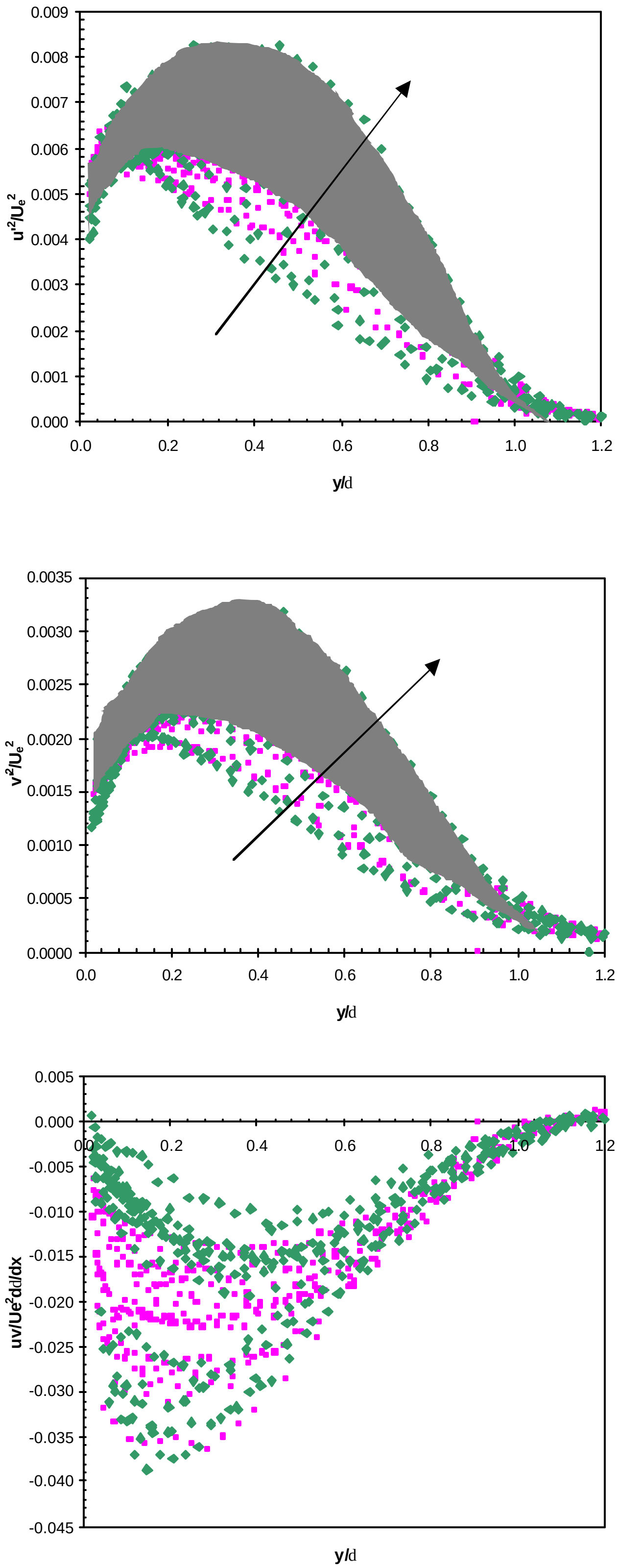

Figure 3 Reynolds stresses u', v' and uv scaled with U_e^2.

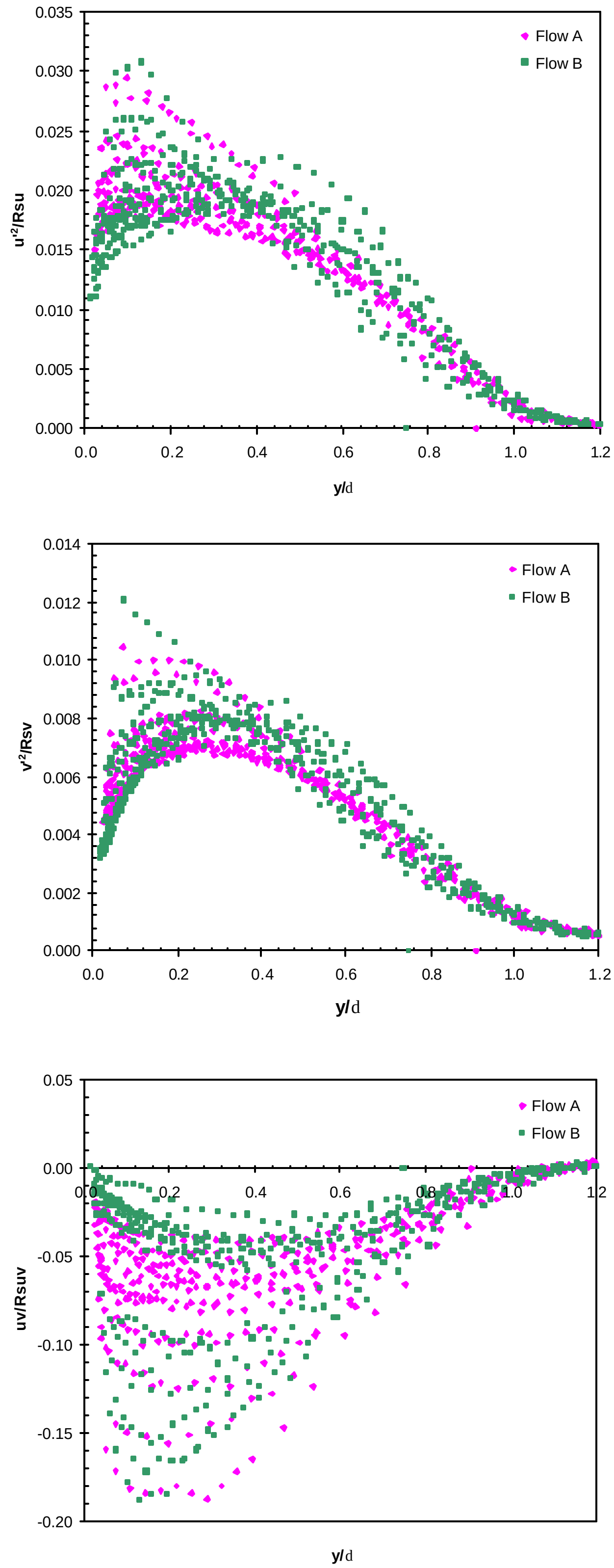

Figure 4 Reynolds stresses u', v' and uv with the corresponding new scaling.