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A Numerical Study of Heat Transfer from a Cylinder in Cross Flow

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Abstract

This paper presents numerical investigations into the characteristics of heat transfer from a cylinder in cross flow. Two dimensional Steady State as well as Unsteady RANS (Reynolds-Averaged Navier Stokes) simulations are presented and compared with experimental results from the literature. Experimental data is compared with numerical results which utilise a standard $k-\omega$ turbulence model as well as a $k-\omega$ model for which the calculation of some turbulent quantities has been modified. Significant improvements upon the prediction of heat transfer were found with the modified $k-\omega$ turbulence model.

Introduction

Heat transfer from a cylinder in cross flow can be found in a multitude of thermal-fluid applications containing heat sinks, heat exchangers or even thermal storage devices and the accurate prediction of such heat transfer using computational methods has not always proved successful. RANS simulations especially have proved difficult to calibrate for accurate heat transfer. The inherently unstable and fluctuating nature of vortex shedding within the velocity field of a cylinder in cross flow makes it additionally difficult to predict instantaneous heat transfer which will also fluctuate in such a flow. Numerical simulations allow excellent visualization and quantification of flow properties however if accurate prediction of heat transfer is not possible their usefulness will be limited. The Reynolds numbers studied in this paper are out of the range of DNS (Direct Numerical Simulation) and thus RANS simulations are used. The ability of RANS simulations to predict heat transfer accurately is dependent, amongst other things, upon the turbulence model used. This paper shows numerical results which include the use of a two-equation RANS turbulence model which was modified to improve heat transfer characteristics. To evaluate success of the simulations, their results are compared with experimental data from the literature. Scholten and Murray [3]and [4] have investigated the mechanism of heat transfer over the circumference of a cylinder in cross flow and some of their results will be used for such comparison.

Numerical Method and Model

The unmodified turbulence model employed in this study is the $k-\omega$ SST turbulence model. For numerical solution of the flow equations a commercial software package FLUENT was employed. FLUENT uses finite volume, implicit techniques to solve the governing equation which were solved sequentially. Flow across the cylinder was solved as an incompressible problem with air being the fluid. Pressure discretization utilised the Standard method whereas velocity-pressure coupling discretization was achieved with the SIMPLEC method. The QUICK discretization scheme was employed for the momentum, turbulent kinetic energy, specific dissipation rate and energy equations. The SST $k-\omega$ turbulence model utilises a calculation of turbulent viscosity which varies slightly from what is known as the standard $k-\omega$ model. Thus the closure coefficients and other relations also differ slightly. Nonetheless for the current study the SST $k-\omega$ turbulence model is considered the ordinary or unmodified turbulence model. The basic $k-\omega$ transport equations used are as follows:

Turbulent Kinetic Energy (k):

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left[(\mu+\frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial x_j}\right]-\rho\overline{u_i'u_j'}\frac{\partial u_j}{\partial x_i}-\rho\beta^* k\omega \tag{1}$$

Specific Dissipation Rate (ω):

$$\frac{\partial}{\partial t}(\rho\omega)+\frac{\partial}{\partial x_i}(\rho\omega u_i)=\frac{\partial}{\partial x_j}\left[(\mu+\frac{\mu_t}{\sigma_\omega})\frac{\partial \omega}{\partial x_j}\right]-\alpha\frac{\omega}{k}\rho\overline{u_i'u_j'}\frac{\partial u_j}{\partial x_i}-\rho\beta\omega^2 \tag{2}$$

The modified $k-\omega$ turbulence model investigated in this study is that proposed by Durbin [1] and Medic and Durbin [2]. The modification was born from the unfortunate tendency of two equation turbulence models to over-predict levels of turbulent kinetic energy close to a stagnation point. In [1] Durbin outlines his method for modifying two equation models based on a realisability constraint for the Reynolds stress tensor ie., that the eigenvalues of the stress tensor should be non negative. The purpose of such turbulence model modifications is to reduce the calculated levels of turbulence by either increasing the production of dissipation ω or decreasing the production of turbulent kinetic energy k. Having had observed spuriously high values of the turbulent time scale T_t as well as k close to stagnation points, Durbin's and Medic's modification develops a limiting criterion for this turbulent time scale. Turbulent time scale T_t appears in the formula for eddy viscosity (or turbulent viscosity) μ_t expressed as:

$$\mu_t = C_\mu \rho u^2 T_t \tag{3}$$

and in an unmodified or ordinary $k-\omega$ turbulence model T_t = $1/(C_\mu\omega)$. Durbin and Medic's turbulent time scale limit is as follows:

$$T_t = \min\left[\frac{1}{C_\mu\omega}, \frac{\alpha}{\sqrt{6}C_\mu|S|}\right] \tag{4}$$

