15th Australasian Fluid Mechanics Conference
The University of Sydney, Sydney, Australia
13-17 December 2004

Gap Effect on Taylor Vortex Size Between Rotating Conical Cylinders

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Abstract

The present paper is concerned with an experimental study of the occurrence of Taylor vortices between conical cylinders, the inner one rotating and the outer one at rest. Gap effect on the non-uniqueness of the Taylor vortex flow mode is investigated. Six different flow modes are observed according to the gap width, the acceleration of the inner rotating conical cylinder and Reynolds number. In one of the observed flow modes very large Taylor vortices are obtained with a wavelength up to 3.5 times the gap width. This kind of Taylor vortices has not been observed before in our knowledge.

Introduction

The non-uniqueness of flow states with Taylor vortices has been largely investigated in the circular Couette flow. Coles [1] conducted a deep investigation on the non-uniqueness of the Taylor vortex flow and showed that a large number of flow modes can be reached. He noticed that these flow states are sensitive to the flow history. The geometric effects have been particularly studied for Couette-Taylor flow systems with finite length [2-4]. The non-uniqueness of the Taylor vortex flow in systems other than circular cylinders has also been discussed. Wimmer [7] obtained different Taylor vortex flow modes between conical cylinders when the inner cylinder was rotating and the outer at rest. In recent years an increasing interest has been accorded to the study of flow mode selection related to Taylor vortices due to the acceleration effect of the inner rotating body in both circular cylinders and conical cylinders. Lim et al. [5] discovered the existence of a Taylor-vortex flow in a region where wavy Taylor vortices were expected according to the Reynolds number. Noui-Mehidi et al. [6] investigated the acceleration effect on the Taylor vortex flow mode observed between conical cylinders when the inner conical cylinder was accelerated linearly with different acceleration rates.

In the classical Couette-Taylor system, different studies have reported that the maximal size of a Taylor vortex was less than 1.1 times the gap width [2-4]. The main objective of this work was to investigate Taylor vortex size between conical cylinders in a wide gap configuration since in the early studies, vortices larger than twice the gap width were observed.

Experimental Setup

The experimental apparatus consists of a stainless steel machined inner conical cylinder and an outer conical cylinder made of Plexiglas. The square outer wall of the outer cylinder permits a good visualization and eliminates the effect of wall curvature leading to mis-observation. The outer stationary conical cylinder has an upper radius R_{oh} = 50 mm. Two inner conical cylinders are used with the upper radius R_{ih}= 34 and 42 mm respectively. The outer conical cylinder and the inner ones have the same apex angle ϕ = 16.38 degrees. Thus two configurations are obtained with axially constant gap of d_1 = 8 mm for configuration I (denoted CI) and d_2 = 16 mm for configuration II (denoted CII) in a horizontal direction respectively. At the top of the flow system the radius ratio is $\eta_1 = R_{ih} / R_{oh}$ = 0.84 for CI and $\eta_2 = R_{ih} / R_{oh}$ = 0.68 for CII. Both top and bottom end plates are fixed in the experiments. The aspect ratio $\Gamma = L / d$ is fixed to 15.62 in CI and 7.81 in CII (L is the vertical height of the fluid column). The Reynolds number estimated with an accuracy better than 2.5%, is defined at the upper base for the largest radius, as:

$$Re = \frac{R_{ih}\,\Omega\, d}{\nu} \tag{1}$$

ν is the kinematic viscosity and Ω the rotational speed of the inner conical cylinder. In CI the working fluid is a 66% solution of glycerol in filtered water and in CII the working fluid is a 33 % solution of glycerol in filtered water. For flow visualization, 2% of Kalliroscope AQ 1000 is added to each of the working fluids. The temperature is measured by a thermo-couple of Copper-Constantan with accuracy better than 0.1 ºC. The kinematic viscosity ν for each fluid solution is respectively 14.82 cS and 4.62 cS for CI and CII at 25 ºC.

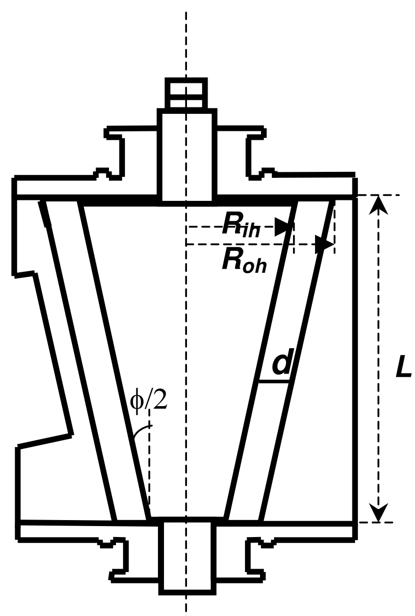

Fig.1. Experimental system.