Placing a limiting criterion on T_t will subsequently limit turbulent viscosity μ_t which will then have an effect on the production and dissipation of k and ω and importantly on the solution of the energy equation and heat transfer. It is not intended here to describe in detail the derivation of the T_t- limit equation and the reader should refer to its original publications [1] and [2]. Solutions utilising this modified $k-\omega$ turbulence model will be hence referred to as the T-limited $k-\omega$ solutions. To solve with the T-limited $k-\omega$ model the T-limiting criterion (equation 4) was coded into FLUENT as a user defined function (UDF) for the turbulent eddy viscosity μ_t.

The simulated model consisted of a 2-dimensional mesh, with the cylinder centered at the origin. The computational domain extended 25D (D = cylinder diameter) in the positive (downstream) X-direction, 15D in the negative (upstream) X-direction and 10D in both positive and negative Y-directions. 180 cells were modelled around the circumference of the cylinder allowing a circumferential resolution of 2^o. Consistent with the literature, convective hear transfer coefficient and surface Nusselt number are calculated as follows:

$$h_{conv} = \frac{q}{T_w - T_\infty} \tag{5}$$

$$Nu = \frac{h_{conv} D}{k_{fluid}} \tag{6}$$

Where k=conductivity, D=diameter and q=heat flux. Reynolds numbers, Prandtl numbers and boundary conditions were all set to preserve similarity with experimental results in [3]. For the three Reynolds numbers simulated Prandtl number was held at Pr=0.7, cylinder wall temperature was maintained at 310^oC and free stream temperature was maintained at 310^oC. To match experimental values, inlet turbulence intensity at different Reynolds numbers was set as follows: $T_u = 1.6\%$ for Re = 7190; $T_u = 0.46\%$ for Re = 21580; and $T_u = 0.34\%$ for Re = 50350.

Vortex Shedding Results

Two comparisons arise from the present results. Firstly steady state results for the Nusselt number (Nu) distribution around the cylinder will be compared to unsteady simulations which were time averaged over a single vortex shedding period. Secondly, results using the T-limited $k-\omega$ turbulence model will be compared with those using the ordinary $k-\omega$ model.

Unsteady simulations were conducted at a Reynolds number of 21580 and depicted vortex shedding well. Figure 1 shows the sinusoidal behaviour of coefficient of drag C_d for the $k-\omega$ solution at Re = 21580 and the constant frequency and amplitude indicate a steady and periodic simulation of vortex shedding. Strouhal number calculated from data of figure 1 is $S_t = 0.2146$ which compares well with the experimental value of $S_t = 0.213$ from [3].

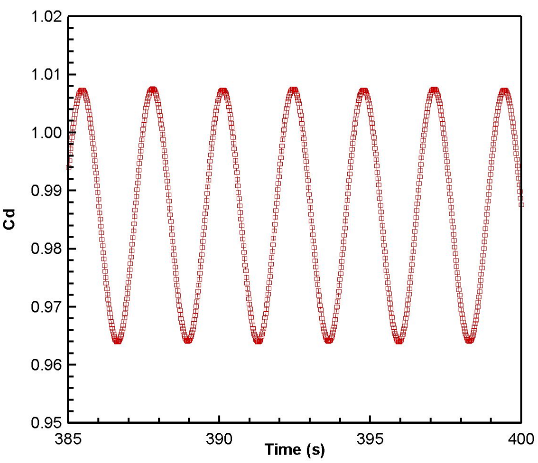

Figure 1: Coefficient of drag (C_d) vs Time for Re = 21580 unsteady $k-\omega$ solution.

Figure 2 shows the clearly sinusoidal behaviour of C_d when the T-limited $k-\omega$ model is employed and these results yield a Strouhal number of $S_{t\ T-limited} = 0.197$ for Re=21580. Although being lower than the relevant experimental Strouhal number from [3] periodic vortex shedding is nonetheless being well represented. Given the objective for the T-limited modification, it is the heat transfer results which will determine the models usefulness. The mean C_d for both ordinary and T-limited $k-\omega$ are slightly higher than the expected values for this Reynolds number but nonetheless still close to the C_d curve for a smooth cylinder.

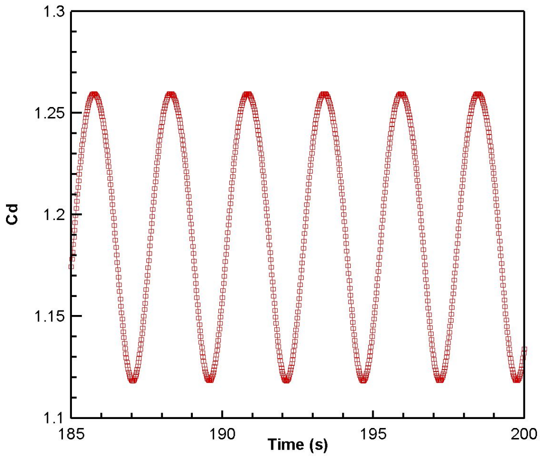

Figure 2: Coefficient of drag (C_d) vs Time for Re = 21580 unsteady T-limited $k-\omega$ solution.

Turbulent Kinetic Energy Results

Figure 3 and figure 4 show the difference in turbulent kinetic energy k when the T-limited $k-\omega$ model is used. The original function of the T-limited model was to restrict high levels of k close to the stagnation point and this can clearly be seen in figure 4 where stagnation point turbulent kinetic energy is two orders of magnitude lower compared to the ordinary $k-\omega$ simulation. Given that both T-limited and ordinary $k-\omega$ cases have equal inlet k, the T-limited $k-\omega$ case predicts a significantly reduced increase in k close to the stagnation point.

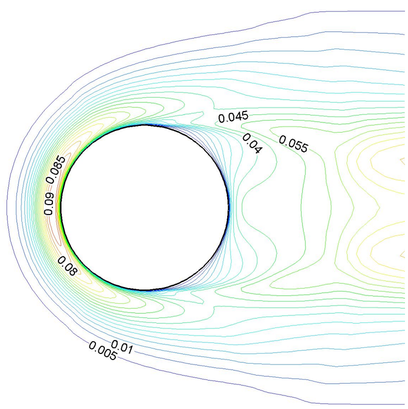

Figure 3: Contours of Turbulent Kinetic Energy (m^2/s^2) for Re = 21580 steady state $k-\omega$ solution.

In the wake however levels of k are comparable for the T-limited and ordinary $k-\omega$ models. Although contours of turbulent kinetic energy are shown only for Re = 21580, for all Reynolds numbers studied it was observed that when using the ordinary

$k-\omega$ model the local maximum of k near the stagnation point was in fact the solution maximum. This stagnation point maximum is precisely the incorrect result which the T-limited $k-\omega$ model ameliorates. When the T-limited $k-\omega$ model is employed levels of k at the stagnation point drop significantly and overall maximums of k occur within the wake region.

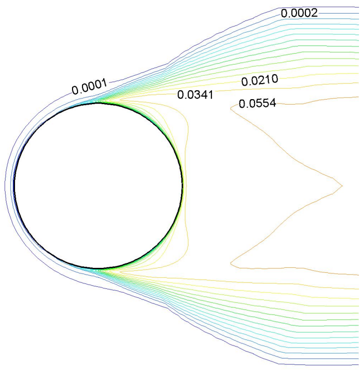

Figure 4: Contours of Turbulent Kinetic Energy (m^2/s^2) for Re = 21580 steady state T-limited $k-\omega$ solution.

By observing k we can witness the T-limited model's effects upon turbulence but it is the heat transfer in which we're ultimately interested.

Heat Transfer Results

In general all cases displayed the correct characteristic shape of surface Nu around the cylinder. A local maximum of Nu at or close to stagnation point is proceeded by a decreasing Nu as one moves towards the top (or bottom) of the cylinder. A minimum Nu occurs close to the top of the cylinder which is associated with separation, where local recirculation could restrict heat transfer away from the surface. Moving further into the wake region Nu increases as the turbulent wake allows heat to again be removed more effectively. Figures 5 to 7 show the Nu profiles for the three Reynolds numbers studied including experimental data from [3] and considering the overall comparison between experiment and simulation, a perfectly accurate matching remains elusive. The horizontal axis in figures 5 to 7 is circumferential angle in degrees, where zero degrees is the leading edge or front stagnation point of the cylinder and 180 degrees is the trailing edge or rear.