In the present study, a computer controls the DC motor rotating the inner body. A program written in Visual Basic permits to fix the acceleration path as an output voltage sent to the motor. Thus the acceleration rate β (rad/s^2) is related to the slope of the linear increase in the angular velocity from zero to the chosen speed. When the desired final value is reached, the inner conical cylinder is rotated with a constant speed. The speed counter gives readings with accuracy better than 1%. Sudden accelerations are not investigated in the present study. The observations are repeated many times to ensure the reproduction of the observed flow states. The regimes investigated here correspond to Reynolds numbers between 0 and 1000.

Experimental Results

Previous studies [6][7] have shown that the laminar-turbulent transition of flow between conical cylinders is more sensitive to the geometrical and dynamical parameters than the flow between concentric cylinders. The transition is mainly marked by a bifurcation branching, which occurs at the early observed instabilities.

Transition to Taylor Vortices

The transition scenario in the present flow system has been previously reported by Noui-Mehidi et al. [6]. The basic flow is three-dimensional, the meridional flow being upward along the inner rotating conical cylinder and downward along the outer fixed one. At the critical Reynolds number of 132, a first vortex rotating inwards to the end plate appears at the top of the flow system where the radius is the largest. When Re is increased further a pair of counter-rotating vortices takes place below the first vortex. For further increase of Re more vortices are observed below the previous ones. When ¾ of the fluid column is filled with these vortices, according to the acceleration rate two transitions can occur when Re is increased: 1. For acceleration rates less than 0.07 rad/s^2 a helical structure takes place in the flow system. The motion is downwards and the helical vortices are counter-rotating two by two. This structure winds around the inner rotating body like a coil giving the effect of the "barber-pole" for a stationary observer. 2. The second transition occurs for acceleration rates higher than 0.07 rad/s^2. The previously observed first vortices move upwards until they fill completely the fluid column. Periodically a new pair of vortices is born at the bottom of the flow system while at the top the third vortex below the end plate disappears due to the upward motion and the two neighbouring cells merge to form a big vortex. For higher Re and acceleration rates, the upward motion stops and regimes of Taylor vortices are observed. The number of Taylor vortices obtained depends on the acceleration rate and the Reynolds number.

Taylor Vortices Wavelength

Taylor vortices have been investigated widely in the system of concentric cylinders. The wavelength limits have been studied numerically and experimentally. Dominguez-Lerma *et al.* [3] showed that the wavelength in the Taylor vortex flow mode can be different from the critical wavelength calculated theoretically when Re is changed slightly near its critical value R_c. The larger wavelength obtained in their experiments did not exceed 2.4 times the gap width.

Taylor vortex size in the flow between conical cylinders is not constant axially [6][7]. The definition of a wavelength is quite delicate since a pair of counter-rotating vortices is formed of one large cell and a one smaller cell, the larger cell being below as can be seen in Figs.2 and 3. The sizes of the large and small cells even vary from one axial position to the other. The vortex at the bottom of the flow system is the largest compared to the other large vortices above. The larger vortices are rotating in the same sense as the meridional flow i.e. upwards along the inner rotating body and downwards along the fixed one. The smaller vortices are counter rotating to the meridional flow. On the other hand the large vortex size decreases from the bottom to the top of the system, the smaller vortices have the same behaviour.

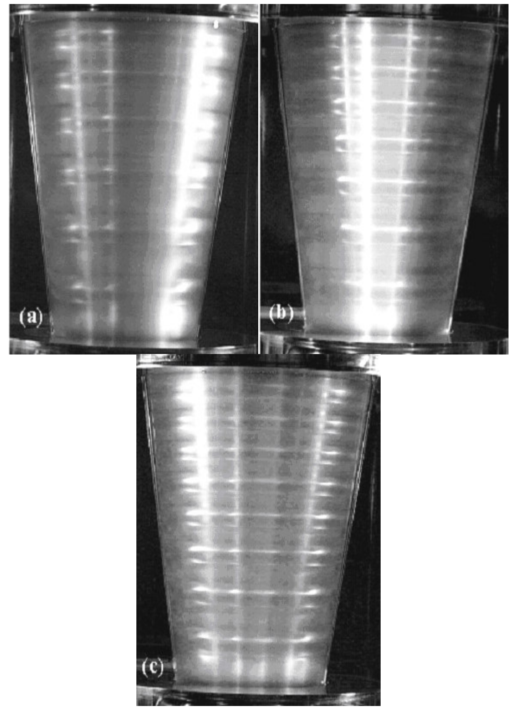

Fig.2. Vortex flow modes observed in CI,
(a) Re=347, (b) Re=516 and (c) Re=672.