A flow's capacity for convective heat transfer will be affected by, amongst other factors, levels of turbulence close to the heat transfer surface. Qualitatively it makes sense that abnormally high levels of turbulence will lead to abnormally high heat transfer due to turbulence affecting the flow's ability to transport heat away from the wall. Thus the ordinary $k-\omega$ turbulence model which over predicts levels of k close to the stagnation point also over predicts stagnation point heat transfer. This can be seen in figures 5, 6 and 7 where ordinary $k-\omega$ cases all display over predicted levels of Nusselt number at the front stagnation point Nu_{fsp}.

Figures 5, 6 and 7 also clearly show the significant reduction in front stagnation point Nusselt number, Nu_{fsp}, when the T-limited $k-\omega$ model is employed. Reduced stagnation point heat transfer is obviously related to the reduced levels of k at the stagnation point but specifically it is through the heat transfer relation that the T-limited model is able to achieve improved heat transfer results. Eddy viscosity μ_t appears in the heat flux

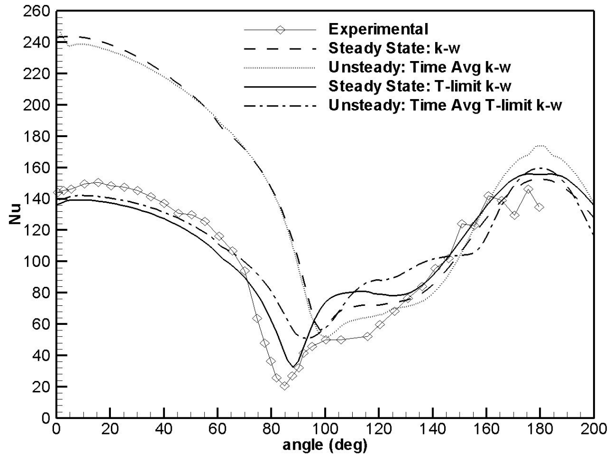

Figure 5: Surface Nusselt number for Re = 21580. Experimental data from [3].

tensor as follows:

$$\dot{q}_t = \frac{\mu_t C_p}{\mathrm{Pr}_t} \nabla T \tag{7}$$

where T=temperature. With its reformulation of μ_t the T-limited model is able to influence heat transfer directly through the heat flux tensor. In this way, combined with its effect on levels of turbulence, the new T-limited turbulent viscosity μ_t plays its part in reducing heat transfer.

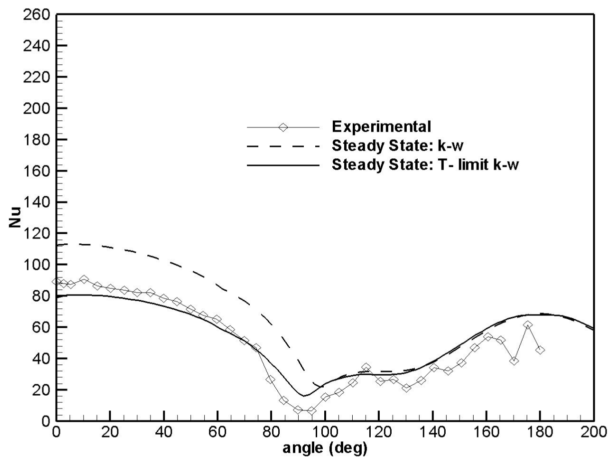

Figure 6: Surface Nusselt number for Re = 7190. Experimental data from [3].

These improvements in the simulation of heat transfer are significant due the erroneously high results obtained using ordinary $k-\omega$ models. Table 1 summarizes the results across the three Reynolds numbers considered. Unsteady results were completed for Re=21580 only. Nu_{fsp} refers to the Nusselt number at the front stagnation point (fsp) and $Nu_{avg\ circ}$ is the circumferentially averaged surface Nusselt number.