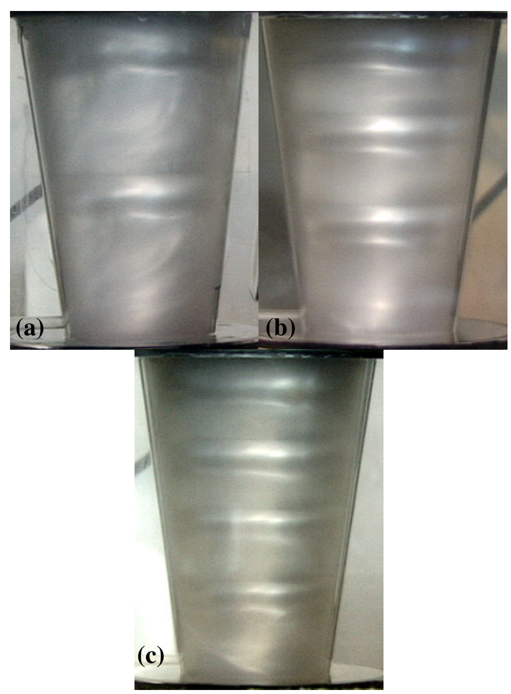

Fig.3. Vortex flow modes obtained in CII,
(a) Re=92, (b) Re=88 and (c) Re=85.

The wavelength can be defined as the size of a vortex pair, one large and the adjacent counter rotating small one. The mid-point of an imaginary line joining the vortex centres is taken as the axial location of the wavelength measurements. The flow modes observed in CI are shown in Fig.2. As can be seen there are three different Taylor vortex flow modes with 6-pair, 7-pair and 8-pair of Taylor vortices. The wavelength axial evolution is presented in Fig.4 (N is the number of vortices). The non-dimensional wavelength λ^* is defined as the ratio of the local wavelength λ to the gap width d. λ^* is plotted against the axial position $z^*=z/L$, L being the vertical fluid column height and z=0 at the bottom of the flow system.

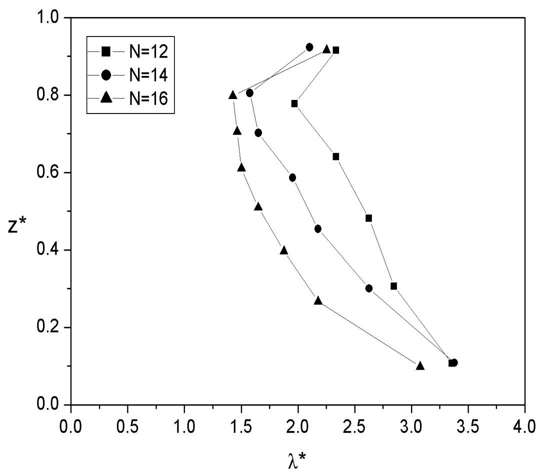

Fig.4. Axial evolution of the non-dimensional wavelength λ^* in CI.

It is worth noting that the size of the first pair of vortices at the bottom of the system in the three modes forms a wavelength varying between 3 and 3.5 times the size of the gap width. The wavelength decreases from the bottom to the top in the three observed modes 6-pair, 7-pair and 8-pair. In the 6-pair flow mode the wavelength decreases from 3.5 to 2.2, which corresponds to values still above the critical wavelength known in the Taylor-Couette system ($\lambda^*=2$). The situation in the 7-pair and 8-pair flow modes is different. In the 7-pair flow mode the wavelength decreases from a value of 3.5 to 1.7 except at the top where the vortex near the end plate has a larger size. In the 8-pair flow mode, λ^* decreases from 3 to values near 1.5 at the top of the flow system except the top vortex at the end plate. The wavelength decrease in the 8-pair flow mode is characteristic to this system since it decreases from a value above the critical λ^* to values lower than the critical one. The top vortex near the end plate in the three modes is related to the Ekman layer thus presents sizes larger than the vortices below.

The experiments performed in the system configuration CII resulted in uncommon vortex flow modes. With a gap size of 16 mm three flow modes could be generated with 2-pair, 4-pair and 6-pair of Taylor vortices only as can be seen in Fig.3.

The same definition of the wavelength is adopted in the configuration CII to represent the axial evolution of the vortices size. As shown in Fig.5 the wavelength decreases from the bottom to the top in both 3-pair and 4-pair flow modes.

In the 3-pair flow mode the wavelength decreases from 3 to 2, which indicates that one pair of vortices has a size larger than the critical value while the two other pairs above have a wavelength corresponding to the critical known value.

The 3-pair flow mode is shown in Fig.3b for Re=88. In the 4-pair vortex mode, obtained at a Reynolds number of 85 (Fig.3c),

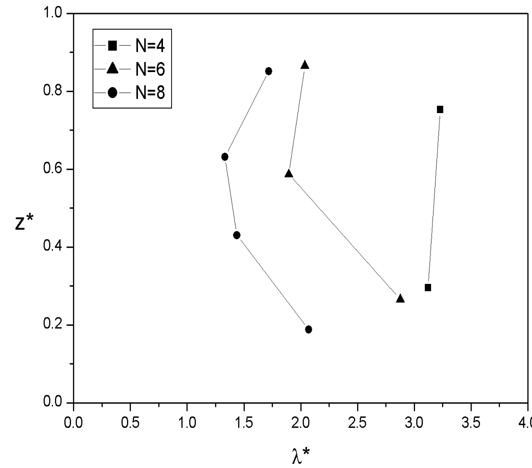

Fig.5 Axial evolution of the non-dimensional wavelength λ^* in CII.

the wavelength decreases from 2 to 1.5 giving values of the wavelength smaller than the critical λ^*. It can be remarked that in both 3-pair and 4-pair modes the wavelength formed by the top vortices is slightly larger than the one below as observed previously in CI.

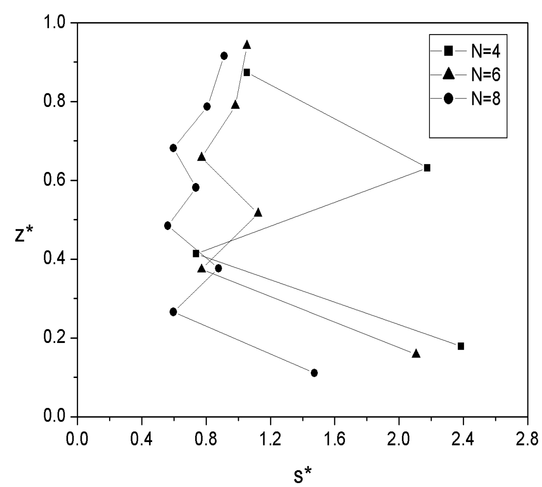

Fig.6 Non-dimensional vortex size axial variation in CII.

The situation is completely different in the case of 2-pair vortex mode. As seen in Fig.5 each of the two pair of vortices forms a very large wavelength λ^* with a value around 3.2. These two pairs of vortices can be seen in Fig. 3a for Re=92. It can be remarked that this type of vortices (2-pair) is obtained for a Reynolds number value higher than the one related to the 3-pair and the 4-pair modes. In order to compare individual vortices obtained in the configuration CII, the non-dimensional vortex size s*=s/d is presented in Fig.6 for each of the three modes 2-pair, 3-pair and 4-pair. In the 2-pair mode, the first vortex at the bottom of the flow system has almost 2.4 times the size of the gap width d, while the next small vortex above has only 0.8 times the size of the gap d. The difference in size between the larger vortices and the smaller ones is less important in the 4-pair flow mode except at the bottom of the system where the vortex is the largest. As deduced from Fig.6 the gap effect on the vortices size can be better expressed in terms of single vortex size than in terms of wavelength presented in Fig.5.

The present results have shown that Taylor vortices with a very large size can be generated between the conical cylinders when the gap is wide. These flow properties are specific to the studied system and have not been observed in other flow systems.

Conclusions

In the present work, flow visualization has shown that Taylor vortices generated between conical cylinders, the inner rotating and the outer at rest, can reach sizes up to 2.4 times the gap width. This kind of vortices has not been observed before between rotating circular cylinders or other rotating systems. The present results constitute a good challenge for more experimental and numerical investigation of the different effects leading to the occurrence of such vortices.

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