It should be noted that higher Reynolds numbers yielded a greater relative error in Nu compared to the experimental values. Furthermore the T-limited $k-\omega$ models bore less favourable results at high Reynolds number. This is also evident

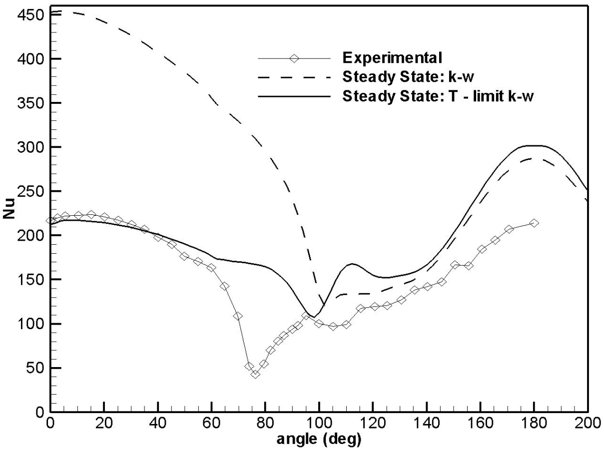

Figure 7: Surface Nusselt number for Re = 50350. Experimental data from [3].

	steady k-ω	unsteady k-ω	steady k-ω T-lim	unsteady k-ω T-lim	Exp [3]
Re=7190					
Nu (fsp)	112.6	–	79.0	–	89.1
$\%\ diff$ $from\ exp$	26.4%		-1.2%		
Nu $(avg\ circ)$	67.3	–	52.5	–	51.0
$\%\ diff$ $from\ exp$	32.0%		2.9%		
Re=21580					
Nu (fsp)	243.1	247	136.2	139.7	148
$\%\ diff$ $from\ exp$	64.2%	66.9%	-8.0%	-6.0%	
Nu $(avg\ circ)$	149.3	148	105.7	108.8	103.4
$\%\ diff$ $from\ exp$	44.4%	43.1%	2.2 %	5.2%	
Re=50350					
Nu (fsp)	453.3	–	212.5	–	216.8
$\%\ diff$ $from\ exp$	109.1%		-2.0%		
Nu $(avg\ circ)$	284.5	–	191.1	—	149.9
$\%\ diff$ $from\ exp$	89.8%		27.5%		

Table 1:

in figure 7 where the Nu plot has a poorer qualitative comparison with the experimental data. Assuming the minimum Nu is associated with separation it is evident from figure 7 that the highest Re simulation displayed the greatest error in the location of separation. Indeed the best alignment of minimum Nu with the experimental minimum occurred for the lowest Reynolds number investigated, Re=7190.

Comparing Unsteady and Steady State Simulations

The purpose of conducting unsteady simulations was to test the solution's ability to depict vortex shedding accurately and to compare time averaged Nu results with their steady state counterparts. Figure 5 shows all unsteady and steady state results together. The unsteady plots of Nu v Theta were time averaged over several vortex shedding periods. Although it was shown earlier how the unsteady results depicted vortex shedding reasonably well this did not transfer into advantageous heat transfer results. For Re=21580 the Nu v Theta plot for the time averaged unsteady ordinary $k-\omega$ case did not differ drastically from the steady state ordinary $k-\omega$ plot. Figure 5 also shows how the unsteady time averaged T-limited $k-\omega$ plot of Nu v Theta is qualitatively worse than the steady state T-limited $k-\omega$ case. However Table 1 shows that the circumferential average and Nu_{fsp} for the unsteady T-limited case is nonetheless well matched with the experimental data.

Concluding Remarks

In general the T-limited $k-\omega$ turbulence model fulfilled its objective to reduce heat transfer close to the stagnation point. Heat transfer in the wake region however was not significantly altered by the T-limited $k-\omega$ model. The reduction in stagnation point heat transfer nonetheless had a significant reducing effect on circumferentially averaged Nusselt numbers due to the fact that stagnation point over-prediction was largely responsible for over predicted averages.

Unsteady simulations of the cylinder depicted vortex shedding behaviour satisfactorily. However when compared with steady-state simulations, and for Reynolds numbers studied, unsteady simulations did not yield significant benefits in predicting heat transfer.

The highest Reynolds numbers studied produced heat transfer results least similar to experiments. This was the case for both the ordinary $k-\omega$ turbulence model at Re=50350 and the T-limited $k-\omega$ model. Thus further high Reynolds number studies would be recommended in order to test the capabilities and limitations of these type of numerical models.

Acknowledgements

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